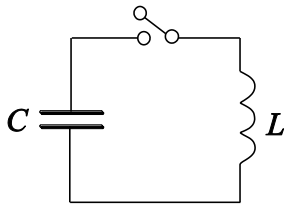
5. Električni nihajni krogi+

5.1 IDEALNI ELEKTRIČNI NIHAJNI KROG (*R* = 0)

$\boxed{U_C(t=0)=U_0}$



$U_L + U_C = 0$

$-L\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{e}{C} = 0 \quad / \frac{\mathrm{d}}{\mathrm{d}t}$

$-L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - \frac{I}{C} = 0$, kjer smo upoštevali $I = \frac{\mathrm{d}e}{\mathrm{d}t}$

$$\boxed{\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = -\frac{1}{LC} I} \qquad (5.1.1a)$$

Nastavek za rešitev:

$$\boxed{I = I_0 \sin(\omega_0 t)} \qquad (5.1.2)$$

$$\frac{dI}{dt} = I_0 \omega_0 \cos(\omega_0 t)$$

$$\frac{d^2 I}{dt^2} = -I_0 \omega_0^2 \sin(\omega_0 t) = -\omega_0^2 I \qquad (5.1.1b)$$

Iz primerjave med enačbama (5.1.1a) in (5.1.1b) sledi:

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$ lastna krožna frekvenca (5.1.3)

$$U_L = -L\frac{dI}{dt} = -L I_0 \omega_0 \cos(\omega_0 t) \qquad (5.1.4)$$

$$U_C = -U_L = L I_0 \omega_0 \cos(\omega_0 t) \qquad (5.1.5)$$

torej:

$$\boxed{U_0 = L I_0 \omega_0} \qquad (5.1.6)$$

- **Skupna energija idealnega električnega nihajnega kroga**

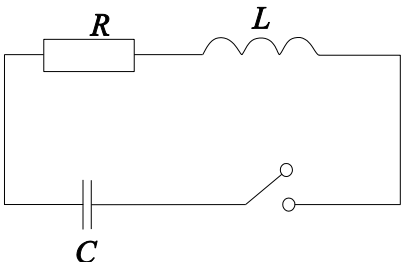
$$W=\frac{1}{2}LI^2+\frac{1}{2}CU_C^2=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}CU_0^2\cos^2(\omega_0 t)=$$
$$=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}CL^2I_0^2\omega_0^2\cos^2(\omega_0 t)=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}LI_0^2\cos^2(\omega_0 t)=$$
$$=\frac{1}{2}LI_0^2\left\{\sin^2(\omega_0 t)+\cos^2(\omega_0 t)\right\}=\frac{1}{2}LI_0^2=\frac{1}{2}CU_0^2,$$

kjer smo upoštevali:

$$I_0=\frac{U_0}{L\omega_0} \quad \text{in} \quad \omega_0^2=\frac{1}{LC} \quad .$$

Zaključek: skupna energija idealnega nihajnega kroga je konstantna.

5.2 DUŠENI ELEKTRIČNI NIHAJNI KROG



$$U_L + U_R + U_C = 0\,, \tag{5.2.1}$$

$$-\ L\ \frac{\mathrm{d}I}{\mathrm{d}t}\ -\ RI\ -\ \frac{e}{C}\ =\ 0 \quad \Big/\frac{\mathrm{d}}{\mathrm{d}t}\ ,$$

$$\boxed{-L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{I}{C} = 0}\ , \tag{5.2.2}$$

kjer smo upoštevali $I = \dfrac{\mathrm{d}e}{\mathrm{d}t}$.

Nastavek za rešitev:

$$\boxed{I\ =\ I_0 e^{-\beta t}\cos\left(\omega_0' t + \delta\right)}\ , \quad \text{kjer je } \omega_0' \text{ krožna frekvenca} \tag{5.2.3a}$$

ali v kompleksnem:

$$I = I_0\, e^{-\beta t}\, e^{i\omega_0' t} = I_0\, e^{\left(i\omega_0' - \beta\right)t} \tag{5.2.3b}$$

torej:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = I_0\left(i\,\omega_0' - \beta\right) e^{\left(i\omega_0' - \beta\right)t}\,, \tag{5.2.4}$$

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = I_0\left(i\omega_0' - \beta\right)^2\ e^{\left(i\omega_0' - \beta\right)t}\ = \left(-\omega_0'^2 - 2\beta\,\omega_0'\, i + \beta^2\right) I_0\, e^{\left(i\omega_0' - \beta\right)t}\ . \tag{5.2.5}$$

Izraze (5.2.3b), (5.2.4) in (5.2.5) vstavimo v enačbo (5.2.2), torej:

$$\begin{aligned} &I_0\,\beta^2 e^{-\beta t} e^{\ i\,\omega_0' t} - I_0\, e^{-\beta t}\omega_0'^2 e^{\ i\,\omega_0' t} - 2I_0\,\beta i\ \omega_0' e^{-\beta t} e^{\ i\,\omega_0' t}\ + \\ &+\ \frac{R}{L}\left(-I_0\beta e^{-\beta t} e^{i\,\omega_0' t} + I_0 e^{-\beta t} i\omega_0' e^{i\,\omega_0' t}\right) + \frac{1}{LC} I_0 e^{-\beta t} e^{i\,\omega_0' t}\ =\ 0 \end{aligned} \tag{5.2.6}$$

Iz enačbe (5.2.6) sledi:

$$I_0 e^{\left(i\omega_0' - \beta\right)t}\left[\beta^2 - \omega_0'^2 - \beta\frac{R}{L} + i\omega_0'\left\{-2\beta + \frac{R}{L}\right\} + \frac{1}{LC}\right] = 0\ ,$$

zato mora v splošnem veljati:

$$\left[\beta^2 - \omega_0'^2 - \beta\frac{R}{L} + \omega_0^2 + i\omega_0'\left\{-2\beta + \frac{R}{L}\right\}\right] = 0\ , \tag{5.2.7}$$

kjer smo upoštevali $\omega_0^2 = \frac{1}{LC}$.

Imaginarni in realni del enačbe (5.2.7) morata biti nič vsak posebej, torej:

$$-2\beta + \frac{R}{L} = 0 \tag{5.2.8}$$

$$\beta^2 - \omega_0'^2 - \beta\frac{R}{L} + \omega_0^2 = 0 \tag{5.2.9}$$

Iz enačbe (5.2.8) sledi:

$$\boxed{\beta = \frac{1}{2}\frac{R}{L}}\ . \tag{5.2.10}$$

Ob upoštevanju enačbe (5.2.10) zapišemo enačbo (5.2.9) v obliki:

$$\beta^2 - \omega_0'^2 - 2\beta^2 + \omega_0^2 = 0\ ,$$

od koder sledi:

$$\boxed{\omega_0' = \sqrt{\omega_0^2 - \beta^2}}\ . \tag{5.2.11}$$

Torej:

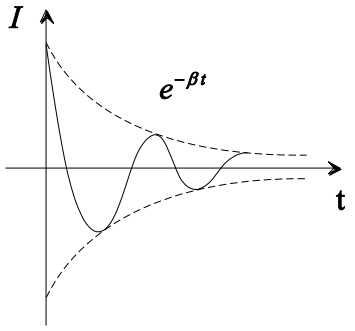
1. če $\beta < \omega_0$: $I = I_0 e^{-\beta t}\cos\left(\omega_0' t + \delta\right)$

2. če $\beta > \omega_0$: $\omega_0' = i\sqrt{\left(\beta^2 - \omega_0^2\right)} = i\alpha, \alpha = \sqrt{\left(\beta^2 - \omega_0^2\right)}$

$$I = I_0 e^{-\beta t} e^{i\omega_0' t} = I_0 e^{-\beta t} e^{-\alpha t} = I_0 e^{-(\alpha+\beta)t}$$

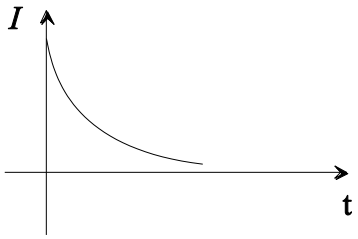
Primer 1 ($\beta < \omega_0$):

ZMERNO DUŠENJE



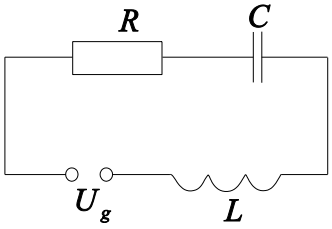
Primer 2 ($\beta > \omega_0$):

MOČNO DUŠENJE



5.3 VSILJENO NIHANJE ELEKTRIČNEGA NIHAJNEGA KROGA

5.3.1 ZAPOREDNA VEZAVA



Napetost generatorja: $U_g = U_0 \cos(\omega t)$

$$U_g + U_C + U_L + U_R = 0 \tag{5.3.1}$$

$$U_g - \frac{e}{C} - L\frac{\mathrm{d}I}{\mathrm{d}t} - IR = 0 \quad / \frac{\mathrm{d}}{\mathrm{d}t}$$

$$\boxed{\frac{\mathrm{d}U_g}{\mathrm{d}t} - \frac{I}{C} - L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} = 0}\ , \tag{5.3.2}$$

kjer smo upoštevali $I = \frac{\mathrm{d}e}{\mathrm{d}t}$. V nadaljevanju iščemo rešitev enačbe (5.3.2). Predpostavimo, da člen zaradi lastnega nihanja, ki je sorazmeren $e^{-\beta t}\cos(\omega_0' t + \delta)$ po zadosti dolgem času izgine. Zato takrat nihajni krog niha le z vsiljeno frekvenco. Torej iščemo rešitev enačbe (5.3.2) z nastavkom (v kompleksnem):

$$I = I_0 e^{i\omega t}\ , \tag{5.3.3}$$

kjer je

$$U_g = U_0 e^{i\omega t}\ . \tag{5.3.4}$$

Vstavimo izraza (5.3.3) in (5.3.4) v enačbo (5.3.2). Tako dobimo:

$$e^{i\omega t} i\omega U_0 - \frac{I_0}{C}\ e^{i\omega t} + L\omega^2 I_0 e^{i\omega t} - R\, i\omega I_0 e^{i\omega t} = 0 \tag{5.3.5}$$

Enačbo (5.3.5) preuredimo v obliko**:**

$$\left(i\omega U_0 - \frac{I_0}{C} + L\omega^2 I_0 - Ri\omega I_0\right) e^{i\omega t} = 0. \tag{5.3.6}$$

Vidimo, da v splošnem velja:

$$i\omega U_0 - \frac{I_0}{C} + L\omega^2 I_0 - Ri\omega I_0 = 0 \ , \tag{5.3.7}$$

torej:

$$U_0 = \left(-\frac{\omega L}{i} + R + \frac{1}{i\omega C} \right) I_0 = \left(R + i\omega L - i\frac{1}{\omega C} \right) I_0 \ . \tag{5.3.8}$$

Vpeljemo impedance:

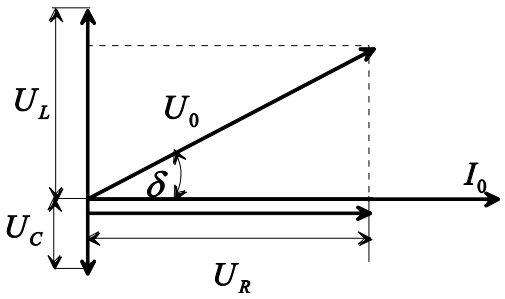
$$Z_R = R \ , \tag{5.3.9}$$

$$Z_L = i\omega L \ , \tag{5.3.10}$$

$$Z_C = -i\frac{1}{\omega C} \ , \tag{5.3.11}$$

Ker teče skozi upornik, tuljavo in kondenzator enak tok predpostavimo, da je I_0 realno število, U_0 pa kompleksno število.

Torej:



Iz slike ter enačbe (5.3.8) sledi:

$$tg\,\delta = \frac{\left(\omega L - \frac{1}{\omega C} \right)}{R} \tag{5.3.12}$$

in

$$\left| U_0 \right|^2 = U_0 U_0^* = I_0^2 \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right] . \tag{5.3.13}$$

Iz enačbe (5.3.13) sledi:

$$I_0 = \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \qquad (5.3.14)$$

Vidimo, da je I_0 maksimalen, če je:

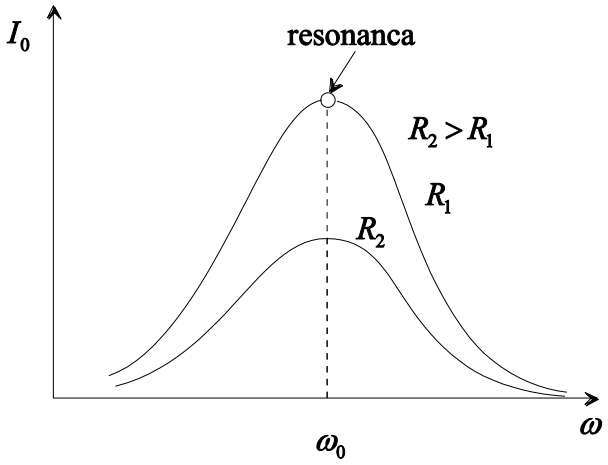
$$\omega L - \frac{1}{\omega C} = 0\,, \qquad (5.3.15)$$

torej

$$\omega L = \frac{1}{\omega C}\,,$$

oziroma

$$\boxed{\omega^2 = \frac{1}{LC} = \omega_0^2\,,} \qquad (5.3.16)$$



Če je $\omega = \omega_0$ velja:

$$tg\,\delta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} = 0\,, \qquad (5.3.17)$$

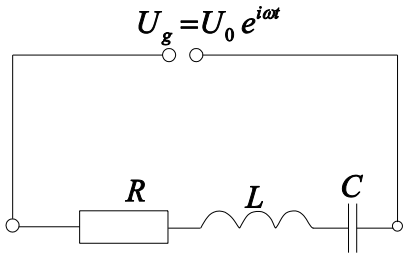
torej $\delta = 0$. Ker je v resonanci $\delta = 0$ in I_0 maksimalen, je takrat porabljena povprečna moč:

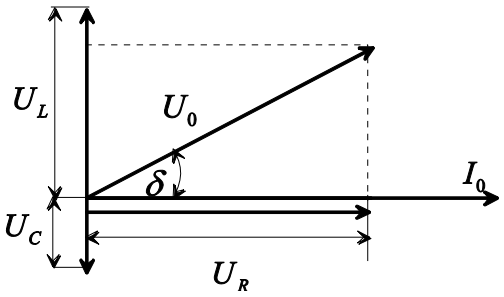
$$\overline{P} = \frac{1}{2} I_0 |U_0| \cos\delta = \frac{1}{2} \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} |U_0| \cos\delta = \tag{5.3.18}$$

$$= \frac{1}{2} \frac{|U_0|^2}{R} \cos\delta = \frac{1}{2} \frac{|U_0|^2}{R}$$

največja možna. V izpeljavi smo upoštevali enačbi (5.3.15) in (5.3.17).

