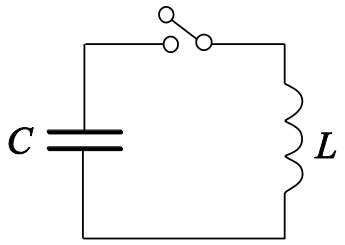
Električni nihajni krogi

IDEALNI ELEKTRIČNI NIHAJNI KROG (*R* = 0)

$$U_C(t=0)=U_0$$



$$U_L+U_C=0$$

$$-L\frac{\mathrm{d}I}{\mathrm{d}t}-\frac{e}{C}=0 \quad / \frac{\mathrm{d}}{\mathrm{d}t}$$

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2}=-\frac{1}{LC}I$$

nastavek za rešitev: $I=I_0\sin(\omega_0 t)$

$$\frac{dI}{dt}=I_0\,\omega_0\,\cos(\omega_0 t)$$

$$\frac{d^2 I}{dt^2}=-I_0\,\omega_0^2\,\sin(\omega_0 t)=-\omega_0^2\,I$$

sledi:

$$\omega_0^2=\frac{1}{LC} \Rightarrow \omega_0=\frac{1}{\sqrt{LC}}$$

$$U_L=-L\frac{dI}{dt}=-L\,I_0\,\omega_0\cos(\omega_0 t)$$

$$U_C=-U_L=L\,I_0\,\omega_0\cos(\omega_0 t)$$

$$U_0=L\,I_0\,\omega_0$$

Skupna energija idealnega električnega nihajnega kroga

$$W=\frac{1}{2}LI^2+\frac{1}{2}CU_C^2=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}CU_0^2\cos^2(\omega_0 t)=$$

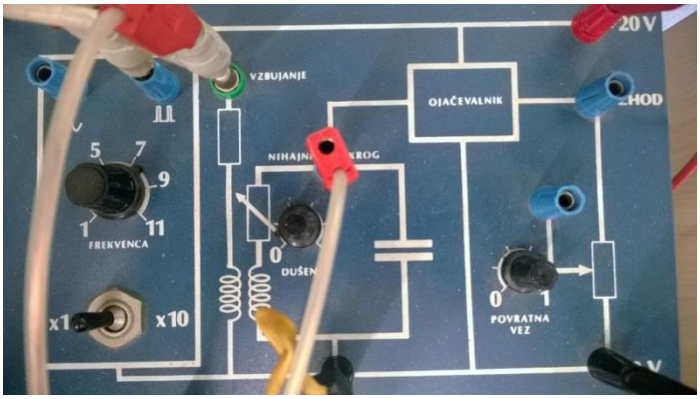
$$=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}CL^2I_0^2\omega_0^2\cos^2(\omega_0 t)=\frac{1}{2}LI_0^2\sin^2(\omega_0 t)+\frac{1}{2}LI_0^2\cos^2(\omega_0 t)=$$

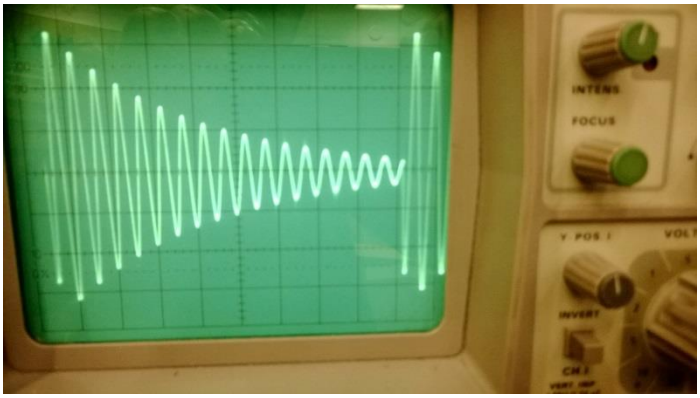
$$=\frac{1}{2}LI_0^2\left\{\sin^2(\omega_0 t)+\cos^2(\omega_0 t)\right\}=\frac{1}{2}LI_0^2=\frac{1}{2}CU_0^2,$$

kjer smo upoštevali: $I_0=\dfrac{U_0}{L\omega_0}$ in $\omega_0^2=\dfrac{1}{LC}$.

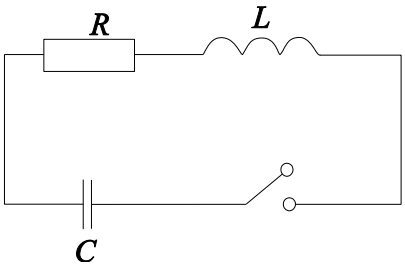
Zaključek: skupna energija idealnega nihajnega kroga je konstantna.

Poskus: DUŠENI ELEKTRIČNI NIHAJNI KROG (osciloskop)





DUŠENI ELEKTRIČNI NIHAJNI KROG (teorija)



$$U_L + U_R + U_C = 0\,,$$

$$-L\frac{\mathrm{d}I}{\mathrm{d}t} - RI - \frac{e}{C} = 0 \quad \Big/ \frac{\mathrm{d}}{\mathrm{d}t}$$

$$-L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{I}{C} = 0 \qquad I = \frac{\mathrm{d}e}{\mathrm{d}t}.$$

Nastavek za rešitev:

$$I = I_0 e^{-\beta t}\cos\left(\omega_0^{'} t + \delta\right)$$

$\omega_0^{'}$ = krožna frekvenca

ali v kompleksnem:

$$I = I_0\, e^{-\beta t}\, e^{i\omega_0^{'} t} = I_0\, e^{\left(i\omega_0^{'} - \beta\right)t}$$

torej:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = I_0\left(i\,\omega_0^{'} - \beta\right) e^{\left(i\,\omega_0^{'} - \beta\right)t}\,,$$

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = I_0\left(i\omega_0^{'} - \beta\right)^2 e^{\left(i\,\omega_0^{'} - \beta\right)t} = \left(-\omega_0^{'2} - 2\beta\,\omega_0^{'}\, i + \beta^2\right) I_0\, e^{\left(i\,\omega_0^{'} - \beta\right)t}\,.$$

Izraze vstavimo v diferencialno enačbo $-L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{I}{C} = 0$:

$$I_0\,\beta^2 e^{-\beta t} e^{\,i\,\omega_0^{'} t} - I_0\, e^{-\beta t}\omega_0^{'2} e^{\,i\,\omega_0^{'} t} - 2I_0\,\beta i\;\omega_0^{'} e^{-\beta t} e^{\,i\,\omega_0^{'} t} +$$

$$+\frac{R}{L}\left(-I_0\beta e^{-\beta t} e^{i\,\omega_0^{'} t} + I_0 e^{-\beta t} i\omega_0^{'} e^{i\,\omega_0^{'} t}\right) + \frac{1}{LC} I_0 e^{-\beta t} e^{i\,\omega_0^{'} t} = 0$$

$$I_0\,\beta^2 e^{-\beta t} e^{\;i\,\omega_0^{'}\,t} - I_0\,e^{-\beta t}\omega_0^{'2} e^{\;i\,\omega_0^{'}\,t} - 2I_0\,\beta i\;\omega_0^{'} e^{-\beta t} e^{\;i\,\omega_0^{'}\,t} \;+$$

$$+\;\frac{R}{L}\left(-I_0\beta e^{-\beta t} e^{i\,\omega_0^{'}\,t} + I_0 e^{-\beta t} i\omega_0^{'} e^{i\,\omega_0^{'}\,t}\right) + \frac{1}{LC} I_0 e^{-\beta t} e^{i\,\omega_0^{'}\,t} \;=\; 0$$

sledi:

$$I_0\,e^{\left(i\,\omega_0^{'}-\beta\right)t}\left[\beta^2 - \omega_0^{'\,2} - \beta\frac{R}{L} + i\,\omega_0^{'}\left\{-2\beta + \frac{R}{L}\right\} + \frac{1}{LC}\right] = 0\,,$$

$$\left[\beta^2 - \omega_0^{'\,2} - \beta\frac{R}{L} + \omega_0^2 + i\,\omega_0^{'}\left\{-2\beta + \frac{R}{L}\right\}\right] = 0\,, \qquad \omega_0^2 = \frac{1}{LC}.$$

TOREJ:

Im : $\quad -2\beta + \frac{R}{L} = 0 \qquad \beta \;=\; \frac{1}{2}\frac{R}{L}$

Re : $\quad \beta^2 - \omega_0^{'2} - \beta\frac{R}{L} + \omega_0^2 = 0$

$$\beta^2 - \omega_0^{'2} - 2\beta^2 + \omega_0^2 = 0\;,$$

$$\omega_0^{'} \;=\; \sqrt{\omega_0^2 \;-\; \beta^2}$$

DVE MOŽNOSTI:

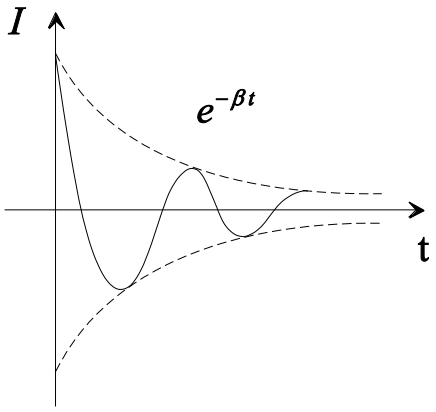
1. $\beta < \omega_0$: $\quad I = I_0\,e^{-\beta t}\cos\left(\omega_0^{'}\,t + \delta\right)$

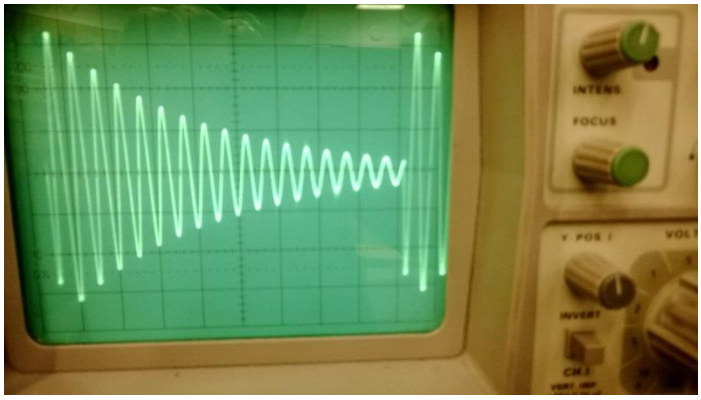
2. $\beta > \omega_0$: $\quad \omega_0^{'} = i\sqrt{\left(\beta^2 - \omega_0^2\right)} = i\,\alpha\,, \alpha = \sqrt{\left(\beta^2 - \omega_0^2\right)}$

$$I = I_0\,e^{-\beta t}\,e^{i\omega_0^{'} t} = I_0\,e^{-\beta t}\,e^{-\alpha t} = I_0\,e^{-(\alpha+\beta)t}$$

