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Fakultet prirodnih znanosti i tehnologije u Ljubljani

Medical Faculty of Ljubljana
Univ. Clinical Hospital for orthopaedics in Ljubljana
Faculty of Natural Science and Technology of Ljubljana

MATHEMATICAL ANALYSIS OF CHIARI OSTEOTOMY*

MATEMATIČKA ANALIZA OSTEOTOMIJE PO CHIARIJU

A. Iglič, F. Srakar, V. Antolič, V. Kralj Iglič and V. Batagelj

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Key words: biomechanics, locomotor system, Chiari osteotomy

SAŽETAK

Konstruiran je jednostavan statički trodimenzionalan matematički model kuka kako bi se omogućila simulacija pelvične osteotomije po Chiariju. U tom se modelu pretpostavlja da je prosječna napetost mišića u jednoj mišićnoj skupini jednaka, što omogućuje da se u model uključi veći broj mišića nego što ima jednadžbi za sile i moment ravnoteže bez uvjeta optimizacije. Proračunom je utvrđeno da se pomicanjem glavice bedrene kosti medijalno za 1 cm oko 10% smanjuje veličina reaktivne sile kuka. Stoga se zaključuje da pri izvođenju pelvične osteotomije po Chiariju glavicu femura treba maksimalno pomaknuti medijalno.

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SUMMARY

A simple static three-dimensional mathematical model of the hip was constructed in order to simulate Chiari pelvic osteotomy. In this model the average muscle tensions are assumed to be equal in the single muscle group, which enables us to include in the model the number of muscles which exceeds the number of model force and momentum equilibrium equations without any optimization condition. It was calculated that shifting of the femoral head medially for 1 cm reduced the magnitude of the hip joint reaction force by approx 10%. Therefore it was concluded that while performing the Chiari pelvic osteotomy the femoral head must be shifted medially as far as possible.

INTRODUCTION

While Chiari osteotomy (1) is performed an extended acetabular roof is obtained by shifting the femoral head medially (Figs. 1, 2). As a consequence of this shifting a femoral head weight bearing surface area is increased and also the hip joint reaction force is changed. In the presented work a simple static three-dimensional model of the hip was constructed in order to evaluate the magnitude of the hip joint reaction force before and after the Chiari osteotomy.

In the last twenty years many biomechanical models of the hip were constructed in order to calculate hip joint reaction force ($\vec{R}$) and muscle forces in the hip and lower extremities during gait (2, 3, 4, 5, 6, 7) or in static body position (8, 9, 10, 11). These dynamic and static hip models and the old Pauwels model of the hip (8) were used for estimation of the effects of different surgical orthopaedic operations and implantations and also for evaluation of the biomechanical status of the hip with different femoral geometry (4, 8, 12, 13, 14).

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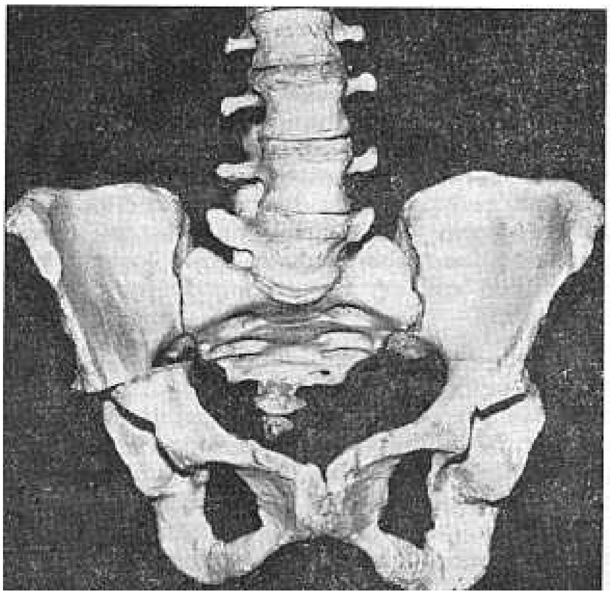
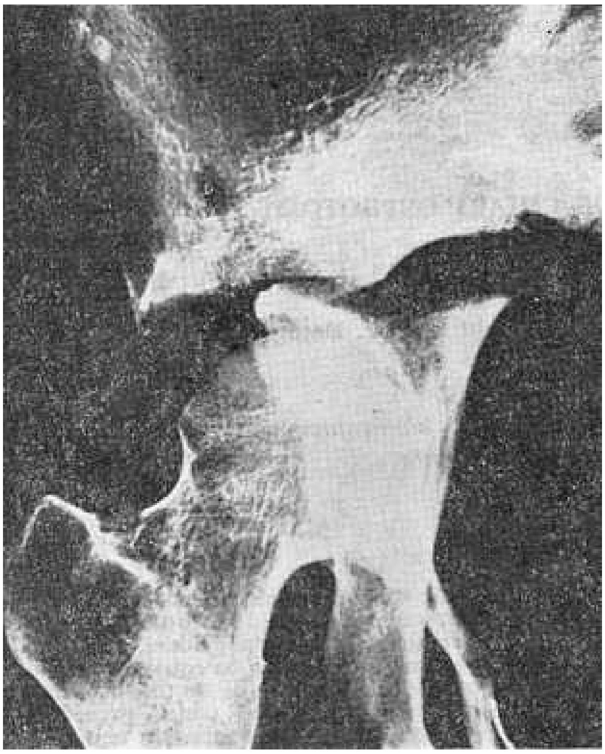