Zaključek:





$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

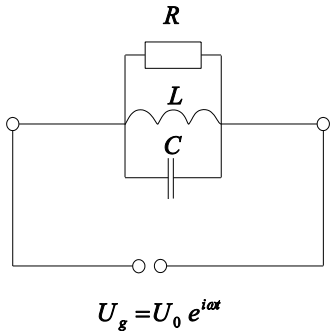
Vzamemo: I_0 je realen, U_0 je kompleksen

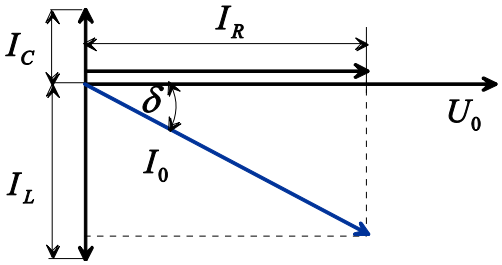
Kompleksne impedance:

$$Z_R = R$$
$$Z_L = i\omega L$$
$$Z_C = -i\frac{1}{\omega C}$$

$$Z = Z_R + Z_L + Z_C$$
$$U_0 = Z I_0$$

5.3.2 VZPOREDNA VEZAVA





$$U_0 = Z\, I_0$$

$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} - \frac{\omega C}{i}$$

Vzamemo: U_0 je realen, I_0 je kompleksen

$$I_0 = \frac{1}{Z} U_0 = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right) U_0 = U_0 \left[\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L} \right) \right]$$

$$|I_0| = \sqrt{I_0 \cdot I_0^*} = U_0 \left[\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{1/2}$$

$$tg\,\delta = \frac{\mathrm{Im}(I_0)}{\mathrm{Re}(I_0)} = \left(\omega C - \frac{1}{\omega L} \right) R$$

5.4 MOČ: IZMENIČNI TOK

Napetost prehiteva tok v fazi za δ.

$I = I_0 \cos \omega t$
$U = U_0 \cos(\omega t + \delta)$

$$P = IU = I_0 U_0 \cos\ \omega t\ \cos(\omega t + \delta) = I_0 U_0 \cos \omega t[\cos \omega t \cos \delta - \sin \omega t \sin \delta] =$$
$$= I_0 U_0[\cos^2 \omega t \cos \delta\ - \sin \omega t\ \cos \omega t\ \sin \delta] \Rightarrow$$
$$\overline{P} = I_0 U_0 \left[\overline{\cos^2 \omega t} \cos \delta - \overline{\frac{1}{2} \sin(2 \omega t)} \sin \delta \right],$$

kjer smo upoštevali:

$$\sin \omega t\ \cos \omega t = \frac{1}{2} \sin(2 \omega t).$$

Velja:

$$\overline{\cos^2 \omega t} = 1/2$$

$$\overline{\frac{1}{2} \sin(2 \omega t)} = 0,$$

torej:

$$\boxed{\overline{P} = \frac{1}{2} I_0 U_0 \cos \delta}$$

oziroma:

$$\overline{P} = I_{ef}\ U_{ef} \cos \delta, \quad \text{kjer} \quad I_{ef} = \frac{I_0}{\sqrt{2}} \quad in \quad U_{ef} = \frac{U_0}{\sqrt{2}}$$

6. EM valovanje in svetloba

6.1 ELEKTROMAGNETNO VALOVANJE

6.1.1. VALOVNA ENAČBA*

Heinrich Rudolf Hertz
(1857 – 1894)

- **Faradayev zakon** (ind. zakon**):**

$$\oint \vec{E}\cdot \mathrm{d}\vec{s} = -\frac{\partial}{\partial t}\int \vec{B}\cdot \mathrm{d}\vec{S} \quad \Rightarrow \quad \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},} \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

- **Amperov zakon:**

$$\int \vec{H}\cdot \mathrm{d}\vec{s} = \int \vec{j}_e \mathrm{d}\vec{S} + \int \frac{\partial \vec{D}}{\partial t}\mathrm{d}\vec{S} \quad \Rightarrow \quad \boxed{\vec{\nabla}\times\vec{H} = \vec{j}_e + \frac{\partial \vec{D}}{\partial t}}$$

če $j_e = 0$ (ni električnih tokov)

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$$

vakuum: $\varepsilon = 1, \quad \mu = 1$

$$\vec{B} = \mu_0 \vec{H}$$
$$\vec{D} = \varepsilon_0 \vec{E}$$

torej:

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$
$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

poseben primer: raven elektromagnetni val, $\vec{E}$ in $\vec{H}$ odvisna samo od koordinate x, $H_x = E_x = 0$:

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(0, -\frac{\partial E_z}{\partial x}, \overset{a}{\downarrow} \frac{\partial E_y}{\partial x} \right) = -\mu_0 \left(0, \frac{\partial H_y}{\partial t}, \overset{a}{\downarrow} \frac{\partial H_z}{\partial t} \right) \qquad (6.1.1a)$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(0, \overset{b}{\downarrow} -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) = \varepsilon_0 \left(0, \overset{b}{\downarrow} \frac{\partial E_y}{\partial t}, \frac{\partial E_z}{\partial t} \right) \qquad (6.1.2a)$$

Iz enačb (6.1.1a) in (6.1.2a) po odvajanju sledi:

$$(6.1.1a) \Rightarrow \frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad / \frac{\partial}{\partial t} \Rightarrow \frac{\partial^2 E_y}{\partial x \partial t} = -\mu_0 \frac{\partial^2 H_z}{\partial t^2} \qquad (6.1.1b)$$

$$(6.1.2a) \Rightarrow -\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t} \quad / \frac{\partial}{\partial x} \Rightarrow -\frac{\partial^2 H_z}{\partial x^2} = \varepsilon_0 \frac{\partial^2 E_y}{\partial t \partial x} \qquad (6.1.2b)$$

torej:

$+\mu_0 \frac{\partial^2 H_z}{\partial t^2} = +\frac{1}{\varepsilon_0} \frac{\partial^2 H_z}{\partial x^2}$, oziroma:

VALOVNA ENAČBA

$$\frac{\partial^2 H_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 H_z}{\partial x^2} \qquad (6.1.3)$$

Prav tako iz enačb (6.1.1a) in (6.1.2a) po odvajanju tudi sledi:

$$(6.1.1a) \Rightarrow \frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad / \frac{\partial}{\partial x} \Rightarrow \frac{\partial^2 E_y}{\partial x^2} = -\mu_0 \frac{\partial^2 H_z}{\partial t \partial x} \qquad (6.1.4)$$

(6.1.2a) $\Rightarrow$ $-\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t}$ $\quad / \frac{\partial}{\partial t}$ $\quad \Rightarrow$ $-\frac{\partial^2 H_z}{\partial t \partial x} = \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$ (6.1.5)

torej:

$-\frac{1}{\mu_0}\frac{\partial^2 E_y}{\partial x^2} = -\varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$, oziroma:

VALOVNA ENAČBA

$$\boxed{\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial^2 E_y}{\partial t^2}} \qquad (6.1.6)$$

Reševanje valovnih enačb (6.1.3) in (6.1.6)

Kot nastavek za rešitev enačbe (6.1.6) vzamemo funkcijo, ki opisuje potujoče valovanje v 1-D: $E_y = E_{y0}\cos\left(\omega\left(t - \frac{x}{c_0}\right)\right) = E_{y0}\cos(\omega t - kx)$, (6.1.7)

kjer je valovni vektor $k = \frac{\omega}{c_0}$ in c_0 hitrost širjenja EM valovanja. Torej:

$\frac{\partial^2 E_y}{\partial t^2} = -\omega^2 E_{y0}\cos(\omega t - kx)$,

$\frac{\partial^2 E_y}{\partial x^2} = -k^2 E_{y0}\cos(\omega t - kx)$, torej:

$-\omega^2 \cos(\omega t - kx) = \frac{-1}{\varepsilon_0 \mu_0} k^2 \cos(\omega t - kx)$, oziroma

$\omega^2 = \frac{1}{\varepsilon_0 \mu_0} k^2$. (6.1.8)

Ker velja $k = \frac{\omega}{c_o}$ iz enačbe (6.1.8) sledi, da je hitrost širjenja EM valovanja v vakuumu:.

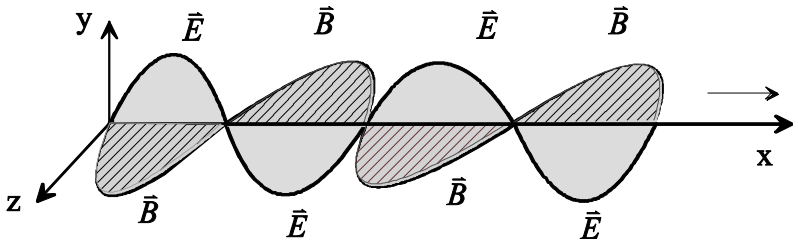
$$\boxed{c_0^2 = \frac{1}{\varepsilon_0 \mu_0}}$$

Influenčna konstanta: $\varepsilon_0 = 8.85 \cdot 10^{-12}$ As/Vm

Indukcijska konstanta: $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am

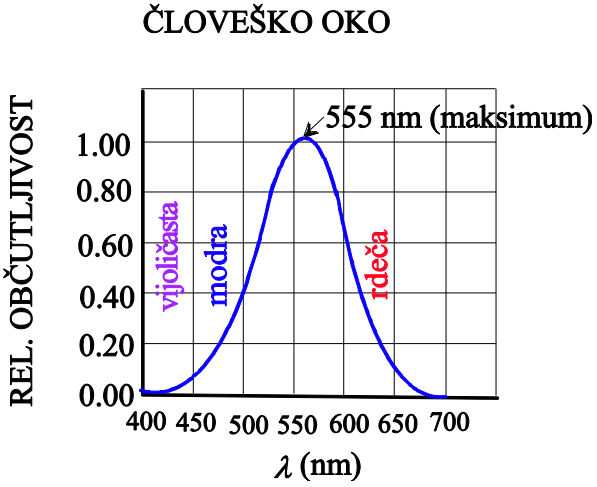
$\Rightarrow c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \cong 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$

Primer: raven elektromagnetni val



VRSTA EM VALOVANJA	VALOVNE DOLŽINE (λ)
radijski valovi	od 1 m do več 1000 km
mikrovalovi	od 0.1 mm do ~ 1 m
infra rdeče sevanje	od ~ 780 nm do 0.1 mm
vidna svetloba	od 380 do 780 nm
UV svetloba	od 5 ~ 380 nm
Rtg žarki	od 1 pm ~ 10 nm
γ žarki	od 0 do 1 pm

6.1.2 SPEKTER ELEKTROMAGNETNEGA VALOVANJA V VIDNEM OBMOČJU



6.1.3 ELEKTROMAGNETNO VALOVANJE V SNOVI – LOMNI KOLIČNIK

$$c = \frac{1}{\sqrt{\varepsilon\,\mu\,\varepsilon_0\,\mu_0}}$$ hitrost EM valovanja

Snovi, po katerih se širi elektromagnetno valovanje, so diamagnetne ali paramagnetne, ki imajo $\mu \approx 1$, torej:

$$\boxed{c = \frac{1}{\sqrt{\varepsilon \ \varepsilon_0 \ \mu_0}} = \frac{c_0}{n}} \ . \qquad (6.1.9)$$

Definiramo lomni kvocient:

$$\boxed{n = \sqrt{\varepsilon}} \qquad (6.1.10)$$

Opomba:

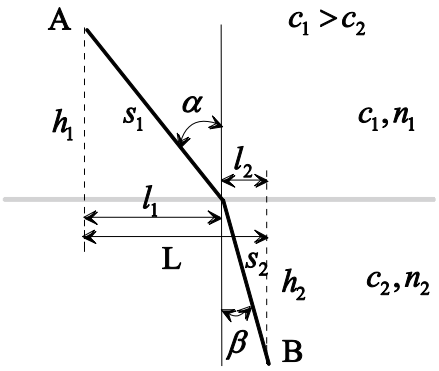
$n = \sqrt{\varepsilon}$ je odvisen od valovne dolžine λ

Iz tega sledi, **da je tudi hitrost EM valovanja** (svetlobe) odvisna od valovne dolžine.

$$\boxed{c = \frac{c_0}{n(\lambda)}} \qquad (6.1.11)$$

6.2 LOM: FERMATOV PRINCIP

Zahteva: minimalen čas za pot svetlobe iz točke A v točko B



$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \Rightarrow$

$t = \frac{h_1}{c_1 \cos\alpha} + \frac{h_2}{c_2 \cos\beta}$, kjer smo upoštevali:

$\cos\alpha = \frac{h_1}{s_1}$, $\cos\beta = \frac{h_2}{s_2}$

$tg\,\alpha = \frac{l_1}{h_1}$, $tg\,\beta = \frac{l_2}{h_2}$

$\mathrm{d}t = \frac{\partial t}{\partial \alpha}\mathrm{d}\alpha + \frac{\partial t}{\partial \beta}\mathrm{d}\beta = 0$ (pogoj za ekstrem)

$\mathrm{d}t = \frac{h_1 \cdot \sin\alpha}{c_1 \cdot \cos^2\alpha}\mathrm{d}\alpha + \frac{h_2 \cdot \sin\beta}{c_2 \cdot \cos^2\beta}\mathrm{d}\beta = 0$, torej:

$$-\frac{d\beta}{d\alpha} = \frac{c_2 h_1}{c_1 h_2} \cdot \frac{\sin\alpha}{\sin\beta}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 \qquad (6.2.1)$$

$L = h_1 \cdot tg\alpha + h_2 \cdot tg\beta$

$dL = h_1 \frac{1}{\cos^2\alpha} d\alpha + h_2 \frac{1}{\cos^2\beta} d\beta = 0$, ker je L konstanta

Torej:

$$\frac{d\beta}{d\alpha} = -\frac{h_1}{h_2}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 \qquad (6.2.2)$$

Upoštevamo enačbi (6.2.1) in (6.2.2);

$\frac{d\beta}{d\alpha} = \frac{d\beta}{d\alpha}$:

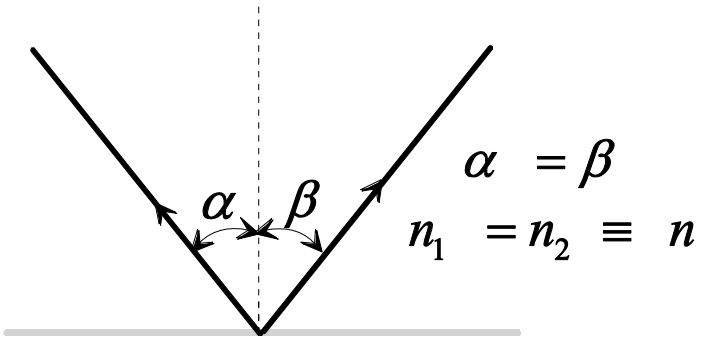
$-\frac{c_2}{c_1} \cdot \frac{h_1}{h_2} \cdot \frac{\sin\alpha}{\sin\beta}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 = -\frac{h_1}{h_2}\left(\frac{\cos\beta}{\cos\alpha}\right)^2$,

torej: $\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}$;

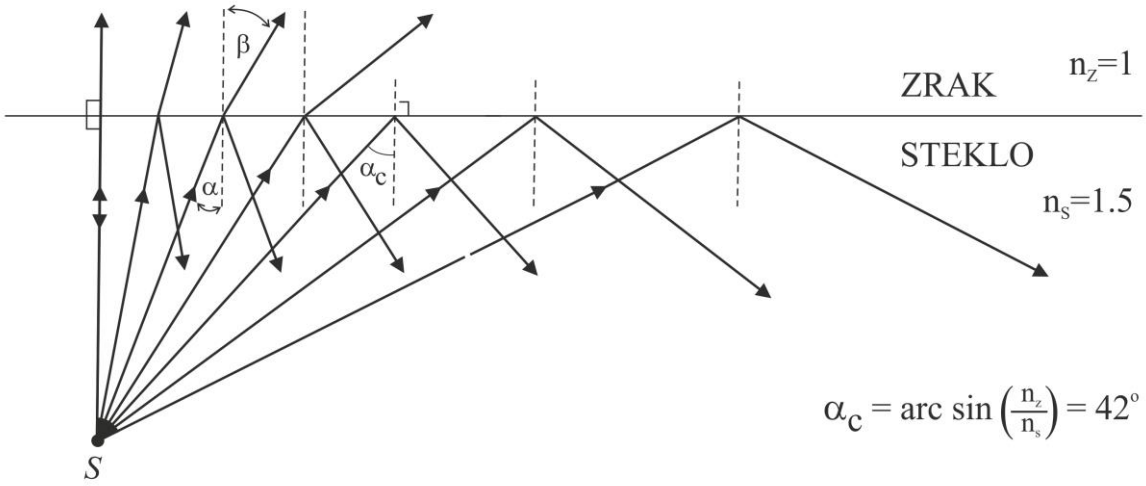
oziroma:

$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$, kjer smo upoštevali $c_1 = \frac{c_0}{n_1}$, $c_2 = \frac{c_0}{n_2}$

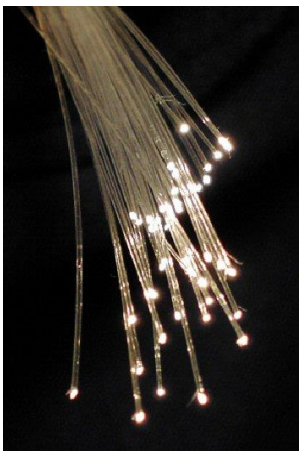
Odboj



Totalni odboj



Optična vlakna



6.3 KOHERENTNOST IZVOROV EM VALOVANJ IN INTERFERENCA

6.3.1 GOSTOTA ENERGIJSKEGA TOKA V ELEKTROMAGNETNEM VALOVANJU

$$\boxed{\bar{j} = \bar{w}\, c_0} \qquad \begin{array}{l} j \equiv \text{gostota energijskega toka EM valovanja} \\ c_0 \equiv \text{hitrost širjenja EM valovanja} \end{array} \tag{6.3.1}$$

Gostota energije

$$\boxed{\begin{array}{l} w_E = \frac{1}{2}\varepsilon_0 E^2 \\ w_B = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\frac{B^2}{\mu_0} \end{array}}$$

$$\boxed{w = w_E + w_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}} \tag{6.3.2}$$

$$\boxed{\vec{E} = \vec{B} \times \vec{c}_0}$$

$$\left.\begin{array}{l} \text{če:} \quad E = E_0 \cos(\omega t - kx) \\ \qquad B = B_0 \cos(\omega t - kx) \end{array}\right\} \Rightarrow \boxed{E_0 = B_0 c_0}$$

$$\bar{w} = \frac{1}{2}\varepsilon_0 E_0^2 \ \overline{\cos^2(\omega t - kx)} + \frac{1}{2\mu_0} B_0^2 \ \overline{\cos^2(\omega t - kx)} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 =$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0}\frac{E_0^2}{c_0^2} \quad \text{torej:}$$

$$\bar{w} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2, \tag{6.3.3}$$

kjer smo upoštevali:

$$\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$$

$$\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$$

$$B_0 = \frac{E_0}{c_0}$$

$$c_0^2 = \frac{1}{\varepsilon_0 \mu_0}$$

- torej:

$$\boxed{\bar{j} = \bar{w} c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 = \frac{1}{2} \frac{B_0^2}{\mu_0} c_0} \qquad (6.3.4)$$

- če antena oddaja izotropno:

$$\boxed{\bar{j} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}} \qquad (6.3.5)$$

- Poyntingov vektor:

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \qquad (6.3.6)$$

podaja:

velikost gostote energijskega toka
smer razširjanja valovanja

Dokaz prve trditve:

če $E = E_0 \cos(\omega t - kx)$, $H = H_0 \cos(\omega t - kx)$ in $B_0 = \mu_0 H_0$ velja:

$$\left|\overline{\vec{P}}\right| = E_0 H_0 \overline{\cos^2(\omega t - kx)} = \frac{1}{2} E_0 H_0 = \frac{1}{2} E_0 \frac{B_0}{\mu_0} = \frac{1}{2} E_0 \frac{E_0}{\mu_0 c_0} \cdot \frac{c_0}{c_0} = \frac{1}{2} E_0 \frac{E_0}{\mu_0 c_0^2} c_0 =$$

$$= \frac{1}{2} E_0 \frac{\varepsilon_0 \mu_0}{\mu_0} E_0 c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 \ .$$

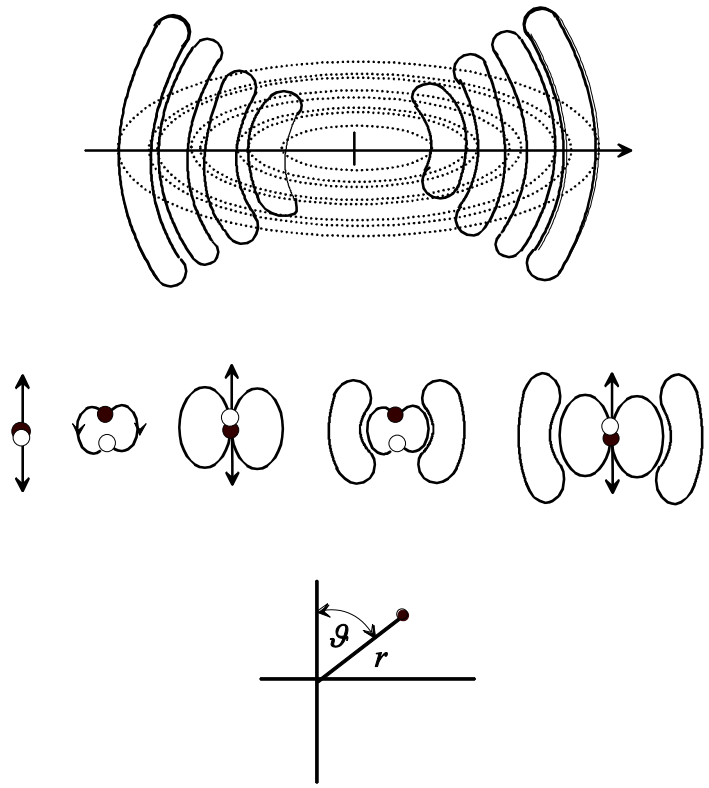
kjer smo upoštevali:

$$B_0 = \frac{E_0}{c_0} \quad \text{in} \quad c_0^2 = \frac{1}{\varepsilon_0 \mu_0} \ .$$

Torej:

$$\boxed{\bar{j} = \frac{1}{2} \varepsilon_0 E_0^2 c_0}$$

6.3.2 SEVANJE DIPOLA



$$\bar{j} = c_0 \bar{w} = \frac{r_0 \omega}{32 \pi^2 \varepsilon_0 c_0^3} \cdot \frac{\sin \vartheta}{r^2}$$
$$\bar{P} = \int \bar{j} \, \mathrm{d}S = \frac{p_0^2 \omega^4}{12 \pi \varepsilon_0 c_0^3} \qquad (6.3.7)$$

6.3.3 MATEMATIČNA FORMULACIJA

○ povprečna gostota energijskega toka v vakuumu:

$$\bar{j} = \bar{w} c_0 = \left(\bar{w}_E + \bar{w}_B\right) c_0 = \varepsilon_0 \overline{E^2} \, c_0 \qquad (6.3.8)$$

○ **eno** ravno EM valovanje: $E = E_0 \cos\left(\omega t - kx\right)$

$$\bar{j} = \bar{w} c_0 = \varepsilon_0 \overline{E_0^2} \, c_0 = \varepsilon_0 E_0^2 \overline{\cos^2\left(\omega t - kx\right)} c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 \ ,$$

kjer smo upoštevali $\overline{\cos^2\left(\omega t - kx\right)} = \frac{1}{2}$.

- **dve** ravni EM valovanji (fazna razlika δ),

V izbrani točki prostora x_0 uporabimo načelo superpozicije:

$$E_1 = E_{01}\cos\left(\omega t - k\,x_0 - \frac{1}{2}\delta\right)$$

$$E_2 = E_{02}\cos\left(\omega t - k\,x_0 + \frac{1}{2}\delta\right)$$

$$E = E_1 + E_2$$

torej:

$$\left(E_1 + E_2\right)^2 = \left[E_{01}\cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right) + E_{02}\cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right)\right]^2 =$$

$$= E_{01}^2\cos^2\left(\omega t - kx_0 - \frac{1}{2}\delta\right) + 2E_{01}E_{02}\cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right)\cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right) + \quad (6.3.9)$$

$$+ E_{02}^2\cos^2\left(\omega t - kx_0 + \frac{1}{2}\delta\right)$$

Poglejmo posebej mešani člen v enačbi (6.3.9):

$$2E_{01}E_{02}\cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right)\cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right) =$$

$$= 2E_{01}E_{02}\left[\cos\left(\omega t - kx_0\right)\cos\left(\frac{1}{2}\delta\right) + \sin\left(\omega t - kx_0\right)\sin\left(\frac{1}{2}\delta\right)\right] \cdot$$

$$\cdot\left[\cos\left(\omega t - kx_0\right)\cos\left(\frac{1}{2}\delta\right) - \sin\left(\omega t - kx_0\right)\sin\left(\frac{1}{2}\delta\right)\right] = \quad (6.3.10)$$

$$= 2E_{01}E_{02}\left[\cos^2\left(\omega t - kx_0\right)\cos^2\left(\frac{1}{2}\delta\right) - \sin^2\left(\omega t - kx_0\right)\sin^2\left(\frac{1}{2}\delta\right)\right] .$$

Iz enačbe (6.3.9) in (6.3.10) sledi (črta nad simboli označuje časovno povprečje):

$$\overline{\left(E_1 + E_2\right)^2} = E_{01}^2\,\overline{\cos^2\left(\omega t - k\,x_0 - \frac{1}{2}\delta\right)} + E_{02}^2\,\overline{\cos^2\left(\omega t - k\,x_0 + \frac{1}{2}\delta\right)} +$$

$$+ 2\,E_{01}E_{02}\left[\overline{\cos^2\left(\omega t - kx_0\right)}\;\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2\left(\omega t - kx_0\right)}\,\overline{\sin^2\left(\frac{1}{2}\delta\right)}\right]. \quad (6.3.11)$$

Če upoštevamo

$$\overline{\cos^2 z} = \frac{1}{2} \quad \text{in} \quad \overline{\sin^2 z} = \frac{1}{2}$$

iz enačbe (6.3.11) sledi:

$$\overline{(E_1+E_2)^2} = \frac{1}{2}E_{01}^2 + \frac{1}{2}E_{02}^2 + E_{01}E_{02}\left[\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2\left(\frac{1}{2}\delta\right)}\right],$$

oziroma

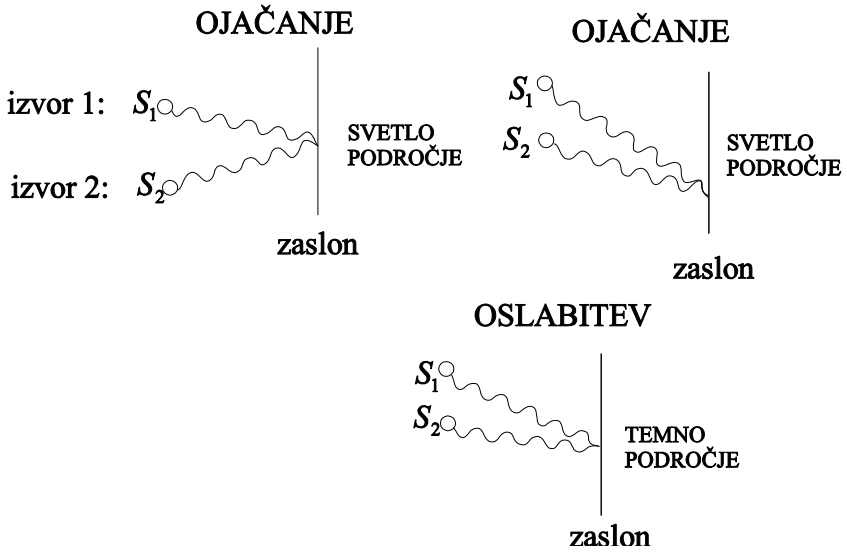
$$\overline{(E_1+E_2)^2} = \frac{1}{2}E_{01}^2 + \frac{1}{2}E_{02}^2 + E_{01}E_{02}\,\overline{\cos\delta}\,, \qquad (6.3.12)$$

Torej:

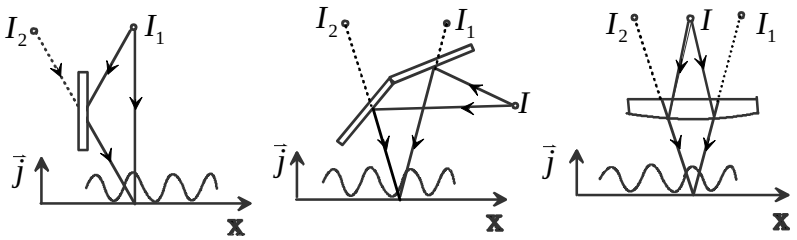
$$\boxed{\bar{j} = \bar{w}c_0 = \varepsilon_0\overline{(E_1+E_2)^2}\,c_0 = \frac{1}{2}\varepsilon_0 E_{01}^2 c_0 + \frac{1}{2}\varepsilon_0 E_{02}^2 c_0 + \varepsilon_0 E_{01}E_{02}c_0\,\overline{\cos\delta}\ .} \qquad (6.3.13)$$

Zaključek:

- nekoherentna izvora, če $\delta = \delta(t)$ naključen $\Rightarrow$ $\overline{\cos\delta(t)} = 0$ $\Rightarrow$ ni interference
- koherentna izvora, če δ stalna $\Rightarrow$ $\overline{\cos\delta(t)} \neq 0$ $\Rightarrow$ je interferenca (glejte slike na naslednji strani)
- **Shematski prikaz interference dveh EM valovanj** (2 koherentna izvora)

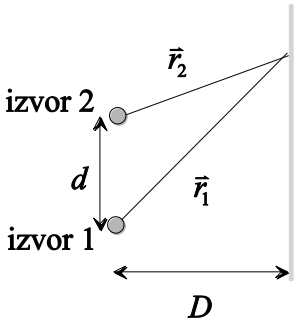


6.3.4 INTERFERENCA DVEH KOHERENTNIH VALOVANJ



Nekaj naprav s katerimi dobimo dva navidezna izvora koherentnih valovanj:
Lloydovo zrcalo (a), Fresnelovi nagnjeni zrcali (b) in Fresnelova biprizma (c).