Primer 1 ($\beta < \omega_0$): $I = I_0\, e^{-\beta t} \cos\left(\omega_0^{'} t + \delta\right)$

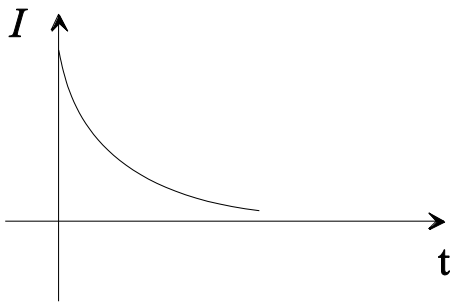
ZMERNO DUŠENJE





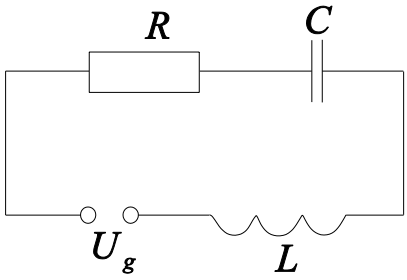
Primer 2 ($\beta > \omega_0$): $I = I_0\, e^{-(\alpha+\beta)t}$

MOČNO DUŠENJE



VSILJENO NIHANJE ELEKTRIČNEGA NIHAJNEGA KROGA

ZAPOREDNA VEZAVA



Napetost generatorja: $U_g = U_0 \cos(\omega t)$

$$U_g + U_C + U_L + U_R = 0$$

$$U_g - \frac{e}{C} - L\frac{\mathrm{d}I}{\mathrm{d}t} - IR = 0 \quad / \frac{\mathrm{d}}{\mathrm{d}t}$$

$$\frac{\mathrm{d}U_g}{\mathrm{d}t} - \frac{I}{C} - L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} = 0 \qquad I = \frac{\mathrm{d}e}{\mathrm{d}t}.$$

člen zaradi lastnega nihanja : $e^{-\beta t}\cos(\omega_0' t + \delta)$ po zadosti dolgem času izgine zato takrat nihajni krog niha le z vsiljeno frekvenco :

$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$

$$I = I_0 e^{i\omega t},$$

$$U_g = U_0 e^{i\omega t}.$$

vstavimo v diferencialno enačbo $\frac{\mathrm{d}U_g}{\mathrm{d}t} - \frac{I}{C} - L\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} - R\frac{\mathrm{d}I}{\mathrm{d}t} = 0$:

$$e^{i\omega t} i\omega U_0 - \frac{I_0}{C} e^{i\omega t} + L\omega^2 I_0 e^{i\omega t} - R\, i\omega I_0 e^{i\omega t} = 0$$

preuredimo **:**

$$\left(i\omega U_0 - \frac{I_0}{C} + L\omega^2 I_0 - R i\omega I_0\right) e^{i\omega t} = 0.$$

torej :

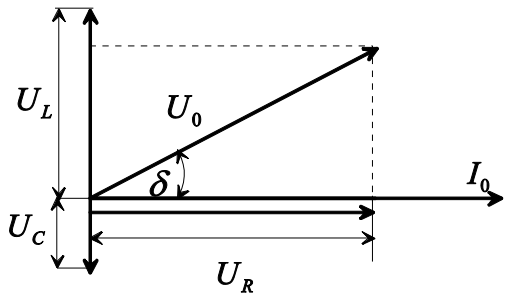
$$i\omega U_0 - \frac{I_0}{C} + L\omega^2 I_0 - R i\omega I_0 = 0 \;,$$

$$i\omega U_0 - \frac{I_0}{C} + L\omega^2 I_0 - Ri\omega I_0 = 0 \ ,$$

$$U_0 = \left(-\frac{\omega L}{i} + R + \frac{1}{i\omega C} \right) I_0 = \left(R + i\omega L - i\frac{1}{\omega C} \right) I_0 \ .$$

$$Z_R = R \ , \qquad Z_L = i\omega L \ , \qquad Z_C = -i\frac{1}{\omega C} \ ,$$

I_0 realno število, U_0 pa kompleksno število



$$tg\,\delta = \frac{\left(\omega L - \frac{1}{\omega C} \right)}{R}$$

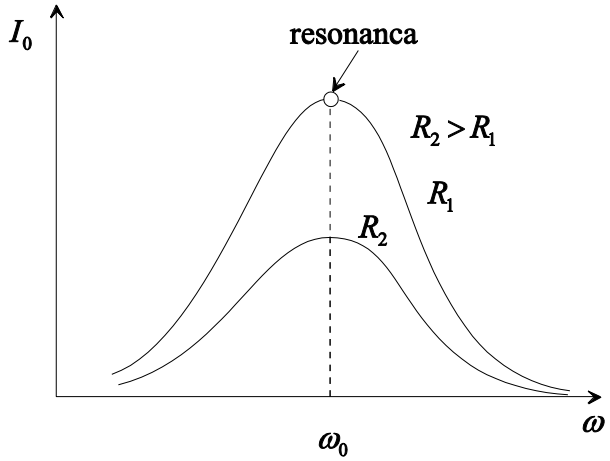
$$|U_0|^2 = U_0 U_0^* = I_0^2 \left[R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right] .$$

$$I_0 = \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}}$$

$$I_0 = \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

I_0 maksimalen, če je:

$$\omega L - \frac{1}{\omega C} = 0 \,, \qquad \omega L = \frac{1}{\omega C} \,, \qquad \omega^2 = \frac{1}{LC} = \omega_0^2 \,,$$



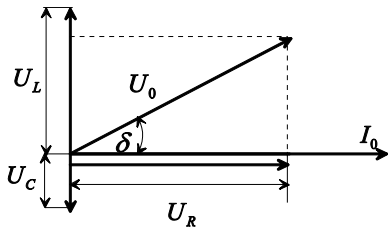
če je $\omega = \omega_0$ velja:

$$tg\,\delta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} = 0 \,, \qquad \text{torej } \delta = 0.$$

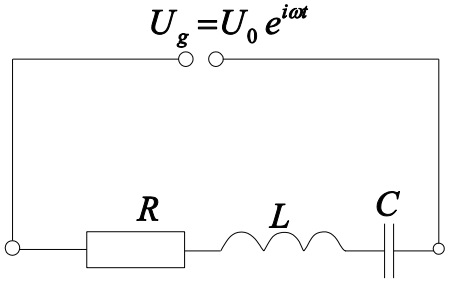
v resonanci torej : $\delta = 0$ in I_0 maksimalen, zato porabljena povprečna moč največja možna :

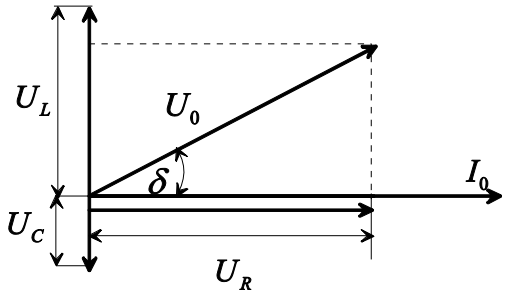
$$\overline{P} = \frac{1}{2} I_0 |U_0| \cos\delta = \frac{1}{2} \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} |U_0| \cos\delta =$$

$$= \frac{1}{2} \frac{|U_0|^2}{R} \cos\delta = \frac{1}{2} \frac{|U_0|^2}{R}$$



Zaključek:



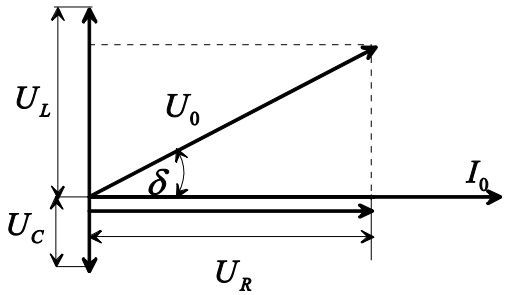


$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$ Vzamemo: I_0 je realen, U_0 je kompleksen

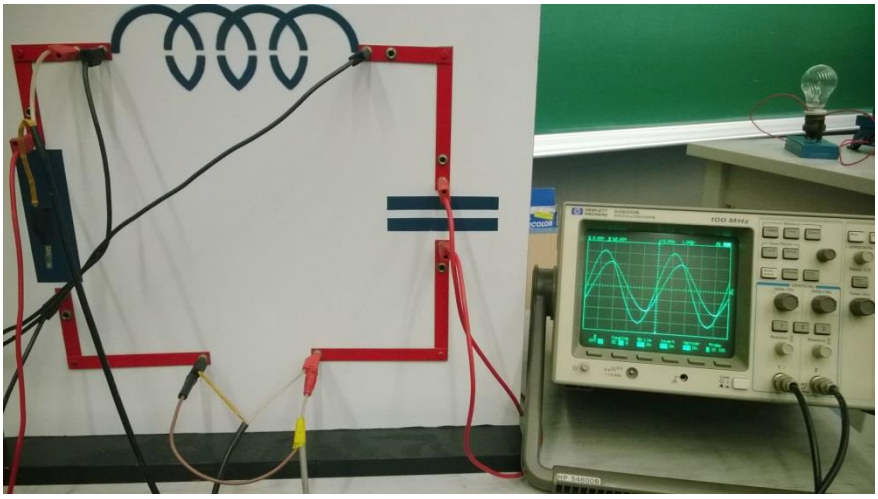
Kompleksne impedance:

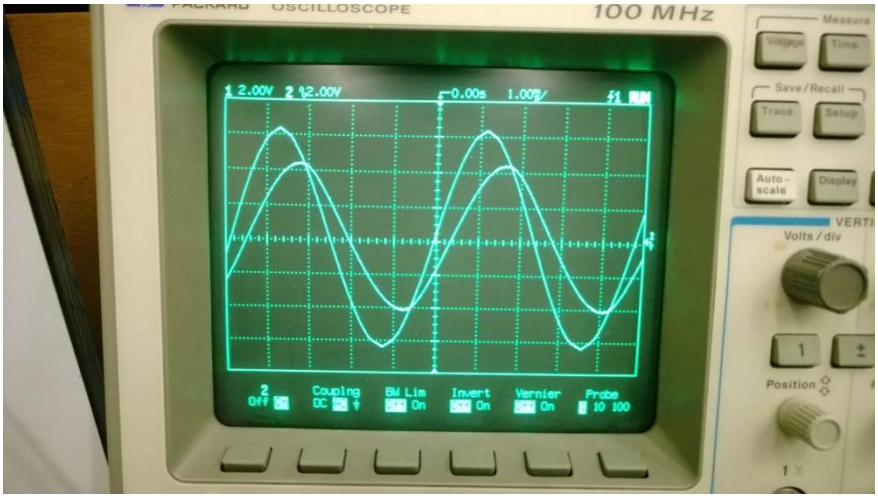
$$Z_R = R$$
$$Z_L = i\omega L$$
$$Z_C = -i\frac{1}{\omega C}$$

$$Z = Z_R + Z_L + Z_C$$
$$U_0 = Z\, I_0$$

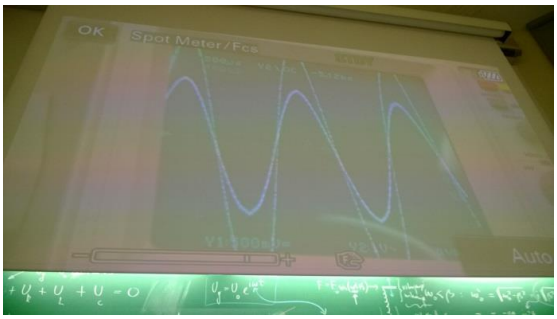


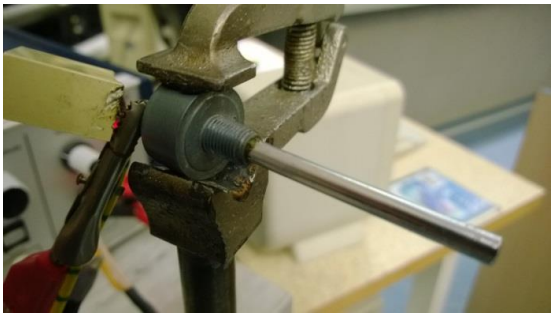
EKSPERIMENT: FAZNI PREMIK MED NAPETOSTJO IN TOKOM PRI R-L-C KROGU (demonstracijska tabla)



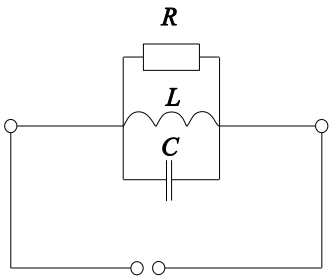


R-L-C NIHAJNI KROG (vsiljeno nihanje - fazni zamik v resonance nič)

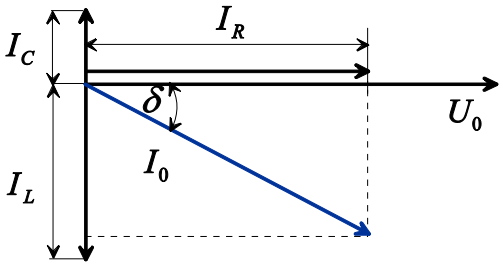




VZPOREDNA VEZAVA



$U_g = U_0\, e^{i\omega t}$



$$U_0 = Z\, I_0$$

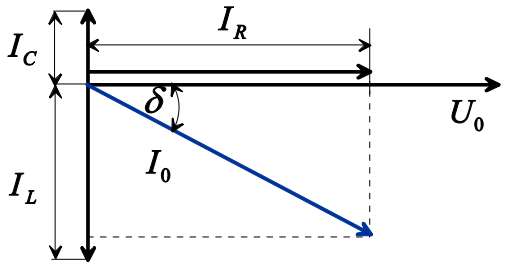
$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} - \frac{\omega C}{i}$$

vzamemo: U_0 je realen, I_0 je kompleksen

$$I_0 = \frac{1}{Z} U_0 = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right) U_0 = U_0 \left[\frac{1}{R} + i \left(\omega C - \frac{1}{\omega L} \right) \right]$$

$$|I_0| = \sqrt{I_0 \cdot I_0^*} = U_0 \left[\left(\frac{1}{R} \right)^2 + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{1/2}$$

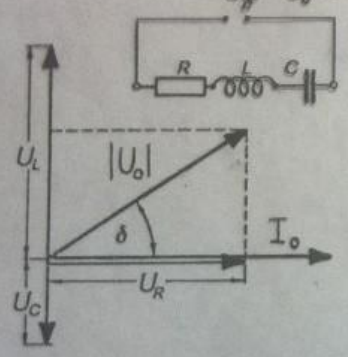
$$tg\,\delta = \frac{\mathrm{Im}(I_0)}{\mathrm{Re}(I_0)} = \left(\omega C - \frac{1}{\omega L} \right) R$$



VSILJENO NIHANJE NIHAJNEGA KROGA

(A)

$U_g = U_0 e^{i\omega t}$



KOMPL. IMPENDANCE :

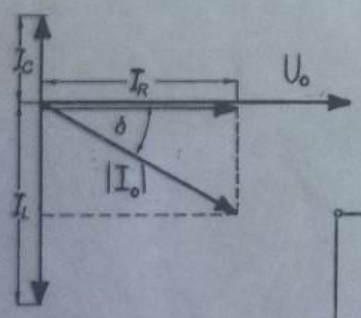
$$\boxed{\begin{aligned} Z_R &= R \\ Z_L &= i\omega L \\ Z_C &= -i\frac{1}{\omega C} \end{aligned}}$$

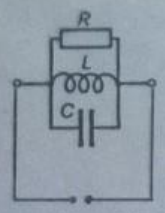
$$\boxed{\begin{aligned} Z &= Z_R + Z_L + Z_C \\ U_0 &= Z\, I_0 \end{aligned}}$$

$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$

$\left.\begin{array}{l} I \text{ je realen} \\ U_0 \text{ je kompleksen} \end{array}\right\}$ vzamemo

(B)





$U_g = U_0 e^{i\omega t}$

$$\boxed{U_0 = Z\, I_0}$$

$$\boxed{\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} - \frac{\omega C}{i}}$$

$\left.\begin{array}{l} U_0 \text{ je realen} \\ I_0 \text{ je kompleksen} \end{array}\right\}$ vzamemo

$$I_0 = \frac{1}{Z} U_0 = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C\right) U_0 = U_0\left[\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)\right]$$

$$|I_0| = \sqrt{I_0 \cdot I_0^*} = U_0\left[\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}$$

$$\boxed{tg\,\delta = \frac{Im(I_0)}{Re(I_0)} = \left(\omega C - \frac{1}{\omega L}\right)\cdot R}$$

EM valovanje in svetloba

ELEKTROMAGNETNO VALOVANJE

VALOVNA ENAČBA

Heinrich Rudolf Hertz
(1857 – 1894)

- **Faradayev zakon** (ind. zakon)**:**

$$\oint \vec{E}\cdot \mathrm{d}\vec{s} = -\frac{\partial}{\partial t}\int \vec{B}\cdot \mathrm{d}\vec{S} \quad \Rightarrow \quad \boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},} \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

- **Amperov zakon:**

$$\int \vec{H}\cdot \mathrm{d}\vec{s} = \int \vec{j}_e \mathrm{d}\vec{S} + \int \frac{\partial \vec{D}}{\partial t}\mathrm{d}\vec{S} \quad \Rightarrow \quad \boxed{\vec{\nabla}\times\vec{H} = \vec{j}_e + \frac{\partial \vec{D}}{\partial t}}$$

če $j_e = 0$ (ni električnih tokov) : $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad \vec{\nabla}\times\vec{H} = \frac{\partial \vec{D}}{\partial t}$

vakuum: $\varepsilon = 1, \quad \mu = 1$

$$\vec{B} = \mu_0 \vec{H}$$
$$\vec{D} = \varepsilon_0 \vec{E}$$

torej:

$$\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$
$$\vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

poseben primer: **raven** elektromagnetni val, $\vec{E}$ in $\vec{H}$ odvisna samo od koordinate x, $H_x = E_x = 0$:

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(0, -\frac{\partial E_z}{\partial x}, \overset{a}{\overset{\downarrow}{\frac{\partial E_y}{\partial x}}} \right) = -\mu_0 \left(0, \frac{\partial H_y}{\partial t}, \overset{a}{\overset{\downarrow}{\frac{\partial H_z}{\partial t}}} \right) \qquad (1a)$$

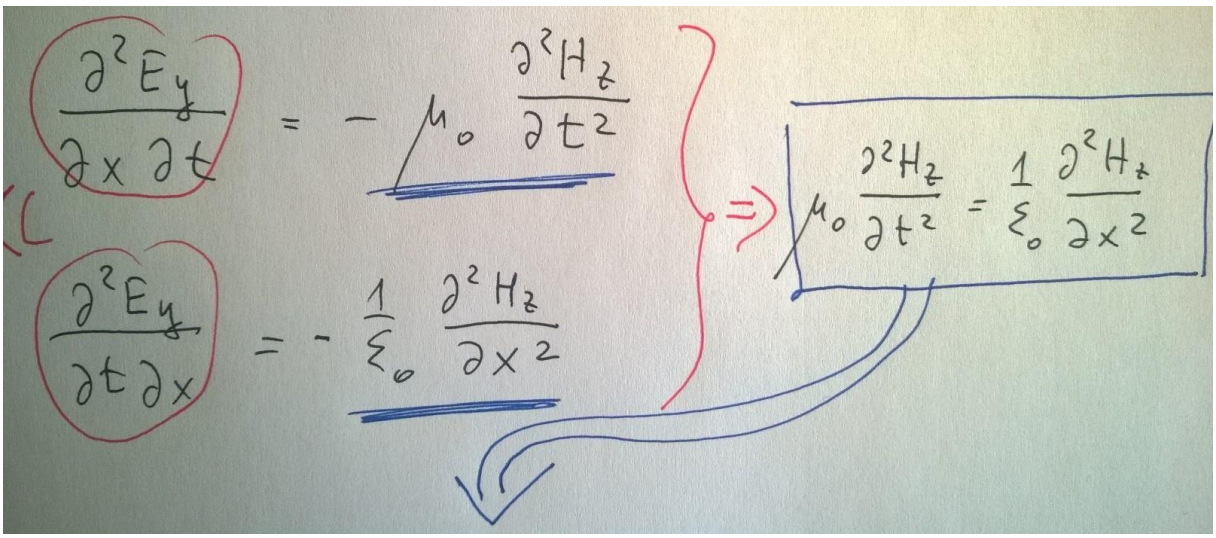
$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(0, \overset{b}{\overset{\downarrow}{-\frac{\partial H_z}{\partial x}}}, \frac{\partial H_y}{\partial x} \right) = \varepsilon_0 \left(0, \overset{b}{\overset{\downarrow}{\frac{\partial E_y}{\partial t}}}, \frac{\partial E_z}{\partial t} \right) \qquad (2a)$$

Iz enačb (1a) in (2a) po **parcialnem** odvajanju po *t* in *x* sledi:

$$(1a) \Rightarrow \frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \quad / \frac{\partial}{\partial t} \quad \Rightarrow \frac{\partial^2 E_y}{\partial x \partial t} = -\mu_0 \frac{\partial^2 H_z}{\partial t^2} \qquad (1b)$$

$$(2a) \Rightarrow -\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t} \quad / \frac{\partial}{\partial x} \quad \Rightarrow -\frac{1}{\varepsilon_0}\frac{\partial^2 H_z}{\partial x^2} = \frac{\partial^2 E_y}{\partial t \partial x} \qquad (2b)$$

torej:



$+\mu_0 \frac{\partial^2 H_z}{\partial t^2} = +\frac{1}{\varepsilon_0}\frac{\partial^2 H_z}{\partial x^2}$, oziroma:

VALOVNA ENAČBA za jakost mag. polja **H**

$$\frac{\partial^2 H_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 H_z}{\partial x^2}$$

Prav tako iz enačb (1a) in (2a)

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(0, -\frac{\partial E_z}{\partial x}, \overset{a}{\downarrow} \frac{\partial E_y}{\partial x} \right) = -\mu_0 \left(0, \frac{\partial H_y}{\partial t}, \overset{a}{\downarrow} \frac{\partial H_z}{\partial t} \right) \qquad (1a)$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(0, \overset{b}{\downarrow} -\frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} \right) = \varepsilon_0 \left(0, \overset{b}{\downarrow} \frac{\partial E_y}{\partial t}, \frac{\partial E_z}{\partial t} \right) \qquad (2a)$$

po parcialnem odvajanju po x in t (v obratnem redu kot prej torej) tudi sledi:

$$(1a) \Rightarrow \frac{\partial E_y}{\partial x} = -\mu_0 \frac{\partial H_z}{\partial t} \qquad / \frac{\partial}{\partial x} \qquad \Rightarrow \frac{1}{\mu_0} \frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 H_z}{\partial t \, \partial x}$$

$$(2a) \Rightarrow -\frac{\partial H_z}{\partial x} = \varepsilon_0 \frac{\partial E_y}{\partial t} \qquad / \frac{\partial}{\partial t} \qquad \Rightarrow -\frac{\partial^2 H_z}{\partial t \, \partial x} = \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

torej:

$$-\frac{1}{\mu_0} \frac{\partial^2 E_y}{\partial x^2} = -\varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}, \quad \text{oziroma:}$$

VALOVNA ENAČBA za jakost električnega polja **E** :

$$\frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 E_y}{\partial x^2} = \frac{\partial^2 E_y}{\partial t^2}$$

Reševanje **VALOVNE ENAČBE** za **E**

$$\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial^2 E_y}{\partial t^2}$$

nastavek za rešitev : potujoče valovanje v 1-D:

$$E_y = E_{y0}\cos\left(\omega\left(t-\frac{x}{c_0}\right)\right) = E_{y0}\cos(\omega t - kx),$$

kjer je valovni vektor $k=\frac{\omega}{c_0}$ in c_0 hitrost širjenja EM valovanja.

nastavek za rešitev odvajamo 2 krat parcialno po času t :

$$\frac{\partial^2 E_y}{\partial t^2} = -\ \omega^2 E_{y0}\cos(\omega t - kx),$$

in nato 2 krat po koordinati x :

$$\frac{\partial^2 E_y}{\partial x^2} = -k^2 E_{y0}\cos(\omega t - kx),$$

oboje vstavimo v valovno enačbo : $\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 E_y}{\partial x^2} = \frac{\partial^2 E_y}{\partial t^2}$ in dobimo

$$-\omega^2 \cos(\omega t - kx) = \frac{-1}{\varepsilon_0 \mu_0} k^2 \cos(\omega t - kx), \quad \text{oziroma} \quad \omega^2 = \frac{1}{\varepsilon_0 \mu_0} k^2 .$$

ker velja $k=\frac{\omega}{c_0}$ $c_0=\frac{\omega}{k}$ iz zgornje enačbe sledi : $c_0^2 = \frac{1}{\varepsilon_0\ \mu_0}$

influenčna konstanta: $\varepsilon_0 = 8.85\cdot 10^{-12}$ As/Vm

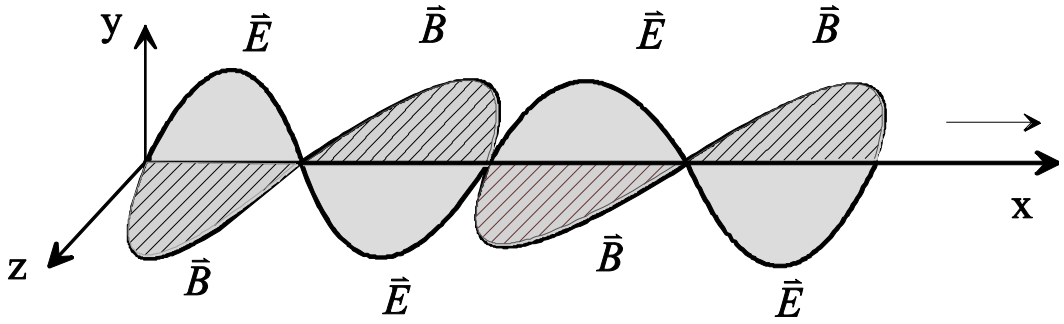
indukcijska konstanta: $\mu_0 = 4\pi\cdot 10^{-7}$ Vs/Am

$$\Big\} \Rightarrow c_0 = \frac{1}{\sqrt{\varepsilon_0\ \mu_0}} \cong 3\cdot 10^8\ \frac{\text{m}}{\text{s}}$$

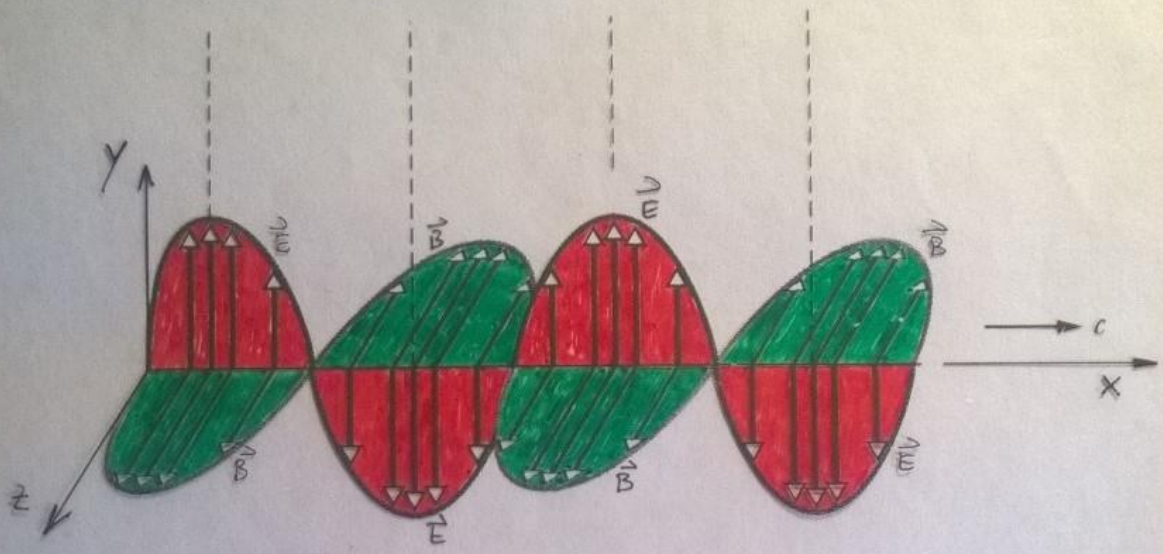
Podobno naredimo za ***B*** iz valovne enačbe za ***B*:**

$$\frac{\partial^2 H_z}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 H_z}{\partial x^2}$$

Rešitev : raven elektromagnetni potujoči EM val :



ELEKTROMAGNETNO VALOVANJE



p. primer: raven elektromagnetni val

$\Downarrow$

Maxwellove enačbe:

$$\left.\begin{aligned} \oint \vec{E}\cdot d\vec{s} &= -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S} \\ \oint \vec{H}\cdot d\vec{s} &= \frac{\partial}{\partial t}\int_S \vec{D}\cdot d\vec{S} \end{aligned}\right\} \Rightarrow \begin{aligned} \frac{\partial^2 E}{\partial t^2} &= \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E}{\partial x^2} \\ \frac{\partial^2 H}{\partial t^2} &= \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 H}{\partial x^2} \end{aligned}$$

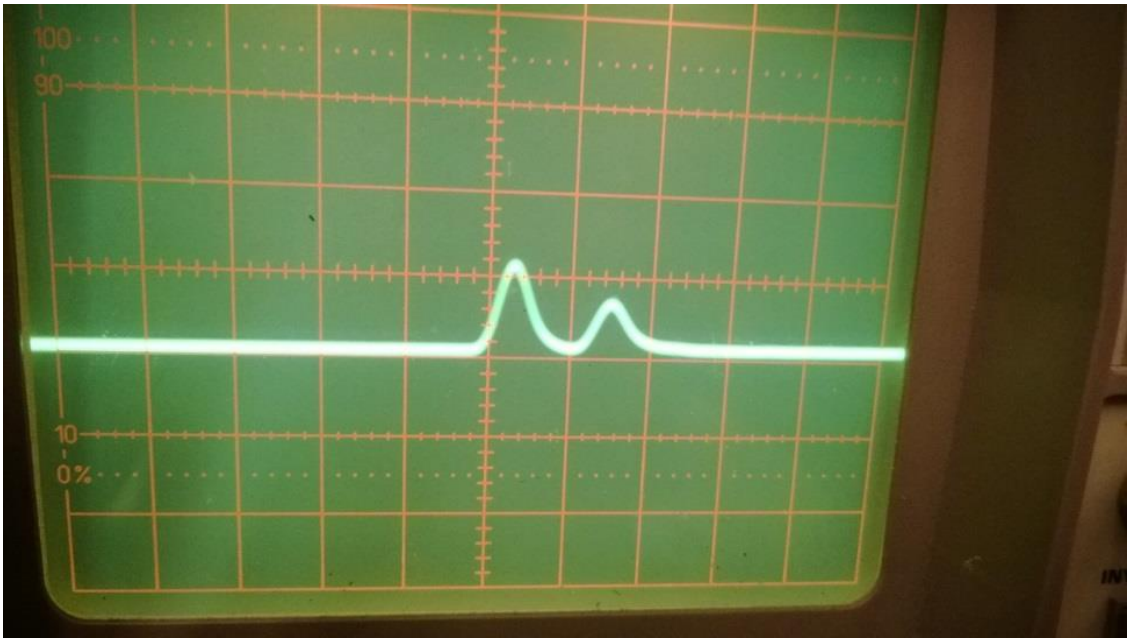
valovna enačba za E in H (1-D)

influenčna konst.: $\varepsilon_0 = 8.85\cdot 10^{-12}\ As/Vm$

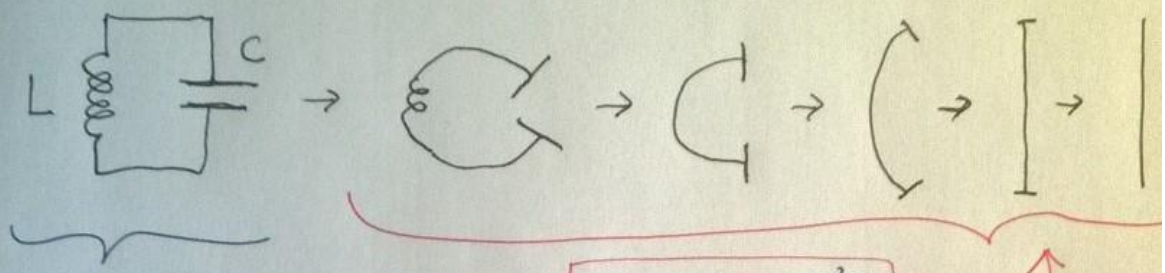
induksijska konst.: $\mu_0 = 4\pi\cdot 10^{-7}\ Vs/Am$

$$\Rightarrow c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \approx 3\cdot 10^8\ \frac{m}{s}$$