Picture 1.

Representation of the Chiari osteotomy on the model plastic pelvis. The distal part of the right osteotomied pelvis is shifted medially. As a consequence acetabular roof is increased.



Picture 2.
X-ray analysis of the hip after Chiari osteotomy. The cut bone surface ought to be the new enlarged femoral head roof. The new hip geometry is fixed by the screw.

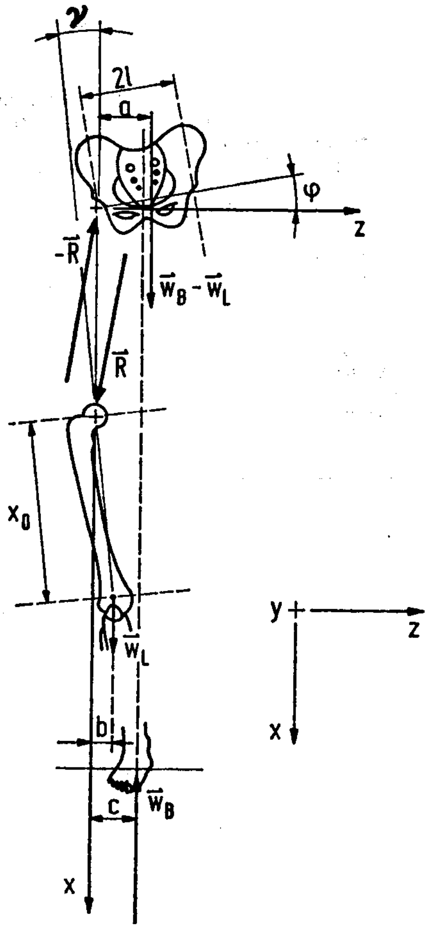
It has been established by direct in vivo measurements with special total hip endoprosthesis (15) that during walking cycle the value of the $\overline{R}$ strongly varies. For this reason many dynamic mathematical models of the hip (2, 3, 4, 5, 6, 7) were constructed in order to simulate the variation of $\overline{R}$ and muscle forces during gait cycle. In these models the number of unknown quantities (i.e. joint and muscle forces) exceeds the number of model equations. Therefore the »optimization technics« had to be used (16, 17) in order to solve model equations. Most of the optimization functions used are physiologicaly badly justified and are chosen merely for practical reasons (3).

In everyday clinical practice the use of dynamic hip models is often too demanding and complicated, therefore static mathematical models as that of Pauwels is fawoured due to their simplicity (8, 12, 13). On the other hand Pauwels hip model seems to be oversimplified because it is two-dimensional and contains practically no direct musculoskeletal anatomical data. Therefore in this work a simple static three-dimensional mathematical model of the hip was constructed in order to simulate Chiari osteotomy, where the hip musculoskeletal anatomical data of Dostal and Andrews (18) and Johnston (14) are taken into account.

METHODS

A simple static three-dimensional mathematical model was used in order to calculate the hip joint reaction force $\overline{R} = (R_x, R_y, R_z)$ in monopodal body position (see Fig. 3) before and after Chiari pelvis osteotomy.