Interferenca dveh koherentnih izvorov EM valovanja:



$\delta = 0$

$$E_1 = E_0\, e^{i(kr_1 - \omega t)}$$
$$E_2 = E_0\, e^{i(kr_2 - \omega t)}$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$
$$e^{-i\alpha} = \cos\alpha - i\sin\alpha$$

$$E_1 = E_0 \cos\left(kr_1 - \omega t\right)$$

$$E_2 = E_0 \cos\left(kr_2 - \omega t\right)$$

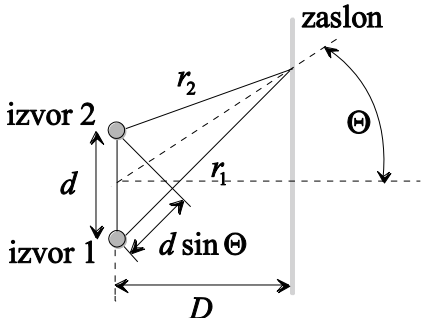
$$\left|E_1 + E_2\right|^2 = \left(E_1 + E_2\right)\left(E_1 + E_2\right)^* = \left(E_1 + E_2\right)\left(E_1^* + E_2^*\right) =$$
$$= E_1E_1^* + E_2E_2^* + E_1E_2^* + E_1^*E_2 = E_0^2\left(2 + e^{ik(r_1 - r_2)} + e^{-ik(r_1 - r_2)}\right) =$$
$$= 2E_0^2 + E_0^2\, 2\cos\left(k\left(r_1 - r_2\right)\right) = 2E_0^2\left(1 + \cos\left(k\left(r_1 - r_2\right)\right)\right) =$$
$$= 4E_0^2 \cos^2\left[\frac{k\left(r_1 - r_2\right)}{2}\right]$$

$1 + \cos 2x = 2\cos^2 x$

Pogoj za maksimum (svetle interferenčne proge):

$$\frac{k\left(r_1 - r_2\right)}{2} = n\pi \Rightarrow r_1 - r_2 = \frac{n2\pi}{k}, n = 0,\, 1,\, 2$$

Definiramo kot Θ :



d ≡ je razdalja med izvoroma

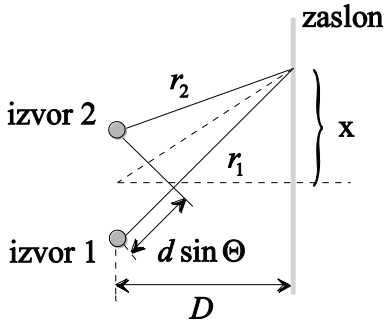
Če upoštevamo $r_1 - r_2 \cong d \sin\Theta$ in $k = \frac{2\pi}{\lambda}$ velja:

$$\left|E_1 + E_2\right|^2 = 4E_0^2 \cos^2\left[\frac{k\left(r_1 - r_2\right)}{2}\right] = 4E_0^2 \cos^2\left[\frac{\pi d \sin\Theta}{\lambda}\right] .$$

Povprečna gostota energijskega toka $\bar{j}$ je sorazmerna $\left|E_1 + E_2\right|^2$, torej:

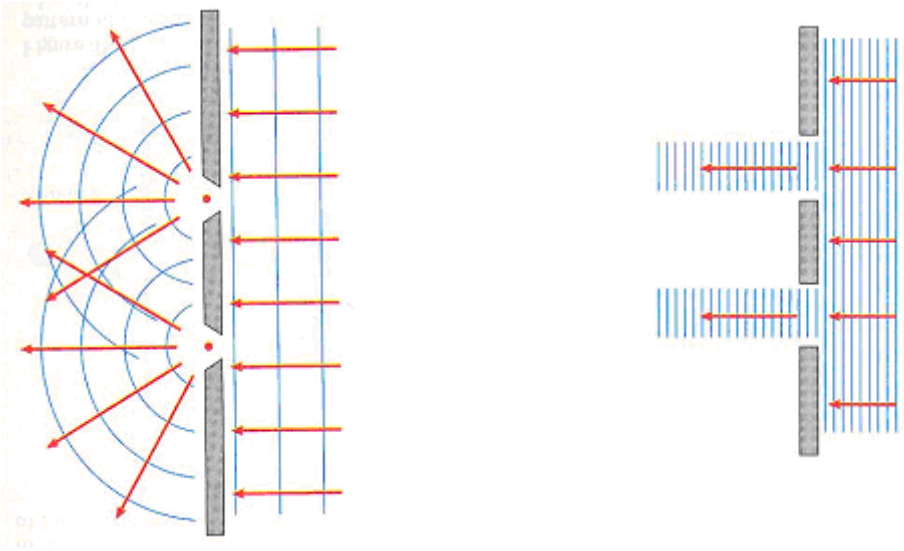
$$\boxed{\bar{j} \propto \left|E_1 + E_2\right|^2 = 4E_0^2 \cos^2\left[\frac{\pi d \sin\Theta}{\lambda}\right]}$$

Če D >> d: $\sin\Theta \approx tg\,\Theta = \frac{x}{D}$, torej: $\boxed{\bar{j} \propto \cos^2\left[\frac{\pi d\, x}{\lambda D}\right]}$



6.4 UKLON SVETLOBE NA REŽAH IN INTERFERENCA

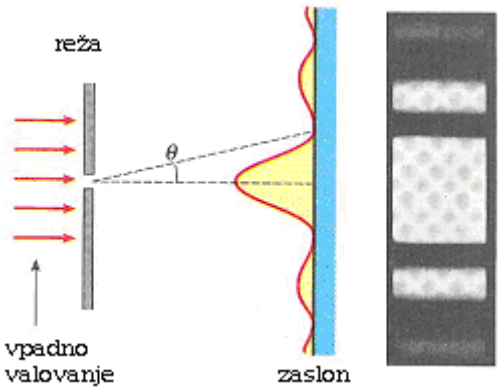
6.4.1 UKLON



sliki: Serway, 1992

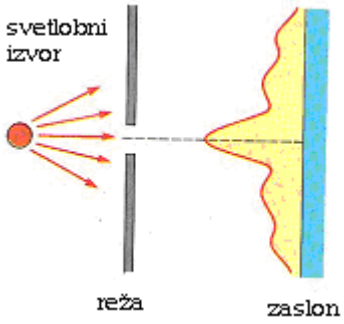
Huygensovo načelo: vsako točko valovne fronte obravnavamo kot izvor krogelnih (krožnih) valov)

Fraunhofer-jev uklon



Zaslon je zelo oddaljen.

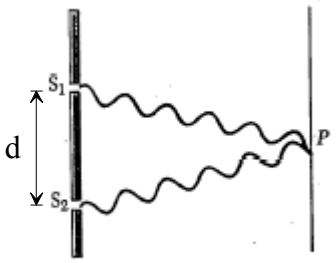
Fresnel-ov uklon



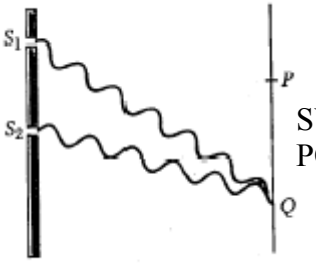
Zaslon je na končni razdalji (delna valovanja, ki interferirajo na izbranem delu zaslona ne smemo imeti za vzporedno).

sliki: Serway, 1992

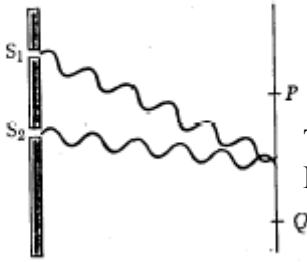
6.4.2 YOUNGOV POSKUS



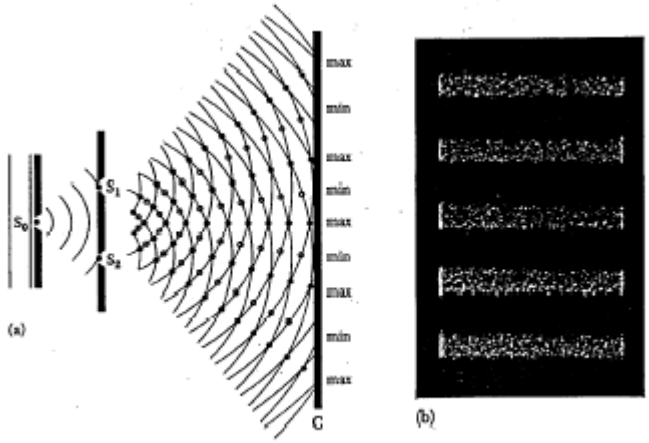
SVETLO
PODROČJE



SVETLO
PODROČJE

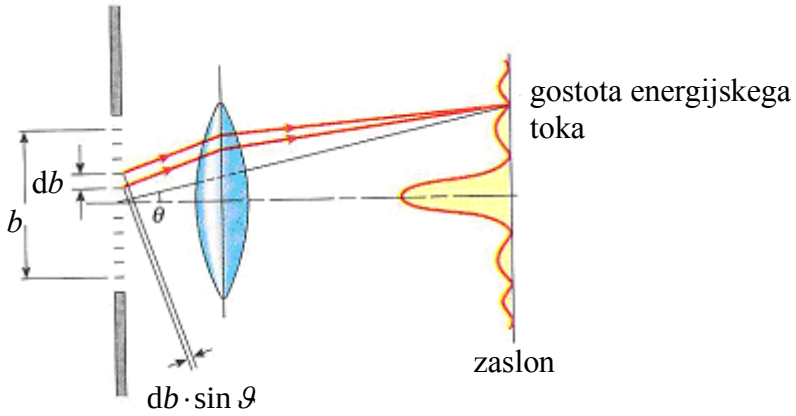


TEMNO
PODROČJE

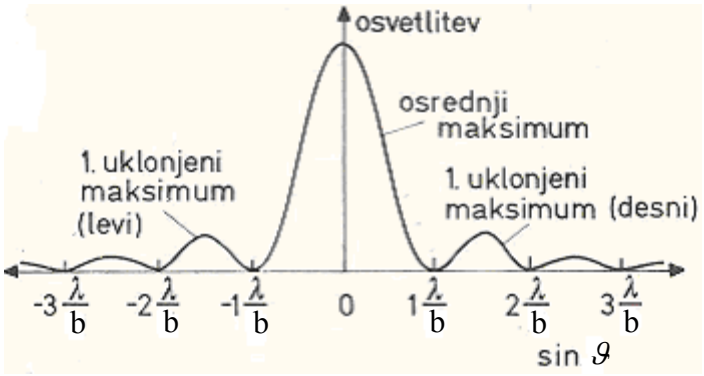


6.4.3 UKLON IN INTERFERENCA NA REŽI

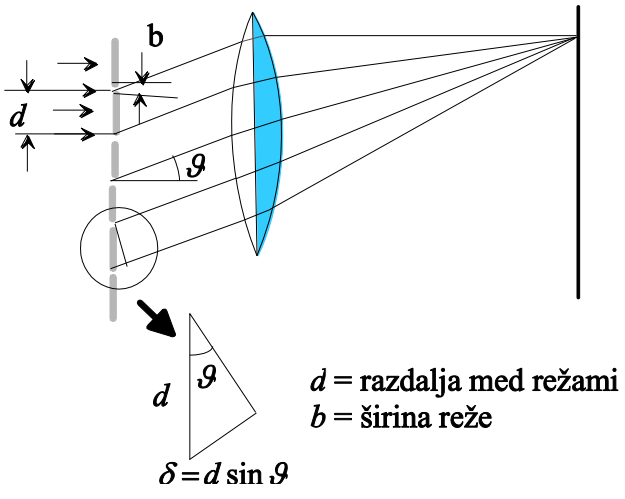
$$j = j_0 \frac{\sin^2(\pi b \sin\vartheta/\lambda)}{(\pi b \sin\vartheta/\lambda)^2}$$

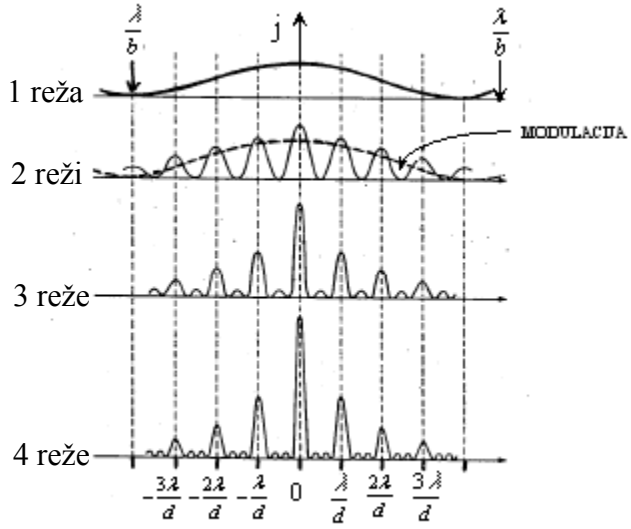


b ≡ širina reže $db \cdot \sin\vartheta$

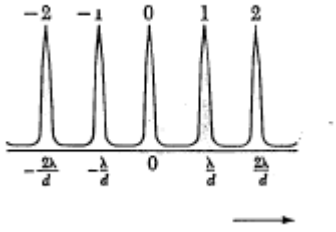


6.4.4 UKLON IN INTERFERENCA NA SISTEMU REŽ V ENAKIH RAZMIKIH





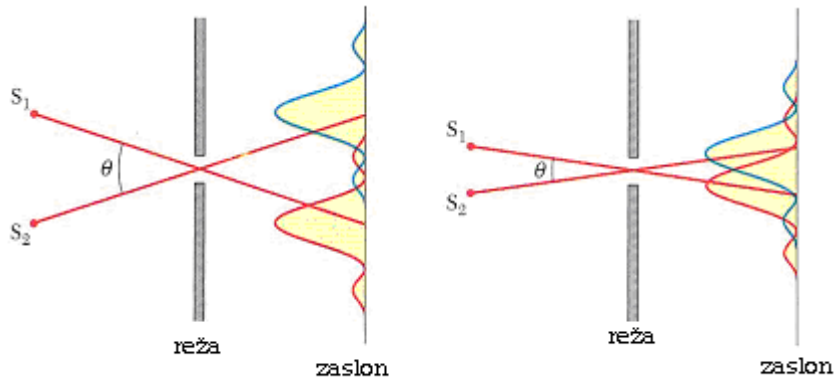
Slika uklonske mrežice:

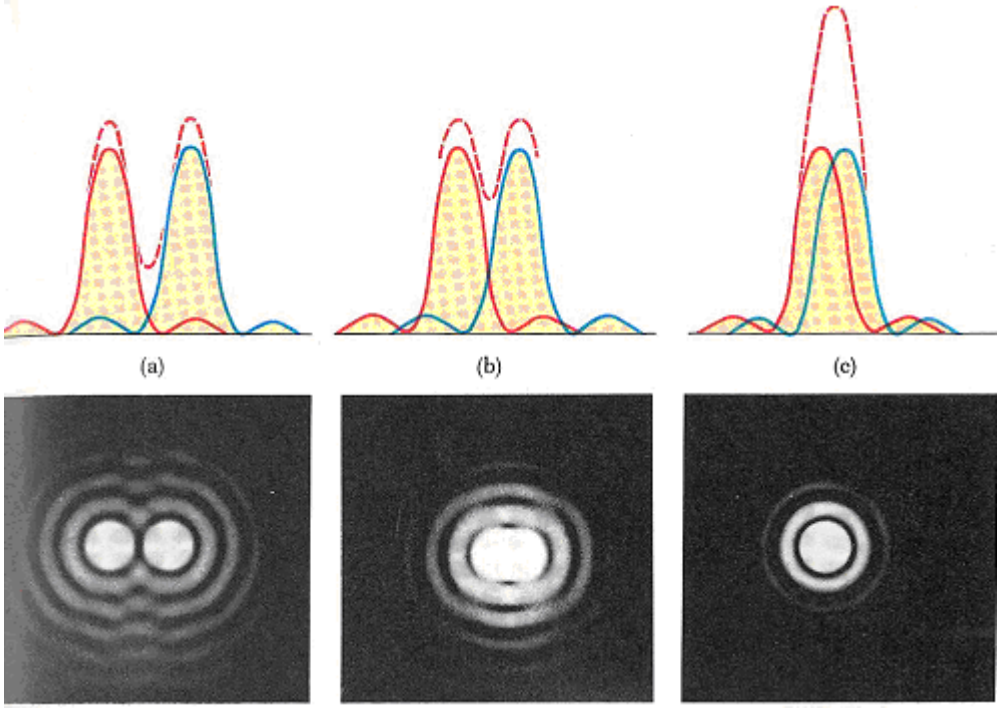


pogoj za ojačanje:

$$d \sin \vartheta = \mathrm{N}\lambda, \quad \mathrm{N} = 0,1,2,3,...$$

6.4.5 LOČLJIVOST (zaradi uklona):





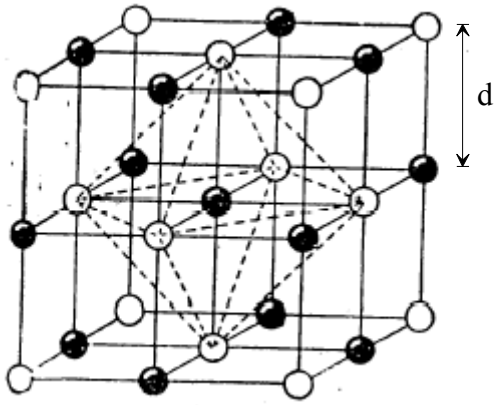
Opomba: ločljivost je pomembna pri mikroskopih