SLIKA: MERITEV SVETLOBNE HITROSTI V ZRAKU - merilni sistem v predavalnici na Jadranski



"TRANSFORMACIJA" elektr. nihajnega kroga v dipolno anteno



idealni nihajni krog: $\boxed{L = \mu\mu_0 \frac{N^2 S_1}{l}}$

$N \equiv$ število ovojev tuljave

$S_2 \equiv$ površina plošče kondenzatorja

$\boxed{C = \varepsilon\varepsilon_0 \frac{S_2}{d}}$

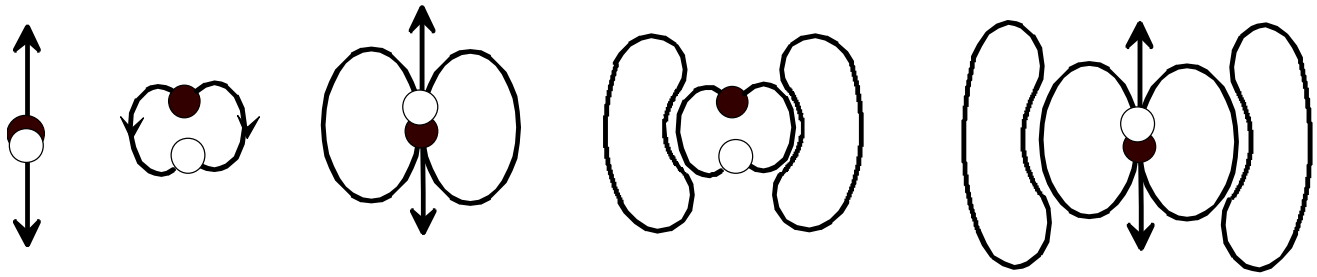
ZAHTEVI:

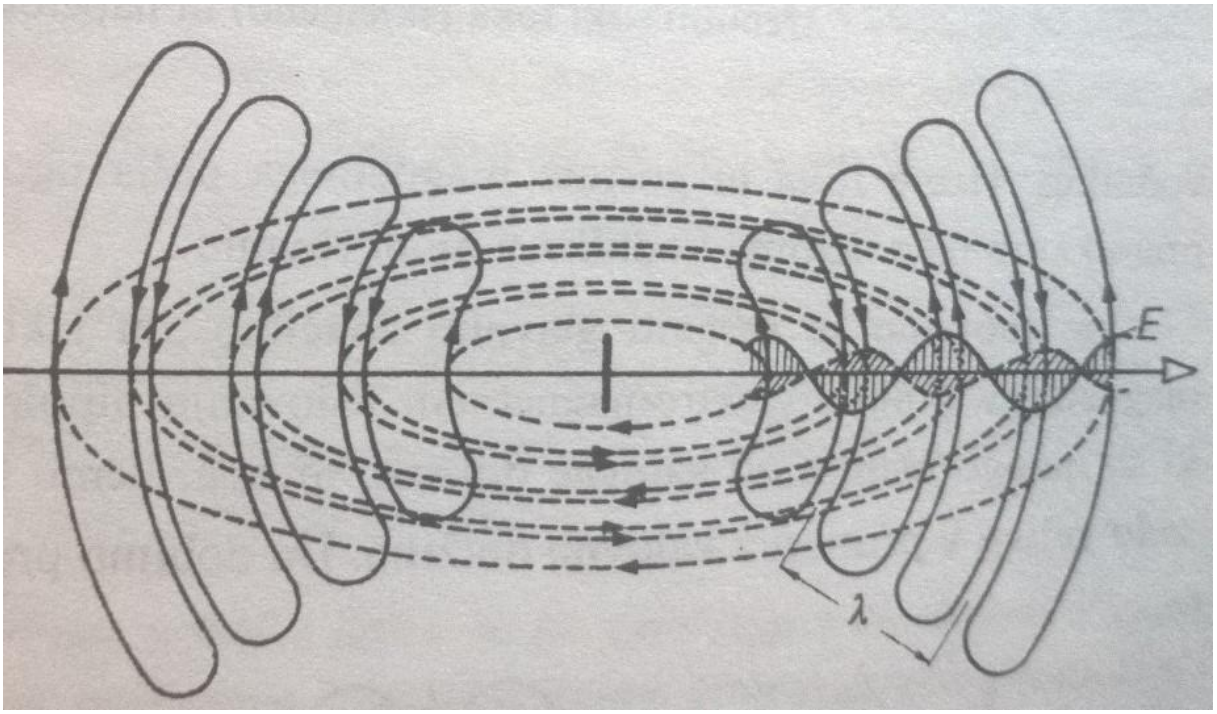
(1) Če hočeš, da gre $\vec{E}$ in $\vec{B}$ tudi izven kondenzatorja in tuljave

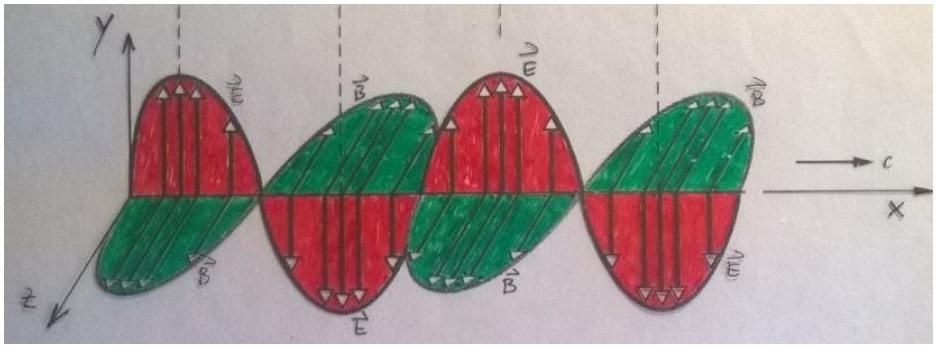
(2) $\omega_0 = \frac{1}{(LC)^{1/2}}$ } Če hočeš, da so lastne frekvence VELIKE, morata biti C in L čim manjša:

- <u>malo</u> ovojev tuljave
- plošči kondenzatorja čim <u>manjši</u>

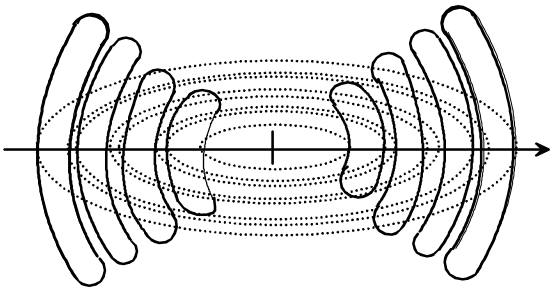
DIPOLNA ANTENA

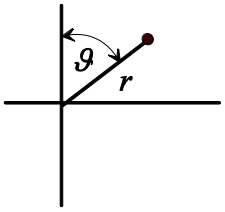






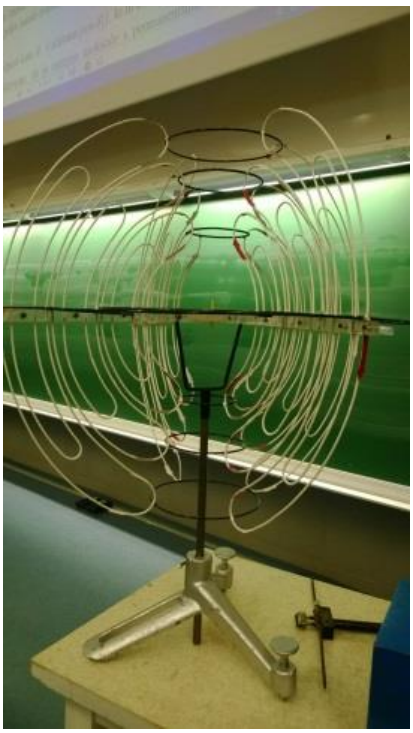
SEVANJE DIPOLA (gostota energijskega toka)





$$\bar{j} = c_0 \bar{w} = \frac{p_0^2 \omega^4}{32 \pi^2 \varepsilon_0 c_0^3} \cdot \frac{\sin^2 \vartheta}{r^2}$$

$$\bar{P} = \int \bar{j} \, \mathrm{d}S = \frac{p_0^2 \omega^4}{12 \pi \varepsilon_0 c_0^3}$$

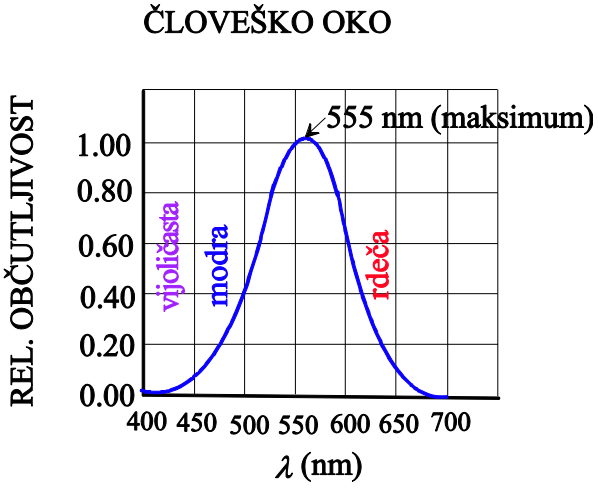


EKSPERIMENT: SEVANJE DIPOLNE ANTENE - **stojno valovanje**
(»otipavanje« električnega in magnetnega polja, dipolna antena kot sprejemnik, zanka z žarnico)