The origin of coordinate system was chosen in the femoral head center (denoted + on Fig. 3) so that x and z axis lie in the frontal plane and x and y axis in the sagital plane of the body. It is assumed that the origin of the hip joint reaction force $\overline{R}$ also lies in the femoral head center. According to the action-reaction force law, the femoral head exerts an opposite force $-\overline{R}$ on the acetabulum. Other forces acting on



Picture 3.
The model of the one-legged stance. Here $\overline{W}_B$ is the total body weight, $\overline{W}_L$ is the weight of the supporting limb, $\overline{R}$ is the hip joint reaction force, 1 is half the distance between the centers of the two acetabula, a is z-coordinate of the force $(\overline{W}_B - \overline{W}_L)$ attachment point, b is z-coordinate of the force $\overline{W}_L$ attachment point, c is z-coordinate of the ground reaction force $-\overline{W}_B$ attachment point and x_0 is the femoral length. The meaning of the angles φ and ν can be seen from figure. Origin of the coordinate systems (x, y, z) coincides with the center of the hip joint (+).

the pelvis are the body weight minus the weight of the loaded leg $(\overline{W}_B - \overline{W}_L)$ and the muscle vector resultant force $\overline{F} = \sum_i \overline{F}_i$ (1)

which is the vector sum of different muscle forces. Individual muscles have one attachment point on the pelvis and the other attachment point on the femur. The attachment of the force $(\overline{W}_B - \overline{W}_L)$ lies at the distance d in the antero-posterior direction from the frontal plane backward, which passes through both femoral head centers. According to Bombelli (12) the value $d = 0$ was used here. The forces acting on the loaded leg are the hip joint reaction force $\overline{R}$, the weight of the loaded leg $\overline{W}_L$ originating above the knee, reaction force exerted by the ground $-\overline{W}_B$ (opposite to the body weight) and the resultant of the muscle forces $-\overline{F}$.

In the applied mathematical model of the hip the muscles piriformis, gluteus medius, gluteus minimus, rectus femoris and tensor fasciae latae are included (see Tab. 1). Since gluteus medius and gluteus minimus are attached to the pelvis over a rather large area, in the presented model each of the two muscles is divided into three parts (18). These segments of gluteus medius and gluteus minimus and the remaining three muscles are clasified in three groups according to their positions: anterior (a), middle (t) and posterior (p) as represented in Table 1.

Table 1.

The muscle icluded in the model are divided into three groups according to their muscle attachment point positions relative to the frontal plane of the body: a (aterior), t (middle) and p (posterior). Here the relative cross- sectional area of the i-th muscle A_i are determined from the data of Johnston et al. (14). The symbol $\overline{F}_i$ denotes the i-th muscle force and f_i the avarage muscle tension in the i-th muscle.

Muscle	Group	i	$\overline{F}_i$	A_i	f_i
gluteus medius-anterior	a	1	$\overline{F}_1$	0,266	f_a
gluteus minimus-anterior	a	2	$\overline{F}_2$	0,113	f_a
tensor fasciae latae	a	3	F_3	0,12	f_a
rectus femoris	a	4	$\overline{F}_4$	0,40	f_a
gluteus medius-middle	t	5	$\overline{F}_5$	0,266	f_t
gluteus minimus-middle	t	6	$\overline{F}_6$	0,113	f_t
gluteus medius-posterior	p	7	$\overline{F}_7$	0,266	f_p
gluteus minimus-posterior	p	8	$\overline{F}_8$	0,113	f_p
piriformis	p	9	$\overline{F}_9$	0,10	f_p

Further it was assumed that any muscle force $\overline{F}_i$ can be approximately written by the following vector expression:

$\overline{F}_i = f_i \cdot A_i \cdot \overline{S}_i \qquad i = 1, .., 9$

where A_i is the relative cross-sectional area of the i-th muscle, f_i is the average tension in the i-th muscle and $\overline{s}_i$ is the unit vector in the direction of the i-th muscle. Unit vector $\overline{s}_i$ is determined by the coordinate vector of the muscle origin point on the pelvis $[\overline{r}_i = (x_i, y_i, z_i)]$ and the corresponding coordinate vector of the muscle insertion on the femur $[\overline{r}_i' = (x_i', y_i', z_i')]$:

$$\overline{s}_i = \frac{\overline{r}_i' - \overline{r}_i}{|\overline{r}_i' - \overline{r}_i|}. \qquad (3)$$

The values of coordinates x_i, y_i, z_i, x_i', y_i', and z_i' for $\varphi = 0$ and $\nu = 0$ (see Fig. 3) are given in Table 2 (18). While using the data for $\overline{r}_i$ and $\overline{r}_i'$, the reference coordinates given in Table 2, must be transformed by the corresponding rotational matrix for given values of angles φ and ν.