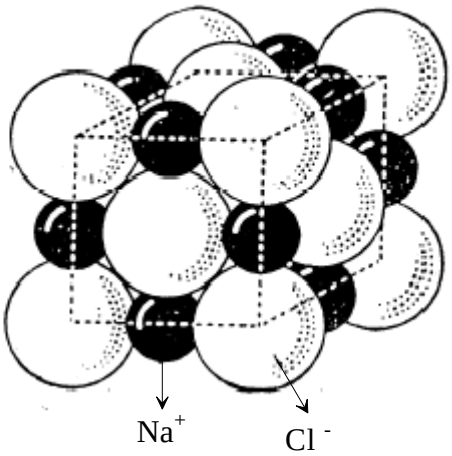
slike: J. Strnad, Fizika II
R. Kladnik, Visokošolska fizika
R.A. Serway, Physics

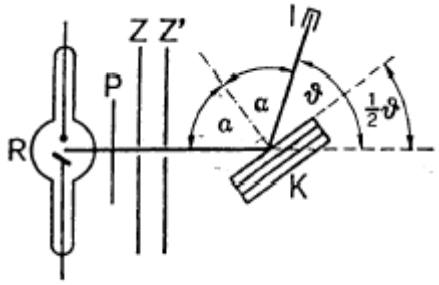
6.5 UKLONSKE SLIKE KRISTALOV

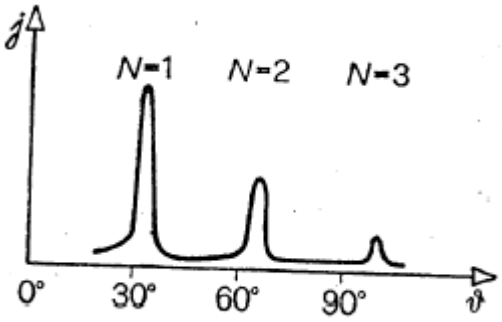
6.5.1 UKLONSKA SLIKA KRISTALA NaCl, BRAGGOVA ENAČBA

Ionska kristalna mreža NaCl









slike: J Strnad, Fizika II

Braggova enačba:

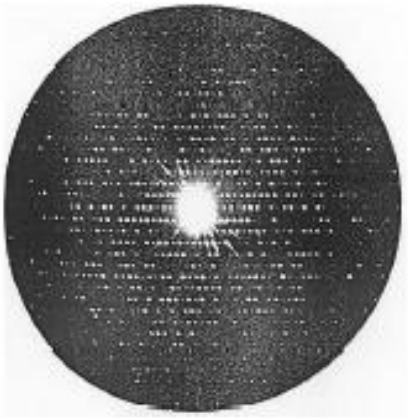
$$\boxed{2d\sin\left(\frac{1}{2}\vartheta\right)=N\lambda}$$

Primer kristala NaCl:

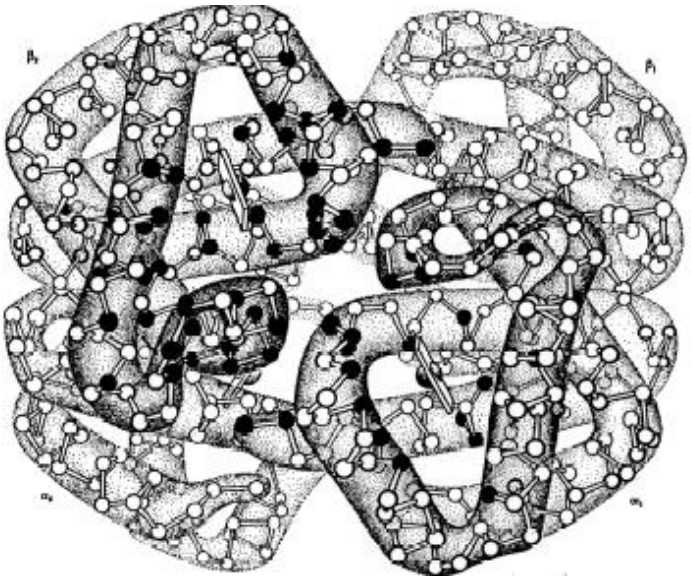
$$N=1,\ \vartheta=31.7^0,\lambda=1.54\,\text{Å}\quad d=\frac{\frac{1}{2}\lambda}{\sin\frac{1}{2}\vartheta}=2.82\,\text{Å}$$

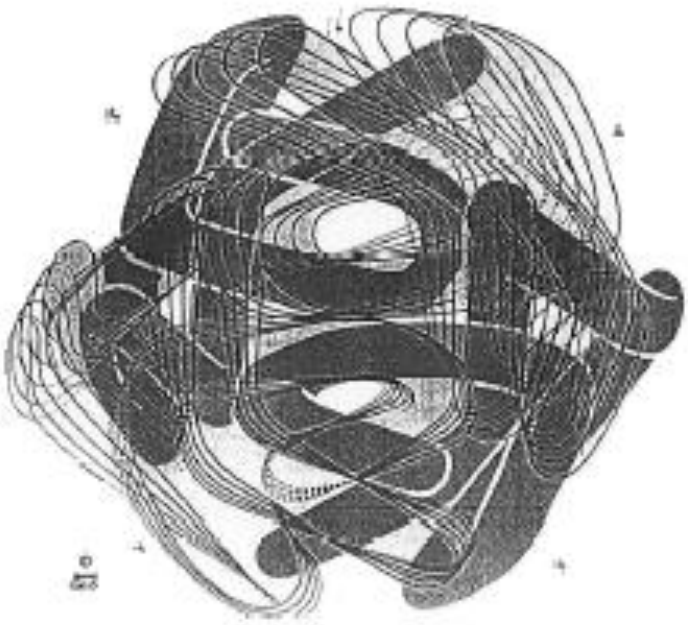
6.5.2 UKLONSKE SLIKE IN STRUKTURE PROTEINOV:

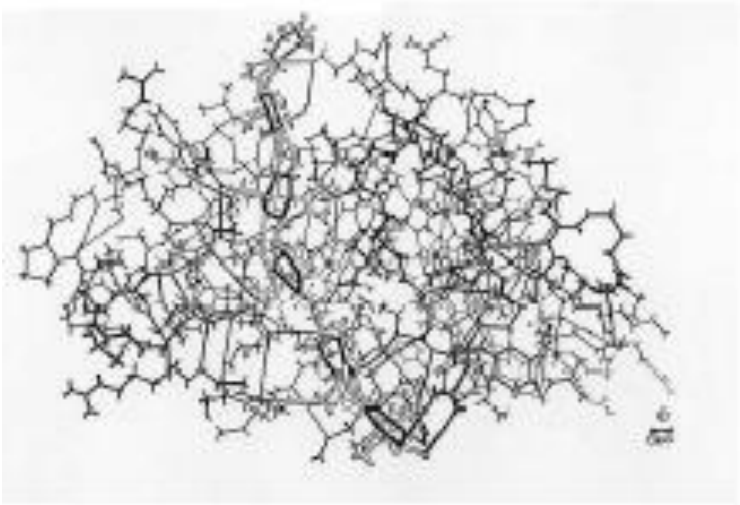
Uklonska slika:

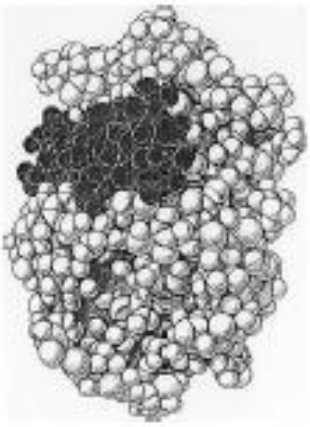


Strukture proteinov, ki so določene s pomočjo uklonskih slik:









slike: A.L.Lehniger, Principles of Biochemistry

6.6 SPREMEMBA FAZE EM VALOVANJA PRI ODBOJU

Študirali bomo odboj EM valovanja na optično redkejši snovi (n' < n) in odboj EM valovanja na optično gostejši snovi (n' > n), kjer sta:

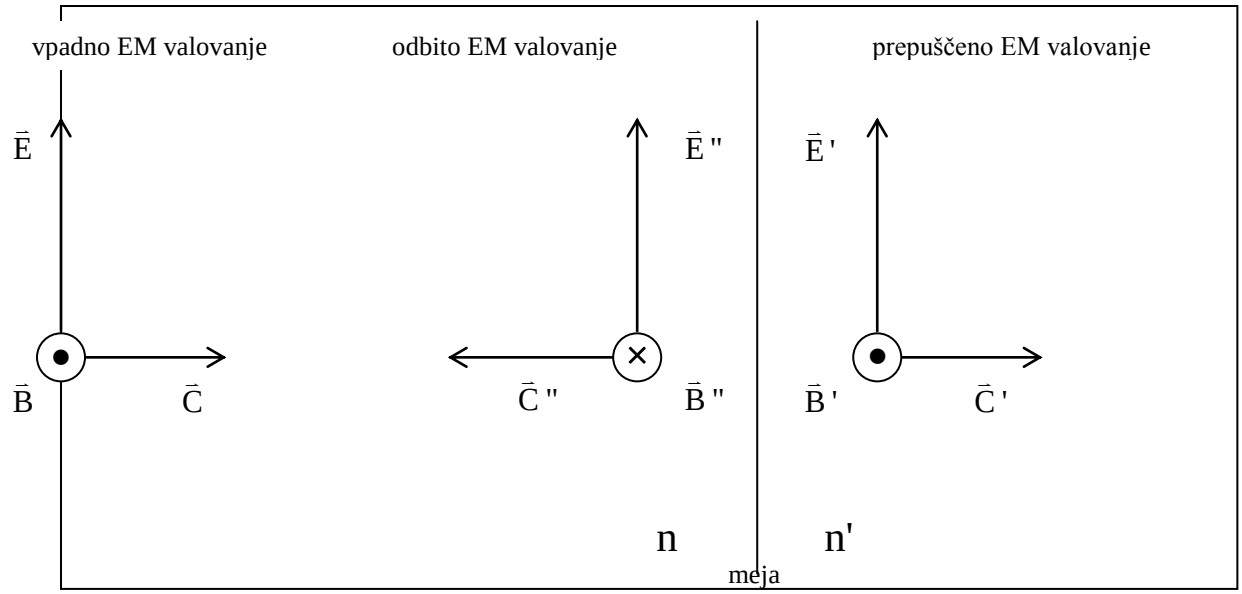
$$n = \sqrt{\varepsilon} \tag{6.6.1}$$

in

$$n' = \sqrt{\varepsilon'} \tag{6.6.2}$$

lomna količnika, ε in ε' pa ustrezni dielektrični konstanti. Upoštevamo μ ≅ 1.

E , B → vpadno EM valovanje
E", B" → odbito EM valovanje
E' , B' → prepuščeno EM valovanje



Slika: Pravokotni vpad EM valovanja na mejo dveh sredstev, $\left|\vec{C}\right| = C = \left|\vec{C}"\right| = C"$.

Hitrosti EM valovanja v snovi na levi strani meje (C, C") in v snovi na desni strani meje (C') sta:

$$C' = \frac{C_0}{n'}, \tag{6.6.3}$$

$$C = C" = \frac{C_0}{n}, \tag{6.6.4}$$

kjer je C_0 hitrost EM valovanja v vakuumu. Upoštevamo robna pogoja na meji dveh sredstev:

$$E + E" = E' , \tag{6.6.5}$$

$$H - H" = H' . \tag{6.6.6}$$

Iz zvez:

$$B = \mu_0 H, \tag{6.6.7}$$

$B' = \mu_0 H'$,
$B'' = \mu_0 H''$,

kjer smo upoštevali $\mu = \mu' = \mu'' \cong 1$, in iz enačbe (6.6.6) sledi

$$B - B'' = B' \tag{6.6.8}$$

Za EM valovanje velja $\vec{E} = \vec{B} \times \vec{C}$, torej

$$\begin{aligned} E &= B\,C, \\ E' &= B'\,C', \\ E'' &= B''C''. \end{aligned} \tag{6.6.9}$$

Iz enačb (6.6.8) in (6.6.9) tako sledi

$$\frac{E}{C} - \frac{E''}{C} = \frac{E'}{C'}, \tag{6.6.10}$$

od tod pa

$$E - E'' = E'\frac{C}{C'}. \tag{6.6.11}$$

Ob upoštevanju enačb (6.6.3) in (6.6.4) iz enačbe (6.6.11) sledi

$$E - E'' = E'\left(\frac{n'}{n}\right), \tag{6.6.12}$$

Enačbi (6.6.5) in (6.6.12) predstavljata sistem dveh enačb za dve neznanki E' in E'' (pri znani vrednosti E, n in n'). Iz enačbe (6.6.5) izrazimo E'':

$$E'' = E' - E \tag{6.6.13}$$

in vstavimo v enačbo (6.6.12). Tako dobimo:

$$E - E' + E = E'\left(\frac{n'}{n}\right),$$

od tod pa sledi:

$$\boxed{E' = \frac{2n}{(n+n')}E\ .} \tag{6.6.14}$$

Iz enačb (6.6.13) in (6.6.14) pa dobimo:

$$\boxed{E'' = \left(\frac{n-n'}{n+n'}\right)E\ .} \tag{6.6.15}$$

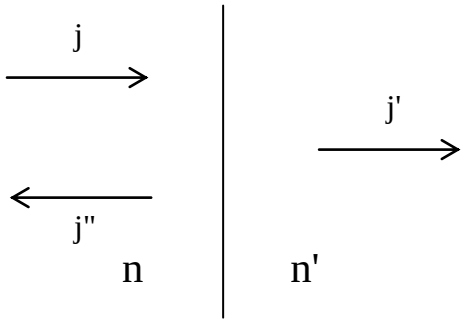
Na osnovi enačbe (6.6.15) torej sklepamo, da*:

1. **se EM valovanje na optično redkejši snovi (n' < n) odbije z enako fazo (če E > 0 ⇒ E" > 0)**

in

2. **se EM valovanje na optično gostejši snovi (n' > n) odbije z nasprotno fazo (če E > 0 ⇒ E" < 0).**

Izračunajmo še odbojnost (a) in prepustnost (b).



odbojnost (albedo) $a = \frac{j''}{j}$ (6.6.16)

prepustnost $b = \frac{j'}{j}$. (6.6.17)

Ob upoštevanju $\bar{j} = \bar{w}c = \frac{1}{2}\varepsilon\varepsilon_0 E_0^2 c$ iz enačb (6.6.14), (6.6.15), (6.6.16) in (6.6.17) sledi:

$$b = \frac{j'}{j} = \frac{\frac{1}{2}\varepsilon'\varepsilon_0 E'^2_0 c'}{\frac{1}{2}\varepsilon\varepsilon_0 E_0^2 c} = \frac{\varepsilon'}{\varepsilon}\cdot\frac{c'}{c}\left(\frac{E'_0}{E_0}\right)^2 = \frac{n'^2}{n^2}\cdot\frac{n}{n'}\left(\frac{E'_0}{E_0}\right)^2 = \frac{n'}{n}\left(\frac{E'_0}{E_0}\right)^2 = \frac{4n\,n'}{(n+n')^2}\quad , \qquad (6.6.18)$$

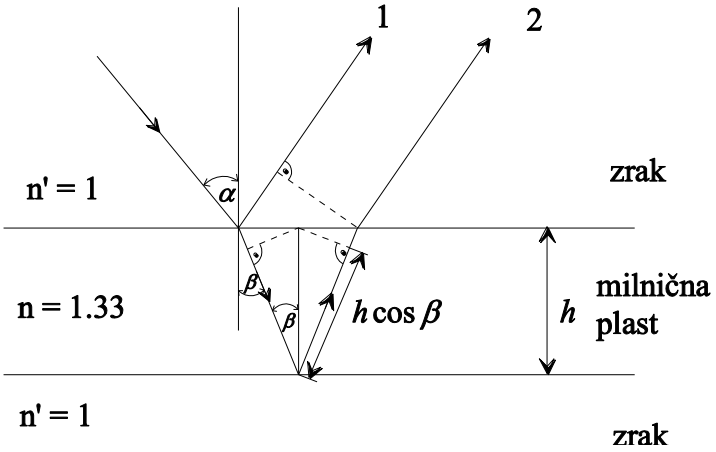
$$a = \frac{j''}{j} = \frac{(n-n')^2}{(n+n')^2}, \qquad (6.6.19)$$

kjer smo upoštevali tudi enačbe (6.6.1) - (6.6.4) .