VRSTE ELEKTROMAGNETNEGA VALOVANJA

VRSTA EM VALOVANJA	VALOVNE DOLŽINE (λ)
radijski valovi	od 1 m do več 1000 km
mikrovalovi	od 0.1 mm do ~ 1 m
infra rdeče sevanje	od ~ 780 nm do 0.1 mm
vidna svetloba	od 380 do 780 nm
UV svetloba	od 5 ~ 380 nm
Rtg žarki	od 1 pm ~ 10 nm
γ žarki	od 0 do 1 pm

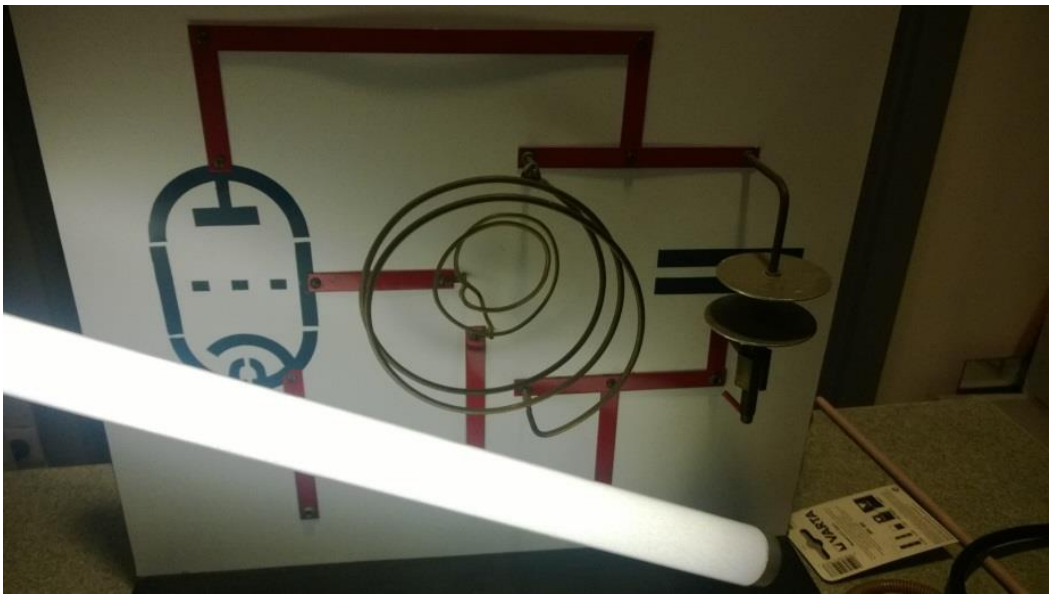
SPEKTER ELEKTROMAGNETNEGA VALOVANJA V VIDNEM OBMOČJU



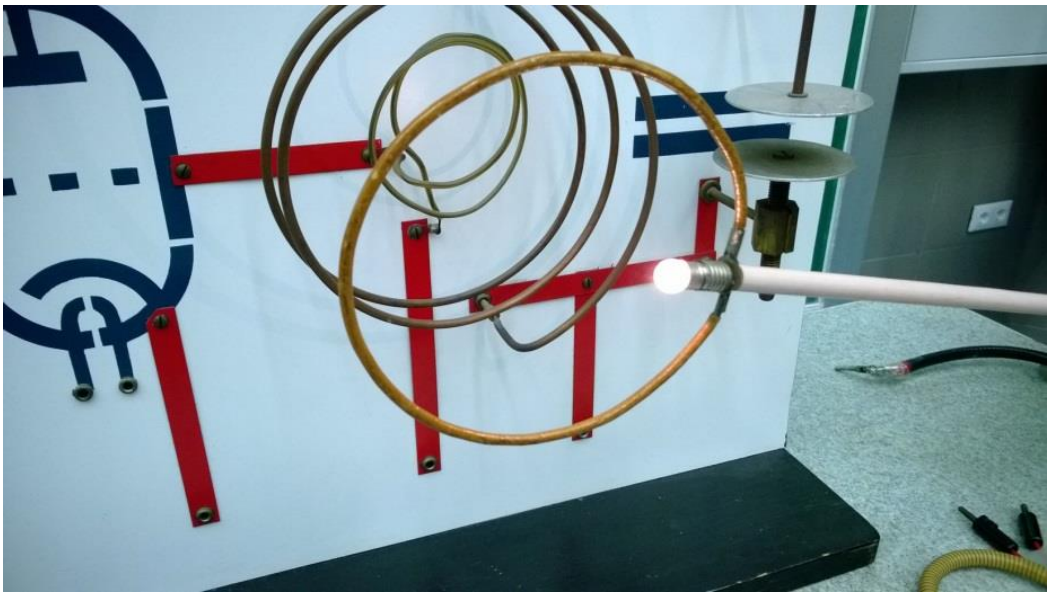
POSKUSI Z VISOKOFREKVENČNIM OSCILATORJEM

SLIKA: uglasitev sprejemnika (nihajni krog z vrtljivim kondenzatorjem)

SLIKA: neonska svetilka kot detektor elektromagnetnega valovanja



SLIKA: zanka z žarnico (na ročaju) kot sprejemnik valovanja



ELEKTROMAGNETNO VALOVANJE V SNOVI – LOMNI KOLIČNIK

$$c = \frac{1}{\sqrt{\varepsilon \mu \varepsilon_0 \mu_0}}$$ = hitrost EM valovanja

Snovi, po katerih se širi EM valovanje, so diamagnetne ali paramagnetne : $\mu \approx 1$, torej:

$$c = \frac{1}{\sqrt{\varepsilon \quad \varepsilon_0 \quad \mu_0}} = \frac{c_0}{n}$$

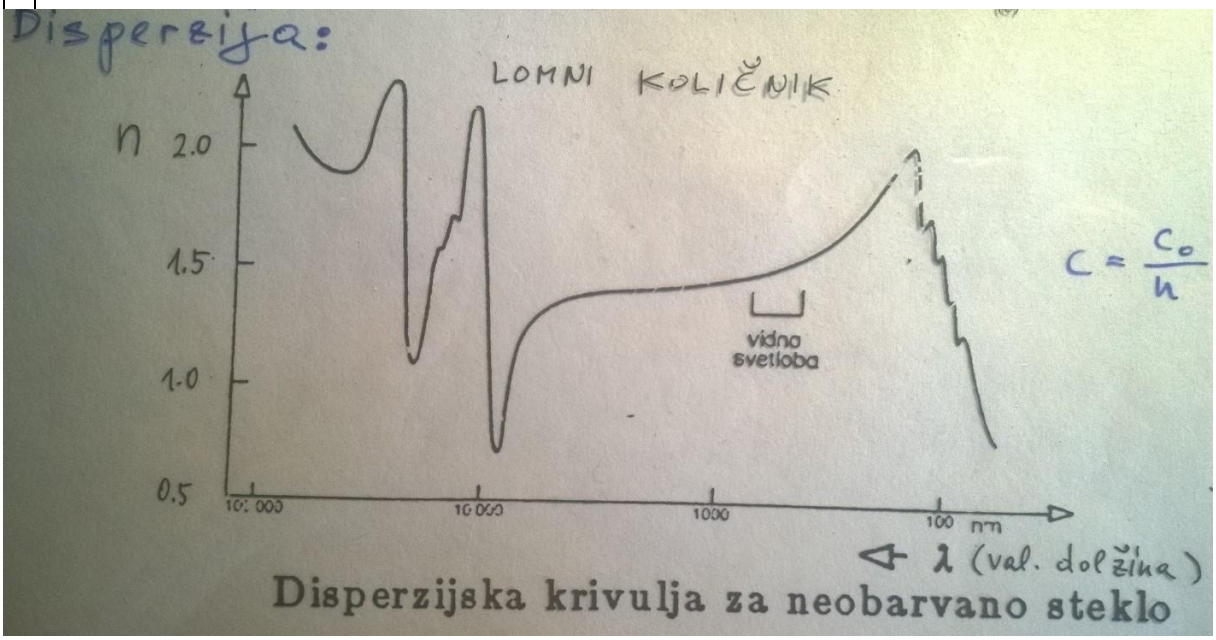
Definiramo lomni kvocient:

$$n = \sqrt{\varepsilon}$$

Opomba: $n = \sqrt{\varepsilon}$ je odvisen od valovne dolžine λ

Iz tega sledi, **da je tudi hitrost EM valovanja** (svetlobe) odvisna od valovne dolžine.

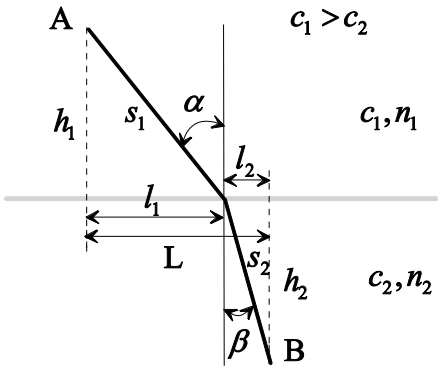
$$c = \frac{c_0}{n(\lambda)}$$



Disperzijska krivulja za neobarvano steklo

LOM: FERMATOV PRINCIP

Zahteva: minimalen čas za pot svetlobe iz točke A v točko B



$$t = \frac{s_1}{c_1} + \frac{s_2}{c_2} \Rightarrow$$

$$t = \frac{h_1}{c_1 \cos\alpha} + \frac{h_2}{c_2 \cos\beta}, \text{ kjer smo upoštevali:}$$

$$\cos\alpha = \frac{h_1}{s_1}, \ \cos\beta = \frac{h_2}{s_2}$$

$$tg\,\alpha = \frac{l_1}{h_1}, \ tg\,\beta = \frac{l_2}{h_2}$$

$$\mathrm{d}t = \frac{\partial t}{\partial \alpha}\mathrm{d}\alpha + \frac{\partial t}{\partial \beta}\mathrm{d}\beta = 0 \quad \text{(pogoj za ekstrem)}$$

$$\mathrm{d}t = \frac{h_1 \cdot \sin\alpha}{c_1 \cdot \cos^2\alpha}\mathrm{d}\alpha + \frac{h_2 \cdot \sin\beta}{c_2 \cdot \cos^2\beta}\mathrm{d}\beta = 0, \text{torej:}$$

$$-\frac{d\beta}{d\alpha} = \frac{c_2 h_1}{c_1 h_2} \cdot \frac{\sin\alpha}{\sin\beta}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 \qquad (1)$$

$$L = h_1 \cdot tg\alpha + h_2 \cdot tg\beta$$

$$dL = h_1 \frac{1}{\cos^2\alpha} d\alpha + h_2 \frac{1}{\cos^2\beta} d\beta = 0, \text{ker je L konstanta}$$

Torej:

$$\frac{d\beta}{d\alpha} = -\frac{h_1}{h_2}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 \qquad (2)$$

Upoštevamo enačbi (1) in (2) in izenačimo $\frac{d\beta}{d\alpha} = \frac{d\beta}{d\alpha}$:

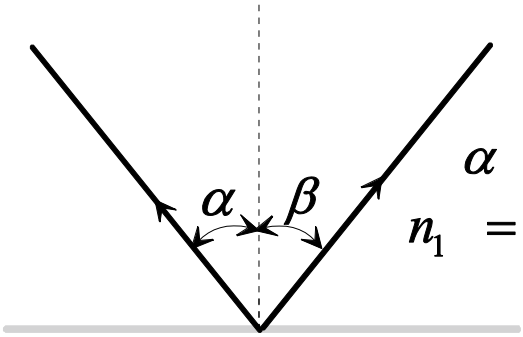
$$-\frac{c_2}{c_1} \cdot \frac{h_1}{h_2} \cdot \frac{\sin\alpha}{\sin\beta}\left(\frac{\cos\beta}{\cos\alpha}\right)^2 = -\frac{h_1}{h_2}\left(\frac{\cos\beta}{\cos\alpha}\right)^2,$$

torej: $$\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2};$$

oziroma:

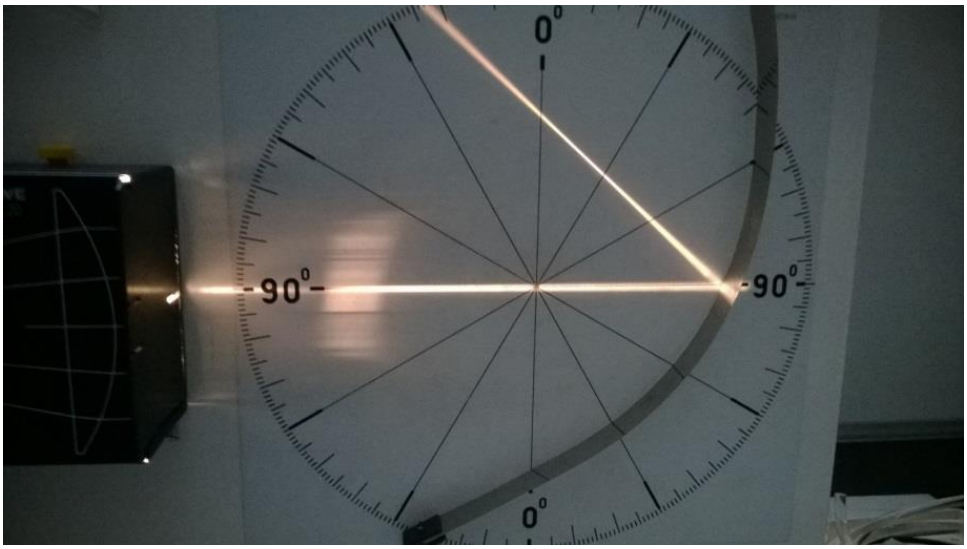
$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}, \text{ kjer upoštevamo } c_1 = \frac{c_0}{n_1}, \quad c_2 = \frac{c_0}{n_2}$$

Odboj

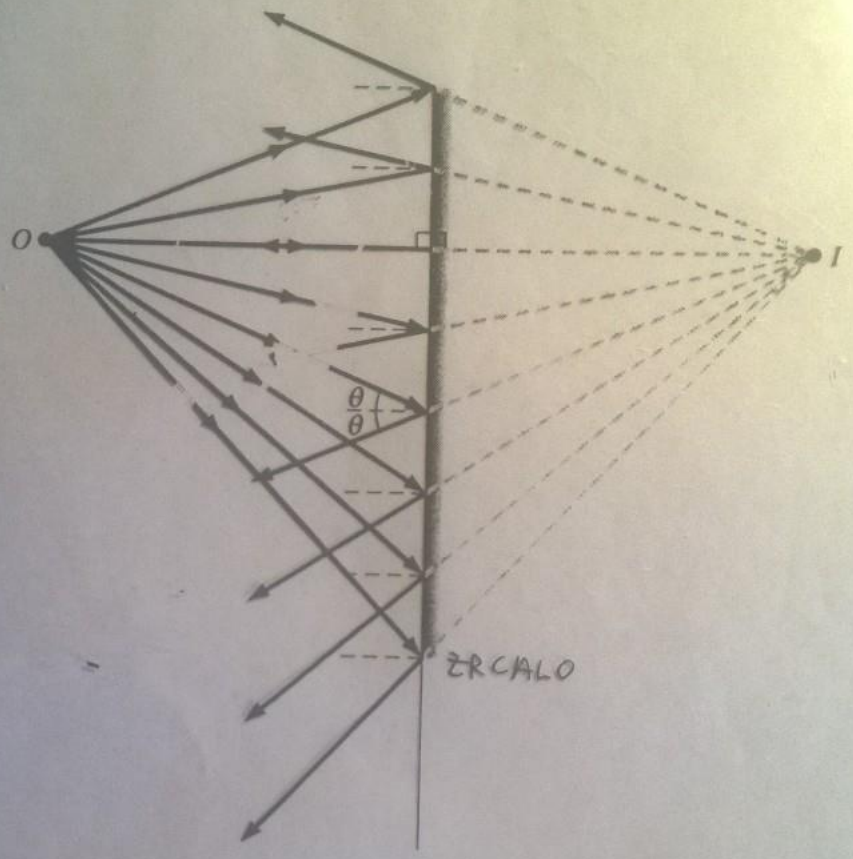


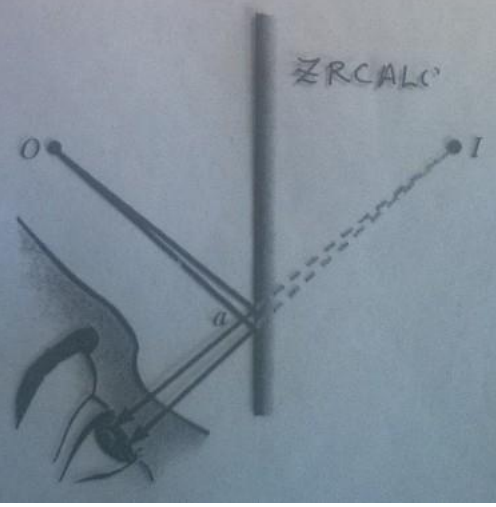
$$\alpha = \beta$$
$$n_1 = n_2 \equiv n$$

SLIKA: ODBOJ IN LOM SVETLOBE (Hartlova plošča)