Table 2.

Coordinates of the muscle pelvis (x_i, y_i, z_i) and femoral (x_i', y_i', z_i') attachment points (cm), where $x_0 = 42{,}3$ cm, $l_{REF} = 8{,}45$ cm, $\varphi = 0$ and $\nu = 0$ (18).

i	x_i	y_i	z_i	x_i'	y_i'	z_i'
1	–10,2	–2,7	–6,2	2,6	1,8	–7,3
2	– 7,3	–2,9	–4,1	2,7	–0,4	–6,9
3	– 7,8	–4,5	–5,6	43,6	–2,2	–3,3
4	– 3,7	–4,3	–2,6	41,5	–4,3	–0,2
5	–13,2	0,2	–1,8	2,6	1,8	–7,3
6	– 8,8	0,4	–2,0	2,7	–0,4	–6,9
7	– 9,7	4,8	1,5	2,6	1,8	–7,3
8	– 7,1	2,6	0,0	2,7	–0,4	–6,9
9	– 5,5	7,8	4,7	0,1	0,1	–5,5

In order to evaluate the hip joint reaction force $\overline{R}$ and muscle resultant $\overline{F}$, the statistic force and momentum equilibrium equations must be solved,

$$\sum_i \overline{F}_i - \overline{R} + (\overline{W}_B - \overline{W}_L) = 0. \qquad (4)$$

$$\sum_i \overline{r}_i \times \overline{F}_i + \overline{a} \times (\overline{W}_B - \overline{W}_L) = 0, \qquad (5)$$

where

$\overline{a} = (0, 0, a)$, and where the distance a (see Fig. 3) is defined by the expression (19)

$$a = \frac{W_B \cdot c - W_L \cdot b}{W_B - W_L}, \qquad (6)$$

where the meaning of the parameter c is defined in Figure 3. The values of b and c were determined according to McLeish and Charnley (19) as a function of the half the distance between the centers of the two acetabula 1:

$$b = 0.48 \cdot 1$$

$$c = 1.01 \cdot 1 \quad (7)$$

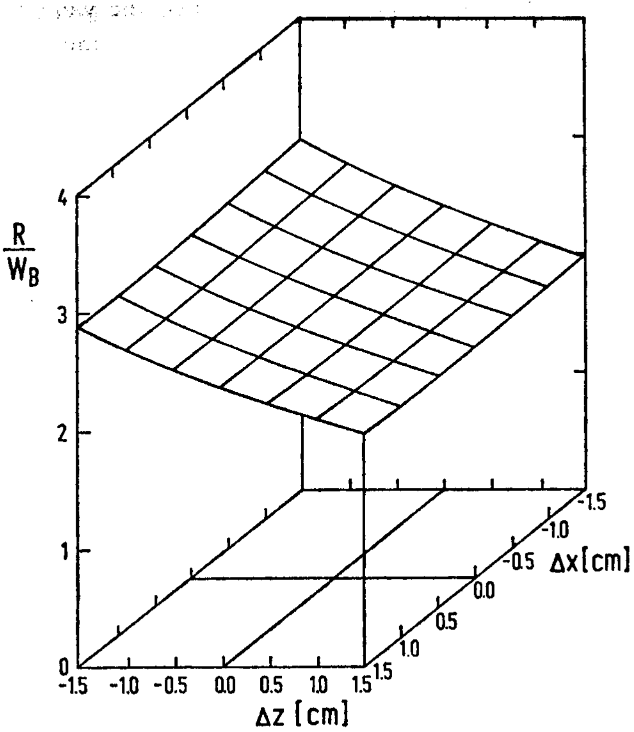
where the value of the angle $\varphi = -0.5^0$. The angle ν is determined by the equation (see Fig. 3):

$$\sin \nu = b/x_0. \quad (8)$$

When determinig the value of W_L, experimentally obtained approximative equation $W_L = 0{,}161 \cdot W_B$ is used (20).