* glejte še knjgo J. Strnad, *Fizika II*

6.7 INTERFERENCA (ODBOJ) EM VALOVANJA NA TANKIH PLASTEH

6.7.1 INTERFERENCA NA MILNIČNI OPNI



V milnični plasti je valovna dolžina svetlobe enaka $\lambda = \lambda_0 / n$. Razlika v poti (δ) med žarkom 1 in 2 je približno (glejte sliko):

$$\delta = 2h\cos\beta + \frac{\lambda}{2},$$

kjer smo z $\lambda/2$ upoštevali, da se žarek 1 odbije na optično gostejši snovi (n'< n). Pogoj za ojačanje zapišemo v obliki:

$$2h\cos\beta + \frac{\lambda}{2} = N\lambda,$$

kjer je N = 1, 2, … Upoštevamo $\lambda = \lambda_0 / n$ in dobimo:

$$2h\cos\beta + \frac{\lambda_0}{2n} = N\frac{\lambda_0}{n},$$

od koder sledi:

$$2nh\cos\beta = \frac{\lambda_0}{2}(2N-1). \qquad (6.7.1)$$

Ob upoštevanju lomnega zakona:

$$\frac{\sin\alpha}{\sin\beta} = n,$$

dobimo:

$$\frac{\sin^2\alpha}{\sin^2\beta}=\frac{\sin^2\alpha}{1-\cos^2\beta}=n^2\ ,$$

od tod pa sledi:

$$\frac{1}{n^2}\sin^2\alpha\ =1-\cos^2\beta\ ,$$

oziroma:

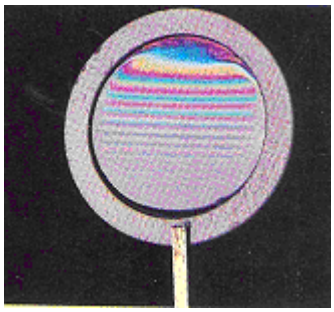
$$\cos\beta=\left(1-\frac{\sin^2\alpha}{n^2}\right)^{1/2}\ . \qquad (6.7.2)$$

Za N = 1 in ob upoštevanju enačbe (6.7.2) iz enačbe (6.7.1) sledi:

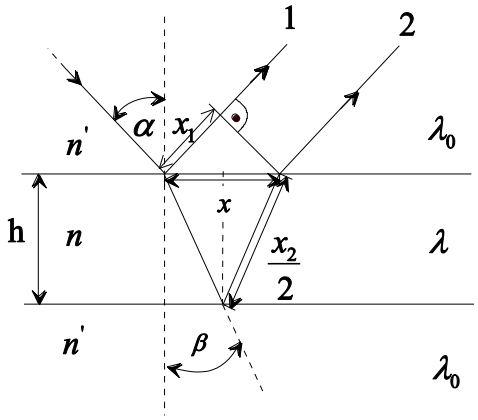
$$2nh\left(1-\frac{\sin^2\alpha}{n^2}\right)^{1/2}=\frac{\lambda_0}{2}\ , \qquad (6.7.3)$$

S pomočjo enačbe (6.7.3) izračunamo debelino milnične plasti:

$$h=\frac{\lambda_0}{4}\left(n^2-\sin^2\alpha\right)^{-1/\ 2} \qquad (6.7.4)$$



slika: Serway, 1992



Daljša izpeljava:

$$E=E_0\cos\left(\omega t-k\,x\right)$$

Pogoj za konstruktivno interferenco žarkov 1 in 2:

$$\pi+\omega t-k_1x_1=\omega t-k_2x_2+N\,2\pi\ , \qquad (D.1)$$

kjer smo upoštevali dodatno fazno razliko $\delta=\pi$, ker je $n>n'$.

Velja:

$$k_2 = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0}; \quad k_1 = \frac{2\pi}{\lambda_0} \ . \tag{D.2}$$

Iz enačbe (D.1) sledi:

$$k_2 x_2 - k_1 x_1 = N\,2\pi - \pi = (2N-1)\pi \ . \tag{D.3}$$

Ob upoštevanju enačb (D.2) iz enačbe (D.3) sledi:

$$\frac{2\pi n}{\lambda_0} x_2 - \frac{2\pi}{\lambda_0} x_1 = (2N-1)\pi \ , \tag{D.4}$$

oziroma:

$$n x_2 - x_1 = \frac{(2N-1)\lambda_0}{2} \ . \tag{D.5}$$

Velja (glejte sliko):

$$x_1 = x \sin\alpha \tag{D.6}$$

$$x_2 = \frac{x}{\sin\beta} \tag{D.7}$$

Ob upoštevanju enačb (D.6) in (D.7) predelamo levo stran enačbe (D.5) v obliko:

$$n x_2 - x_1 = n\frac{x}{\sin\beta} - x\sin\alpha = n\frac{x}{\sin\beta} - x\,n\sin\beta =$$

(kjer smo upoštevali lomni zakon $\sin\alpha = n\sin\beta$)

$$= n\,2h\,tg\,\beta\left(\frac{1}{\sin\beta} - \sin\beta\right) =$$

(kjer smo upoštevali $x = 2h\,tg\,\beta$)

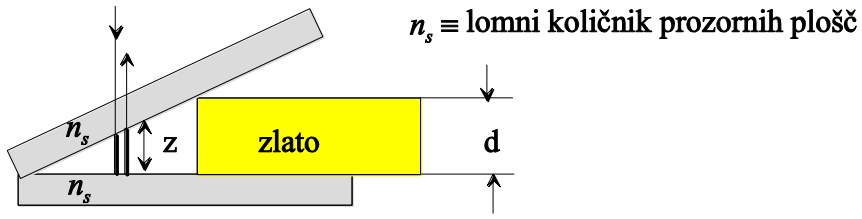
$$= n\,2h\left(\frac{1}{\cos\beta} - \frac{\sin^2\beta}{\cos\beta}\right) = n\,2h\frac{(1-\sin^2\beta)}{\cos\beta} = 2\,n\,h\cos\beta \ . \tag{D.8}$$

Ob upoštevanju enačbe (D.8) iz enačbe (D.5) sledi:

$$n x_2 - x_1 = 2n\,h\cos\beta = \frac{(2N-1)\lambda_0}{2} \ ,$$

kar se ujema z rezultatom v enačbi (6.7.1).

6.7.2 MERJENJE DEBELINE ZLATEGA LISTIČA



Pogoj za ojačanje:

$$\delta = 2z + \frac{\lambda_0}{2} = N\lambda_0 \ , \quad N = 1, 2, 3, \ldots \tag{6.7.5}$$

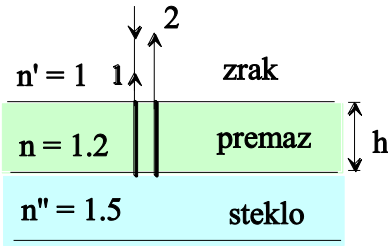
Iz (6.7.5) sledi:

$$z + \frac{\lambda_0}{4} = N\frac{\lambda_0}{2} \ , \tag{6.7.6}$$

torej:

$$\boxed{z = \frac{1}{4}(2N-1)\lambda_0}$$

6.7.3 INTERFERENČNI FILTER



n" > n > n'

Pogoj za oslabitev odbitih žarkov 1 in 2:

$$2h = N\frac{\lambda}{2} \ , \qquad N = 1,\ 3,\ 5 \tag{6.7.7}$$

kjer je

$$\lambda = \frac{\lambda_0}{n} \tag{6.7.8}$$

Torej:

$$\boxed{2h = N\,\frac{\lambda_0}{2n}\,,} \quad N = 1,\ 3,\ 5 \tag{6.7.9}$$

Oziroma:

$$2h = (2N+1)\,\frac{\lambda_0}{2n}\,, \quad N = 0,\ 1,\ 2. \tag{6.7.10}$$

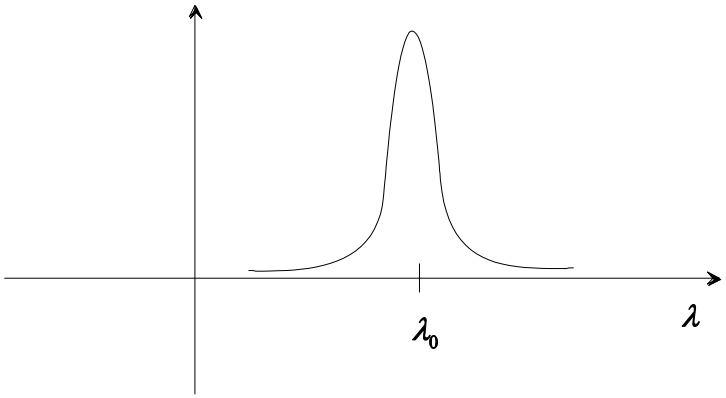
Debeline vrhnje plasti z lomnim količnikom n, ki ustrezajo oslabitvam odbite svetlobe:

$$\boxed{h = (2N+1)\,\frac{\lambda_0}{4n}\,,} \quad N = 0,\ 1,\ 2. \tag{6.7.11}$$

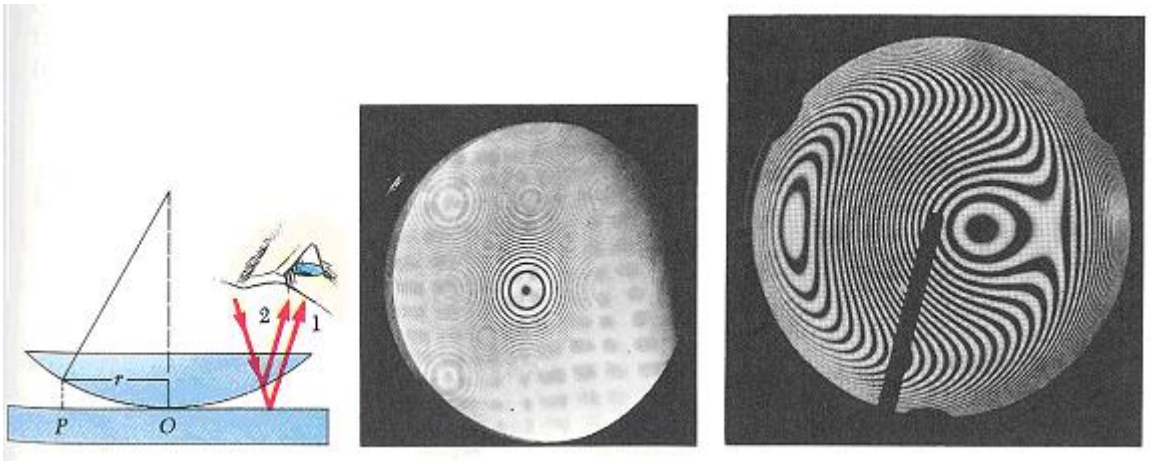
Najmanjša debelina je za N = 0:

$$\boxed{h_{\min} = \frac{\lambda_0}{4n}\,,} \tag{6.7.12}$$

Prepustnost interferenčnega filtra



6.7.4 NEWTONOVI KOLOBARJI

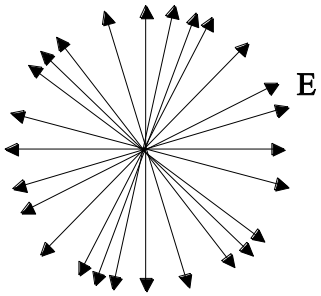


slike: Serway, 1992

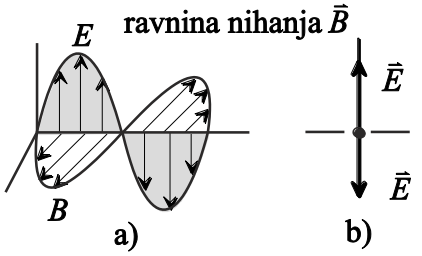
6.8 POLARIZACIJA SONČNE SVETLOBE

6.8.1 POLARIZIRANA IN NEPOLARIZIRANA SVETLOBA

- **Nepolarizirana svetloba (EM valovanje)**



- **Linearno polarizirano elektromagnetno valovanje (svetloba)**

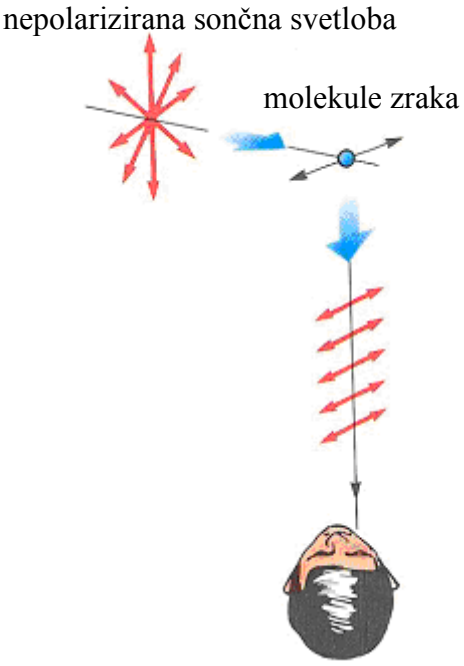


Polarizacija v mehaniki: Nepolarizirano (np) in linearno polarizirano (lp) valovanje na prožni vrvici. P polarizator, A analizator. Za analizatorjem ni valovanja. Daljice kažejo smer gibanja delov vrvice.

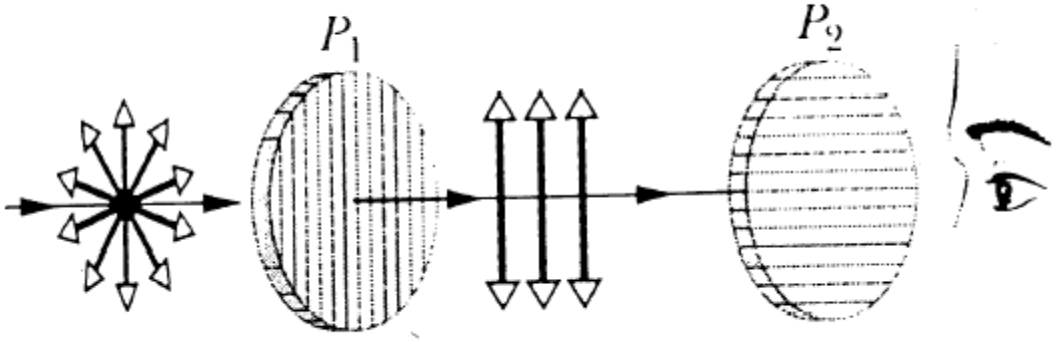
6.8.2 SIPANJE SONČNE SVETLOBE

Sipana svetloba (sevanje molekul kot dipolov)

$$\left.\begin{aligned} j &\propto \omega^4 \propto \frac{1}{\lambda^4} \\ j &\propto \sin^2 \vartheta \end{aligned}\right\}$$ gostota energijskega toka

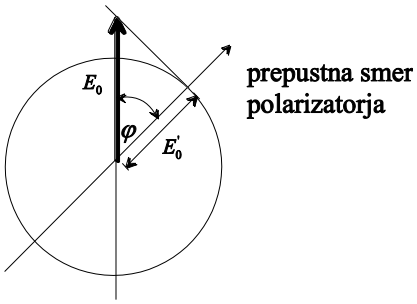


6.8.3 PREPUSTNOST POLARIZATORJA



popolnoma nepolarizirano EM valovanje

linearno polarizirano EM valovanje



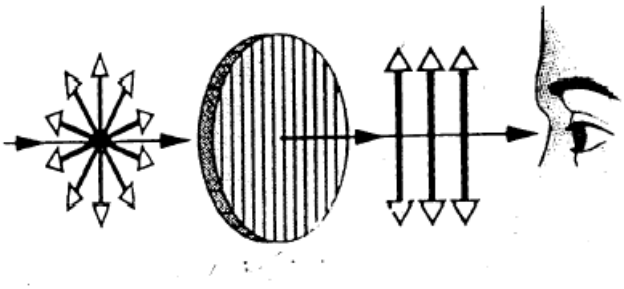
$$E_0^{'} = E_0 \cos\varphi$$

$$j_{prep} = \frac{1}{2}\varepsilon_0 E_0^{'2} c_0 = \frac{1}{2}\varepsilon_0 E_0^2 \left(\cos^2\varphi\right) c_0$$

$$j_{vpad} = \frac{1}{2}\varepsilon_0 E_0^2 c_0$$

prepustnost polarizatorja $= \dfrac{j_{prep}}{j_{vpad}} = \cos^2\varphi$

Primer: vpadno valovanje je popolnoma nepolarizirano

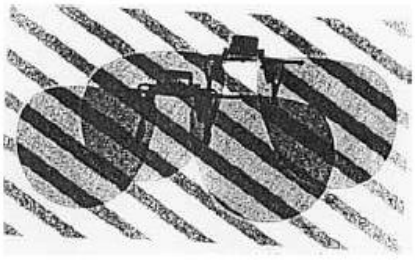


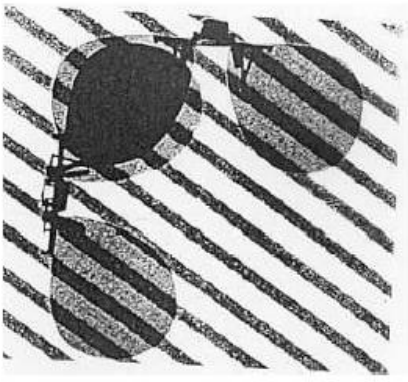
$$\bar{j}_{prep} = \frac{1}{2}\varepsilon_0 E_0^2 \overline{\cos^2\varphi} c_0 = \frac{1}{2}\varepsilon_0 E_0^2 \frac{1}{2} c_0 ,$$

ker $\overline{\cos^2\varphi} = \dfrac{1}{2}$

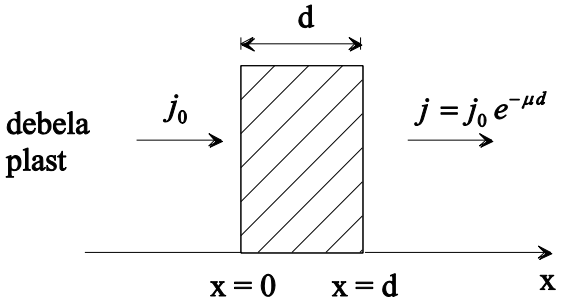
Torej: prepustnost je $\dfrac{1}{2}$.

6.8.4 PRIMER: POLARIZACIJSKA OČALA



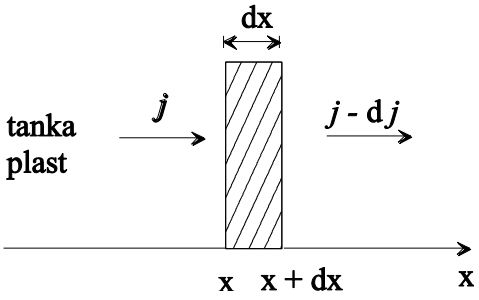


6.9 ABSORPCIJA EM VALOVANJA (svetlobe)



μ ≡ absorpcijski koeficient

Izpeljava:



$$\mathrm{d}j = -\mu\, j\, \mathrm{d}x, \tag{6.9.1}$$

$$\frac{\mathrm{d}j}{j} = -\mu\, \mathrm{d}x\,, \tag{6.9.2}$$

$$\int_{j_0}^{j(x)} \frac{\mathrm{d}j}{j} = -\mu \int_0^x \mathrm{d}x\,, \tag{6.9.3}$$

$$\ln j \Big|_{j_0}^{j(x)} = -\mu x\,, \tag{6.9.4}$$

$$\ln j(x) - \ln j_0 = -\mu x\,, \tag{6.9.5}$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\mu x\,, \tag{6.9.6}$$

$$\boxed{j(x) = j_0\, e^{-\mu x}} \tag{6.9.7}$$

Razpolovna debelina:

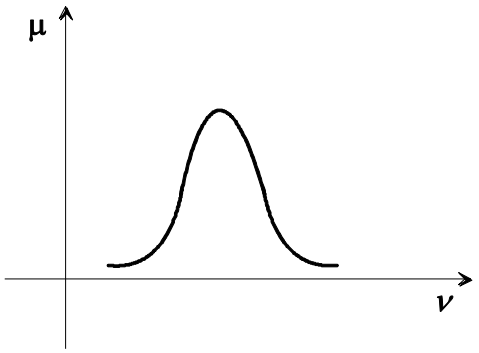
$$\frac{j_0}{2} = j_0 e^{-\mu x_{1/2}} ,$$

$$\frac{1}{2} = e^{-\mu x_{1/2}} ,$$

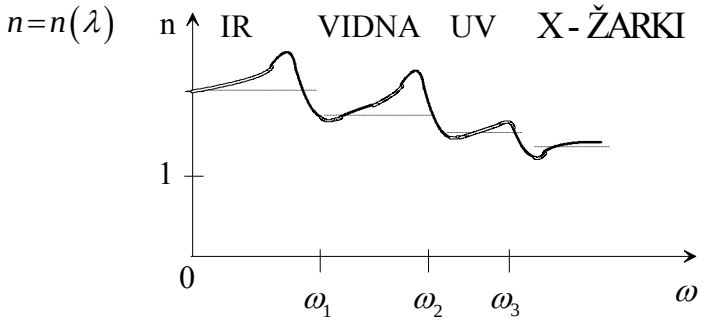
$$\ln\left(\frac{1}{2}\right) = -\mu x_{1/2} ,$$

$$\boxed{x_{1/2} = \frac{\ln 2}{\mu}}$$

Najmočnejša absorpcija EM valovanja je pri frekvenci EM valovanja (ν), ki se ujema z eno izmed lastnih frekvenc:



6.10 ODVISNOST LOMNEGA KOLIČNIKA OD VALOVNE DOLŽINE

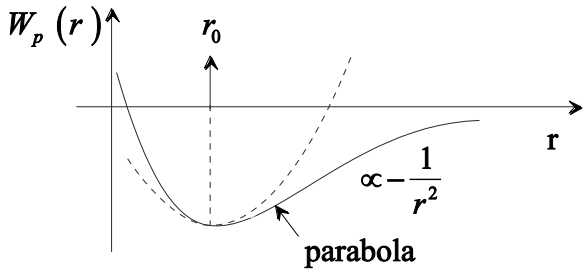


6.10.1 PRIMER: LOMNI KOLIČNIK RAZREDČENEGA VODIKOVEGA (H) PLINA*

pozitivni del naboja: proton ($m_p \equiv$ masa protona)
negativni del naboja: elektron ($m_e \equiv$ masa elektrona)

$\boxed{m_e << m_p}$

r ≡ razdalja med protonom in elektronom
$W_p \equiv$ potencialna energija med protonom in elektronom



$$\boxed{W_p \cong \frac{1}{2} K (r - r_0)^2 - C}$$

sila: $F = -\dfrac{\partial W_p}{\partial r} = -K(r - r_0)$

če $\boxed{r = r_0 + s}$, kjer je s odstopanje od ravnovesne razdalje

$$W_p = \frac{1}{2} K s^2 - C$$

$$\boxed{F = -Ks} \qquad (6.10.1)$$

Sistem opišemo v težiščnem sistemu, kjer je:

$$r_T = \frac{m_p r_p + m_e r_e}{\left(m_p + m_e\right)} = 0 , \tag{6.10.2}$$

torej

$$m_p r_p + m_e r_e = 0 . \tag{6.10.3}$$

Enačbo (6.10.3) odvajamo po času:

$$m_p \frac{\mathrm{d}r_p}{\mathrm{d}t} + m_e \frac{\mathrm{d}r_e}{\mathrm{d}t} = 0 ,$$

oziroma

$$m_p v_p + m_e v_e = 0 . \tag{6.10.4}$$

kjer je $v_p = \frac{\mathrm{d}r_p}{\mathrm{d}t}$ hitrost protona in $v_e = \frac{\mathrm{d}r_e}{\mathrm{d}t}$ hitrost elektrona. Iz enačbe (6.10.4) sledi:

$$v_p = -\frac{m_e}{m_p} v_e \quad . \tag{6.10.5}$$

Ker je $\frac{m_e}{m_p} << 1$ od tod sledi, da je v težiščnem sistemu protona in elektrona

$$\left|v_p\right| << \left|v_e\right| \tag{6.10.6}$$

Zato v prvem približku vzamemo, da proton miruje in zapišemo Newtonov zakon samo za elektron:

$$\boxed{m_e a = -Ks - e_0 E} , \tag{6.10.7}$$

kjer smo upoštevali $F = -Ks$ (enačba (6.10.1)).

$a = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ ≡ pospešek elektrona

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$ ≡ hitrost elektrona

E ≡ jakost zunanjega električnega polja

Predpostavimo, da je jakost zunanjega električnega polja na mestu atomov posledica EM valovanja s krožno frekvenco ω, kjer je

$$E = E_0 \sin(\omega t)\,, \tag{6.10.8}$$

električno polje na mestu vodikovega atoma. Enačbo (6.10.8) vstavimo v enačbo (6.10.7) in dobimo:

$$\boxed{m_e \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + K s + e_0 E_0 \sin(\omega t) = 0} \tag{6.10.9}$$

Rešitev enačbe (6.10.9) iščemo z nastavkom:

$$s = s_0 \sin(\omega t + \delta)\ . \tag{6.10.10}$$

Enačbo (6.10.10) in

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -s_0 \omega^2 \sin(\omega t + \delta) \tag{6.10.11}$$

vstavimo v enačbo (6.10.9) in dobimo:

$$-m_e\, s_0\, \omega^2 \sin(\omega t + \delta) + K s_0 \sin(\omega t + \delta) = -e_0\, E_0 \sin(\omega t)\ . \tag{6.10.12}$$

V enačbo (6.10.12) vstavimo:

$$\sin(\omega t + \delta) = \sin(\omega t)\cos\delta + \cos(\omega t)\sin\delta$$

in dobimo:

$$\sin(\omega t)\left[\left(\omega_0^2 - \omega^2\right) s_0 \cos\delta + \frac{e_0 E_0}{m_e}\right] + \left(\omega_0^2 - \omega^2\right) s_0 \sin\delta \cos(\omega t) = 0\,, \tag{6.10.13}$$

kjer $\boxed{\omega_0^2 = \frac{K}{m_e}}$ lastna krožna frekvenca nihanja elektrona.

Enačba (6.10.13) mora veljati posebej za člen s $\sin(\omega t)$ in člen s $\cos(\omega t)$. Torej

$$\left(\omega_0^2 - \omega^2\right) s_0 \cos\delta + \frac{e_0 E_0}{m_e} = 0\,, \tag{6.10.14}$$

$$\sin\delta = 0,\quad \delta = 0, \pm\pi, \pm 2\pi \ldots . \tag{6.10.15}$$

Iz enačbe (6.10.14) sledi vrednost amplitude s_0 v obliki :

$$s_0 = \frac{e_0 E_0}{m_e\left(\omega_0^2 - \omega^2\right)}\,, \tag{6.10.16}$$

kjer smo upoštevali $\delta = -\pi$. Izračunajmo še polarizacijo $P_0 = n_0\, p_e$:

$$P_0 = n_0\, p_e = n_0\, e_0\, s_0 = \frac{n_0\, e_0^2\, E_0}{m_e\left(\omega_0^2 - \omega^2\right)} \cong \left(\varepsilon - 1\right)\varepsilon_0\, E_0 \ , \tag{6.10.17}$$

kjer je $p_e = e_0\, s_0$ inducirani dipolni moment atoma, n_0 pa volumska gostota atomov.
Upoštevali smo zvezo med susceptibilnostjo (χ) in dielektričnostjo (ε) : $\chi = \varepsilon - 1$.
Iz enačbe (6.10.17) sledi:

$$\boxed{\varepsilon = 1 + \frac{n_0\, e_0^2}{m_e\, \varepsilon_0\left(\omega_0^2 - \omega^2\right)}} \ . \tag{6.10.18}$$

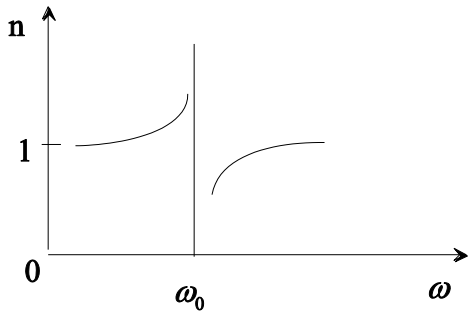
Iz zveze med lomnim količnikom in dielektrično konstanto ε dobimo:

$$n = \sqrt{\varepsilon} = \left(1 + \frac{n_0\, e_0^2}{m_e\, \varepsilon_0\left(\omega_0^2 - \omega^2\right)}\right)^{1/2} \ . \tag{6.10.19}$$

S pomočjo razvoja $\left(1 + x\right)^{1/2} = 1 + \frac{1}{2}x + ...$ iz enačbe (6.10.19) sledi:

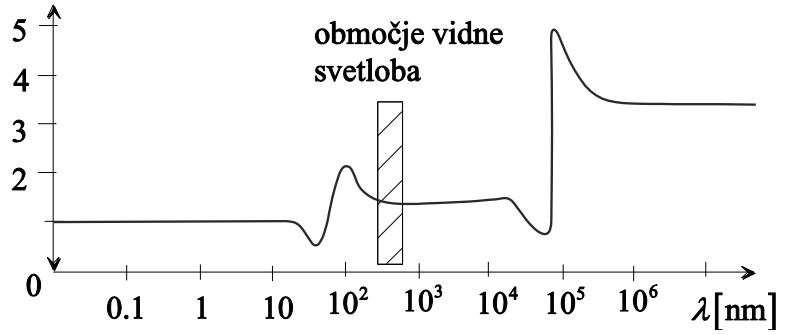
$$\boxed{n \cong 1 + \frac{1}{2}\frac{n_0\, e_0^2}{m_e\, \varepsilon_0\left(\omega_0^2 - \omega^2\right)}} \ . \tag{6.10.20}$$

Opomba: ker v enačbi (6.10.7) nismo upoštevali člena zaradi dušenja, zgornji izraz za lomni količnik n pri $\omega = \omega_0$ divergira.