In the presented model it is assumed that the average muscle tensions of individual muscles are equal in the single muscle group (see Table 1). This presumption enables us to include in the model the number of muscles which exceeds the number of model force and momentum equilibrium equations without any optimization method.

The variation of femoral head center position ($x\Delta$, Δz) was simulated by changing the inter-hip distance $1 \longrightarrow 1_{ref} - \Delta z$ ($1_{ref} = 8.45$ cm) and by changing the coordinate of all muscle attachment points included in the represented model. The value of the angle φ was kept constant during the variation of pelvis configuration.



Picture 4.

Calculated magnitude of the hip joint reaction force with respect to the body weight (R/W_B) as a function of the femoral head shifts in medio-lateral direction (Δz) and superior-inferior direction (Δx). Parameters used in these calculations are $\varphi = -0.5^0$(19), $x_0 = 42.3$ cm and $1_{REF} = 8.45$ cm (18). The reference values of the muscle attachment point coordinates for the given of φ and calculated value of ν (see eq. 8) are determined by the rotational transformation of $\bar{r}$ and $\bar{r}'$ values given in Table 2.

RESULTS

Figure 4 shows the dependence of magnitude of the hip joint reaction force on the femoral head shifts in the medio-lateral and superior-inferior directions, where the femoral head shifts are presumed to be the consequence of the Chiari osteotomy. It can be seen from Figure 4 that the magnitude of the hip joint reaction force (and therefore also the corresponding pressure on the femoral head) decreases as the femoral head is shifted medially, while it increases as the femoral head is shifted laterally in accordance with the results of Johnston et al. (14). On the other hand it can be seen from Figure 4 that femoral head shifts in superior-inferior directions have nearly no influence on the magnitude of the hip joint reaction force.

CONCLUSIONS

It is shown that the simulted femoral head shifts in medio-letaral directions have significant effect on the hip joint reaction force and therefore also on the corresponding pressure on the femoral head bearing area. It was calculated that the shifting of the femoral head medially for 1 cm reduces the magnitude of the hip joint reaction force by approx 10%. It seems that long lasting overexertion of the hip joint reaction force causes development of osteoarthritis. Since medialization of femoral head as a consequence of Chiari osteotomy causes a decrease of the magnitude of the hip joint reaction force, this appears to be favourable. We suggest that reduction of the hip joint reaction force after medialization at Chiari osteotomy causes supplementary protection from cartilage degeneration in addition to the primary effect of the increased weight bearing surface area after medialization. This is especially important when a hip with an incipient coxarthrosis is considered. Therefore it could be concluded that while performing the Chiari pelvic osteotomy, the femoral head must be shifted medially as far as possible in order to reduce the hip joint reaction force and the corresponding pressure on the femoral head.

Symbols

$\overline{R}$ hip joint reaction force

R magnitude of the hip joint reaction force

R_j j-th component of the hip joint reaction force, j = x, y, z

$\overline{W_L}$ weight of the supporting limb

$\overline{W_B}$ total body weight

$\overline{F_i}$ i-th muscle force, i = 1, ..., 9

F_i magnitude of the i-th muscle force

A_i relative cross-sectional area of the it-th muscle force

f_i average muscle tension of the i-th muscle force

$\overline{s_i}$ unit vector in direction of the i-th muscle

$\overline{r_i}$ $= (x_i, y_i, z_i)$, position vector of the i-th muscle pelvis attachment point

$\overline{r_i}'$ $= (x_i', y_i', z_i')$, position vector of the i-th muscle femoral attachment point

$\overline{F}$ vector sum of the muscle forces acting on the pelvis

l half the distance between the centers of the two acetabula

a z-coordinate of the force $(\overline{W_B} - \overline{W_L})$ attachment point

b z-coordinate of the force $\overline{W_L}$ attachment point

c z-coordinate of the ground reaction force — W_B attachment point

x_0 femoral length

Δx, Δz shifts of the femoral head in superior--inferior and medio-lateral directions

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