6.10.2 DRUGI PRIMERI

1. Odvisnost lomnega količnika n od valovne dolžine (λ) za kameno sol (NaCl):



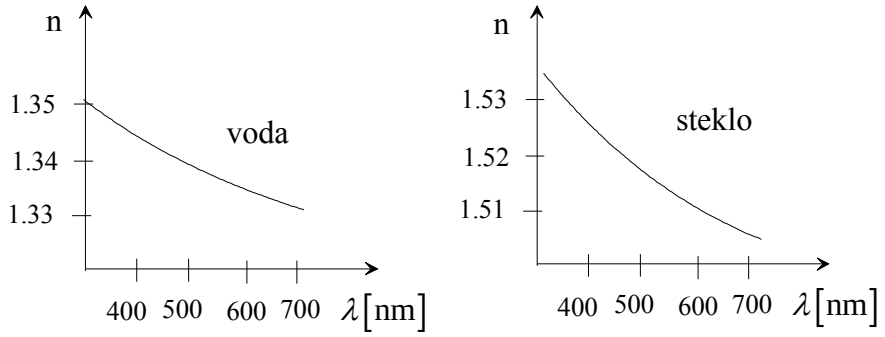
$$c = \nu\lambda$$

$$\omega = 2\pi\upsilon$$

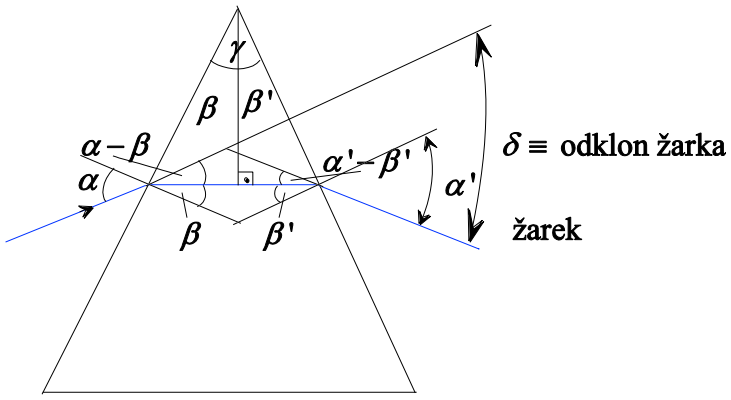
$$c = \frac{c_0}{n}$$

$$\lambda = \frac{2\pi c_0}{\omega}$$

2. Odvisnost lomnega količnika od valovne dolžine (λ) za vodo in steklo:



6.10.3 LOM SVETLOBE NA PRIZMI



$$\frac{\sin\alpha}{\sin\beta} = n$$

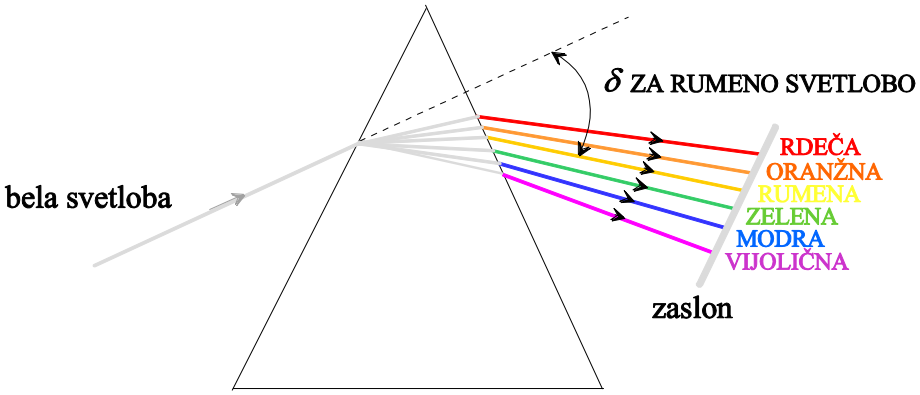
$$\frac{\sin\alpha'}{\sin\beta'} = n$$

$$\gamma = \beta + \beta'$$

$$180^0 = 180^0 - \delta + \alpha - \beta + \alpha' - \beta' \Rightarrow$$

$$\delta=\alpha-\beta+\alpha'-\beta'=\alpha+\alpha'-\gamma$$.

Ker $n=n(\lambda)$ je odklon (δ) posameznih komponent bele svetlobe na prizmi različen:

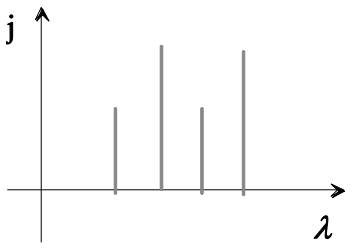


6.11 FOTOMETRIJA

6.11.1 VRSTE SPEKTROV

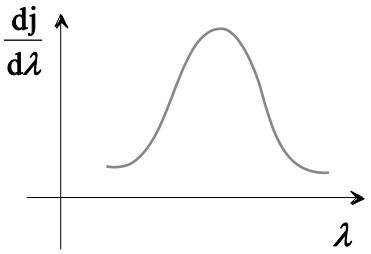
- **emisijski** spekter: spekter, ki ga svetila (izvori EM valovanja) sevajo
- **absorpcijski** spekter
- **odbojni** spekter

- **črtasti** spektri (v plinih)



$j \equiv$ gostota energijskega toka
$\lambda \equiv$ valovna dolžina

- **zvezni** spektri



dj je delež gostote energijskega toka j v intervalu $\left[\lambda - \frac{1}{2}\mathrm{d}\lambda\ ,\ \lambda + \frac{1}{2}\mathrm{d}\lambda\right]$

Skupna gostota energijskega (svetlobnega) toka:

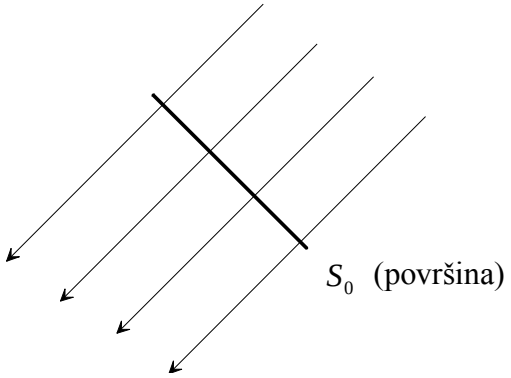
$$j = \int \left(\frac{\mathrm{d}j}{\mathrm{d}\lambda}\right) \mathrm{d}\lambda$$

6.11.2 OSVETLJENOST (E)

Gostota energijskega (svetlobnega) toka:

$$j = \frac{P}{S_0} \tag{6.11.1}$$

kjer je P svetlobni (energijski) tok

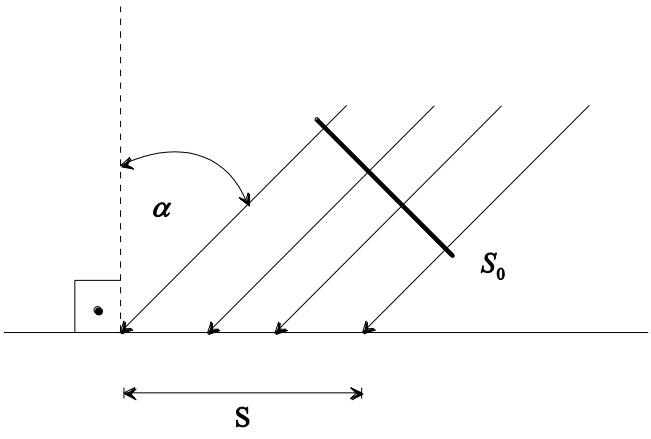


Osvetljenost dela ravne ploskve s površino S je (glejte sliko):

$$E = \frac{P}{S} = \frac{P}{(S_0/\cos\alpha)} = \left(\frac{P}{S_0}\right)\cos\alpha \ . \tag{6.11.2}$$

Ob upoštevanju enačbe (6.11.1) iz enačbe (6.11.2) sledi:

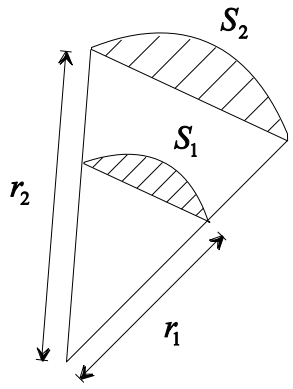
$$\boxed{E = j\cos\alpha} \tag{6.11.3}$$



$$\cos\alpha = \frac{S_0}{S} \Rightarrow S = \frac{S_0}{\cos\alpha}$$

6.11.3 SVETILNOST TOČKASTEGA SVETILA

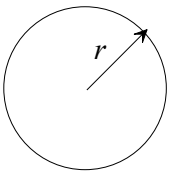
Definicija prostorskega kota



$$\frac{S_1}{r_1^2} = \frac{S_2}{r_2^2}$$

prostorski kot $\Omega = \frac{S}{r^2}$

Poln prostorski kot je 4π :



$$\Omega_{\text{poln}} = \frac{S_{\text{krogle}}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

Infinitezimalni prostorski kot:

$$\boxed{\mathrm{d}\Omega = \frac{\mathrm{d}S}{r^2}} \quad . \tag{6.11.4}$$

Točkasto svetilo, ki seva enakomerno na vse strani:

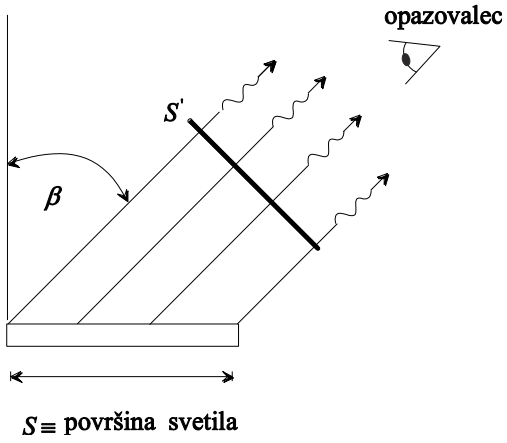
$$\boxed{j = \frac{P}{S} = \frac{P}{4\pi r^2} = \frac{I}{r^2}} \quad , \tag{6.11.5}$$

kjer je svetilnost točkastega svetila:

$$\boxed{I = \frac{P}{4\pi}} \quad . \tag{6.11.6}$$

6.11.4 SVETILNOST IN SVETLOST SVETILA

- **Ploskovno svetilo**



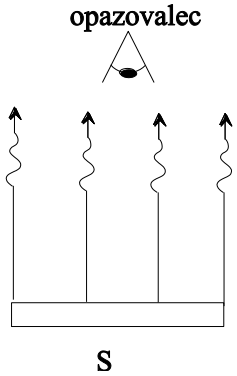
$$\cos\beta = \frac{S'}{S} \Rightarrow S' = S\cos\beta$$

svetilnost je enaka: $\boxed{I = B(\beta)S' = B(\beta)S\cos\beta}$, (6.11.7)

kjer B definiramo kot **svetlost** svetila

poseben primer: opazovanje ploskovnega svetila pod pravim kotom:

$\boxed{I = B(\beta = 0)\, S}$ (6.11.8)



posplošitev:

$\boxed{\mathrm{d}I = B(\beta)\cos\beta\,\mathrm{d}S}$ če svetilo ni ravna ploskev in sevanje ni izotropno (6.11.9)

Lambertov zakon:

$$B(\beta) = B_0 = \text{konst.} \qquad (6.11.10)$$

$$\mathrm{d}I = B_0 \cos\beta \,\mathrm{d}S \qquad (6.11.11)$$

Zaključek:

$$\mathrm{d}j = \frac{\mathrm{d}I}{r^2}$$

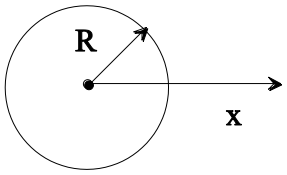
$$\mathrm{d}I = \frac{\mathrm{d}P}{\mathrm{d}\Omega}$$

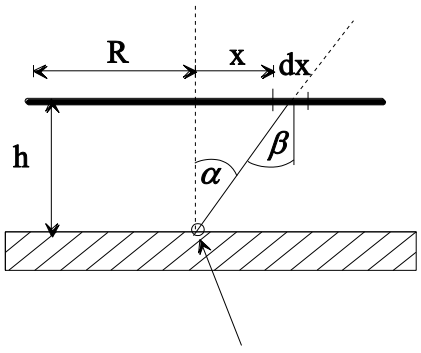
$$\mathrm{d}E = \frac{\mathrm{d}I}{r^2}\cos\alpha$$

$$\mathrm{d}I = B\,\mathrm{d}S\cos\beta$$

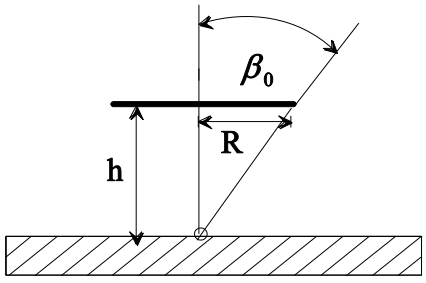
6.11.5 PRIMER: Okroglo ploskovno svetilo s polmerom R seva po Lambertovem zakonu na spodaj ležečo površino.

Tloris: stranski ris:





računamo osvetljenost E v točki



$$\cos\beta = \frac{h}{r} \Rightarrow r = \frac{h}{\cos\beta}$$

$$tg\,\beta = \frac{x}{h}$$

$$\frac{1}{\cos^2\beta}\,\mathrm{d}\beta = \frac{\mathrm{d}x}{h}$$

$$\mathrm{d}x = \frac{h}{\cos^2\beta}\,\mathrm{d}\beta$$

Velja:

$$\mathrm{d}I = B_0 \cos\beta \; 2\pi x \,\mathrm{d}x\,, \tag{6.11.12}$$

$$\mathrm{d}E = \frac{\mathrm{d}I}{r^2} \cos\alpha = \frac{B_0}{r^2} \cos^2\beta \;\; 2\pi x \,\mathrm{d}x \;\; . \tag{6.11.13}$$

S slike razberemo, da je v obravnavanem primeru $\alpha = \beta$. Torej:

$$E = \int \mathrm{d}E = \int \frac{B_o \;\; \cos^4\beta 2\pi h \sin\beta h \mathrm{d}\beta}{\mathrm{h}^2 \cos\beta \cos^2\beta} = \int 2\pi B_0 \sin\beta \cos\beta \mathrm{d}\beta = \pi B_0 \int \sin(2\beta)\,\mathrm{d}\beta =$$

$$= \frac{\pi B_0}{2} \int_0^{2\beta_0} \sin\omega \mathrm{d}\omega = \frac{\pi B_0}{2}(-\cos\omega)\Big|_0^{2\beta} = \frac{\pi B_0}{2}(-\cos 2\beta_0 + 1) = \frac{\pi B_0}{2}(1 - \cos 2\beta_0). \tag{6.11.14}$$

Če je ploščato svetilo zelo veliko je $\beta_0 \;\; \rightarrow \;\; \frac{\pi}{2}$ in iz enačbe (6.11.14) sledi:

$$\boxed{E \;\rightarrow\; \pi B_0} \tag{6.11.15}$$