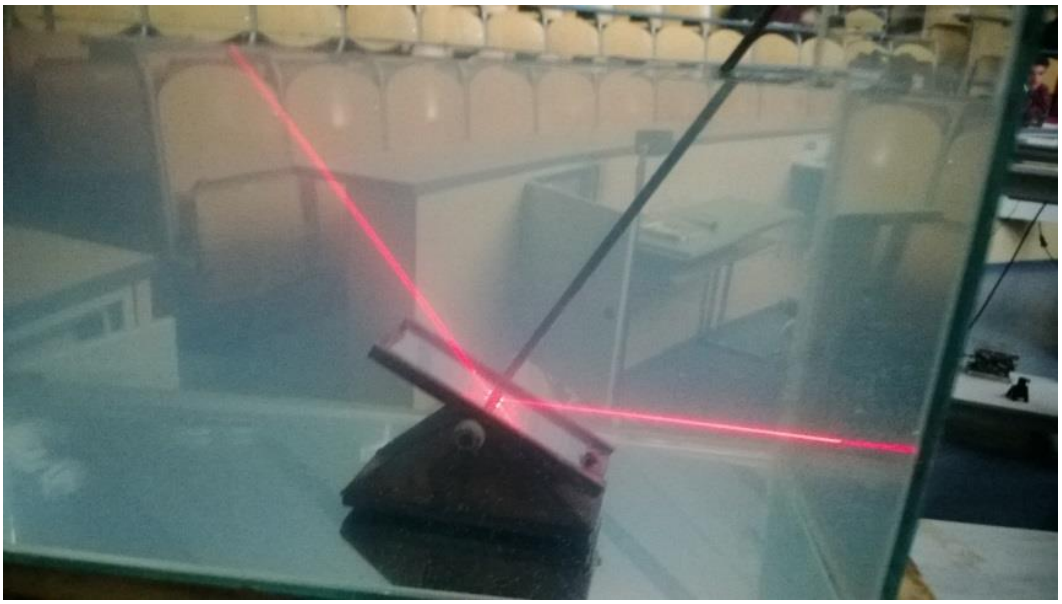
ZRCALO

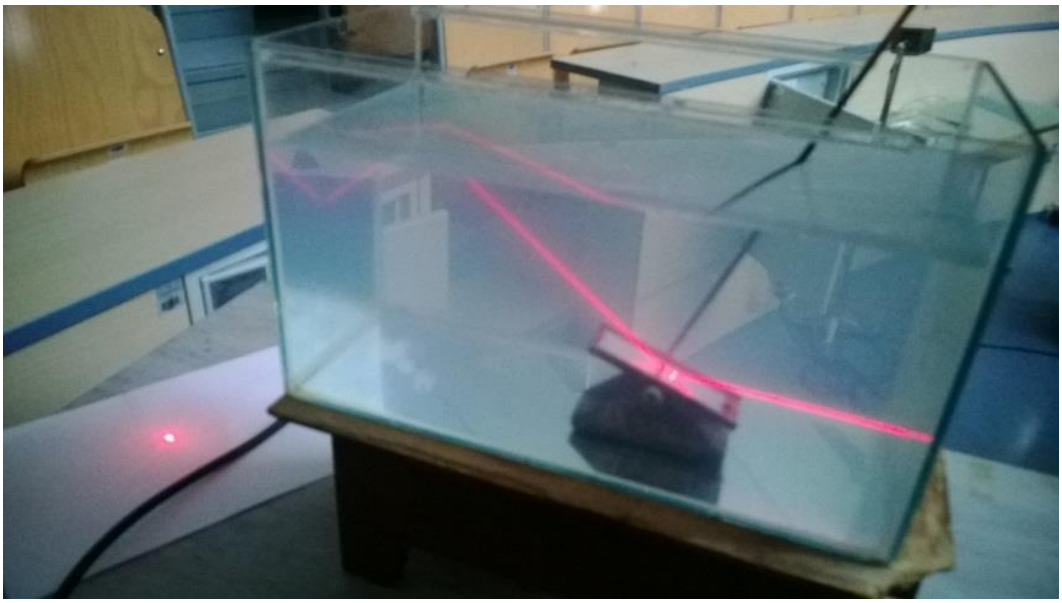


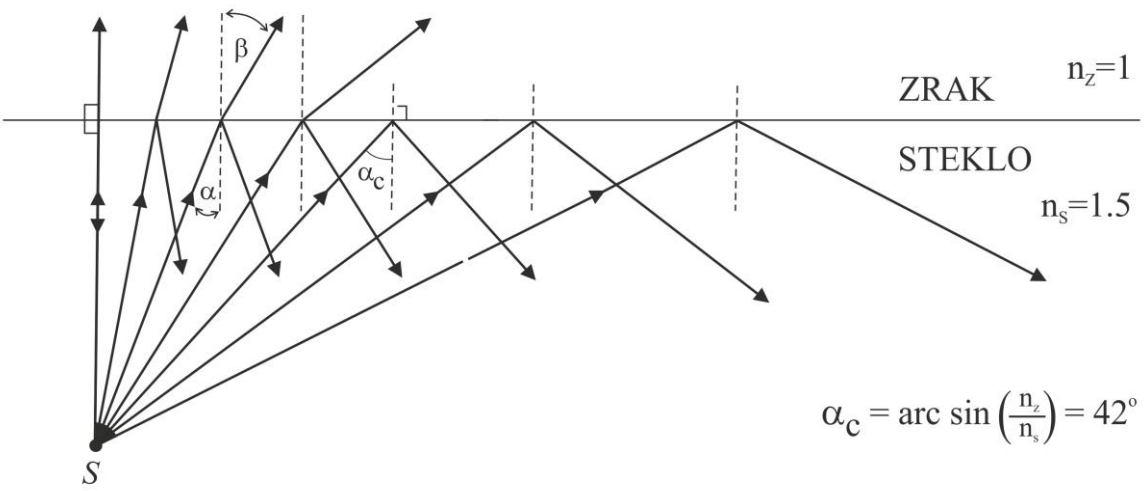


Totalni odboj

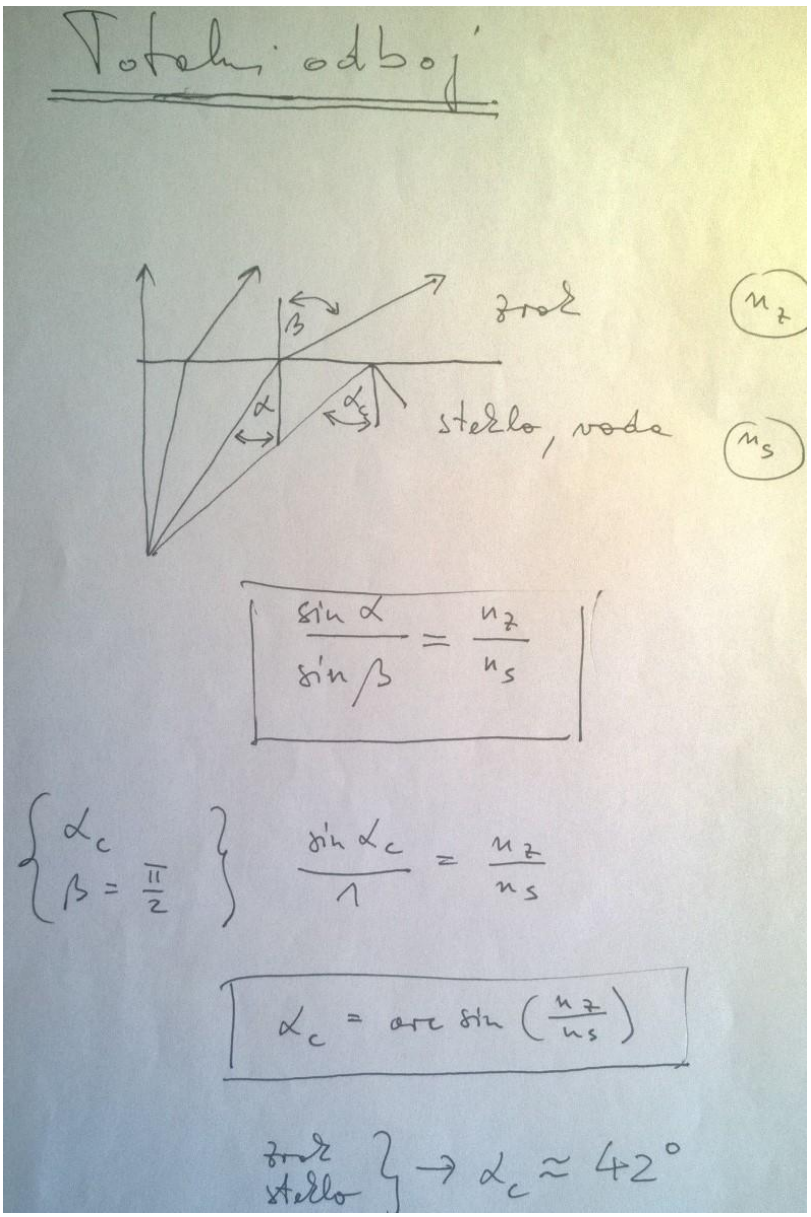
SLIKA: LOM IN TOTALNI ODBOJ V MOTNI VODI (laserski žarek)





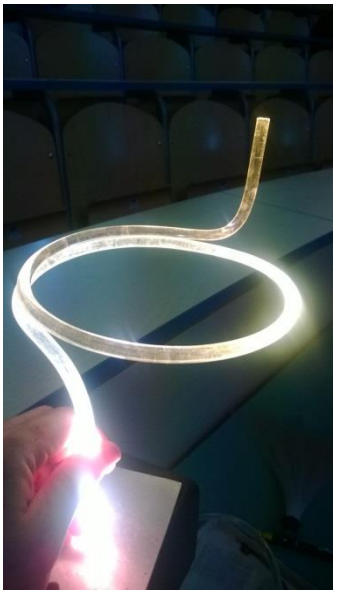


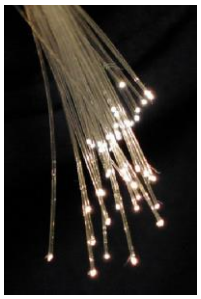
$$\alpha_c = \arcsin\left(\frac{n_z}{n_s}\right) = 42^\circ$$



APLIKACIJA: **optična vlakna**

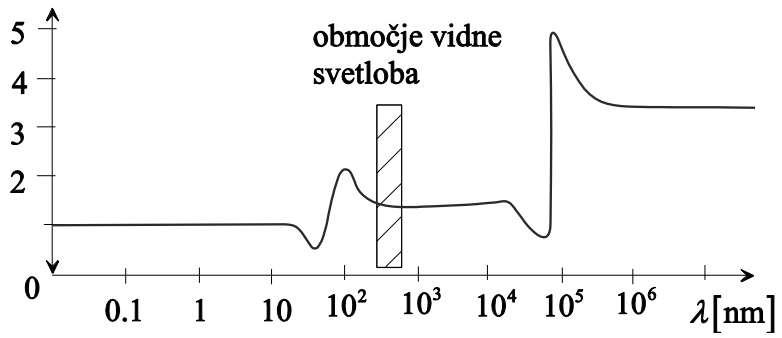
SLIKA: SVETLOBNI VODNIKI (optična vlakna) - totalni odboj





DISPERZIJA : odvisnost lomnega količnika n od valovne dolžine (λ)

1. Odvisnost lomnega količnika n od valovne dolžine (λ) za kameno sol (NaCl):



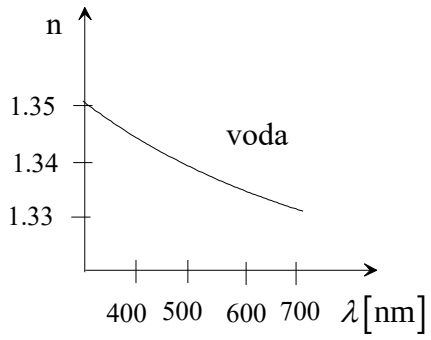
$$c = \nu \lambda$$

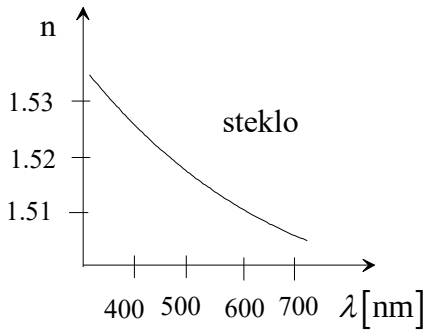
$$\omega = 2\pi \upsilon$$

$$c = \frac{c_0}{n}$$

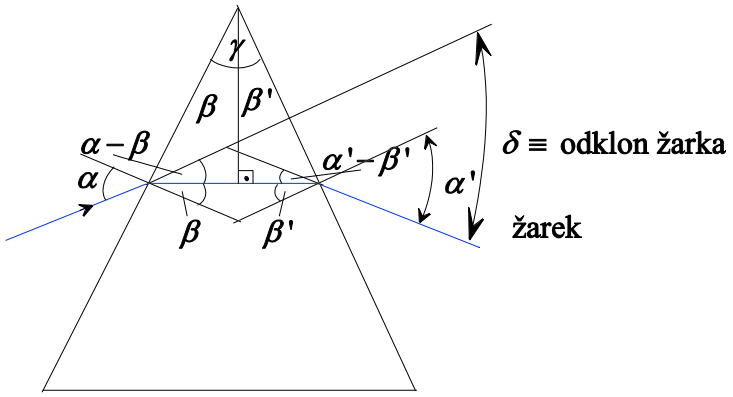
$$\lambda = \frac{2\pi c_0}{\omega}$$

2. **Odvisnost lomnega količnika od valovne dolžine (λ) za vodo in steklo**:





LOM SVETLOBE NA PRIZMI

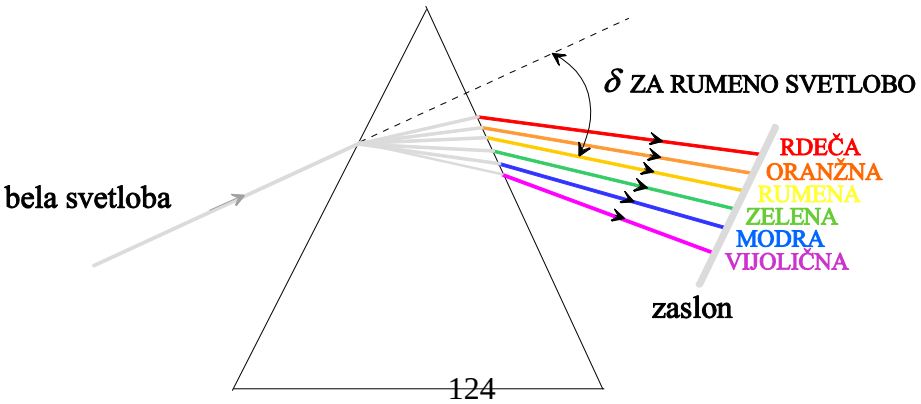


$$\boxed{\frac{\sin\alpha}{\sin\beta}=n} \qquad \boxed{\frac{\sin\alpha'}{\sin\beta'}=n} \qquad \boxed{\gamma=\beta+\beta'}$$

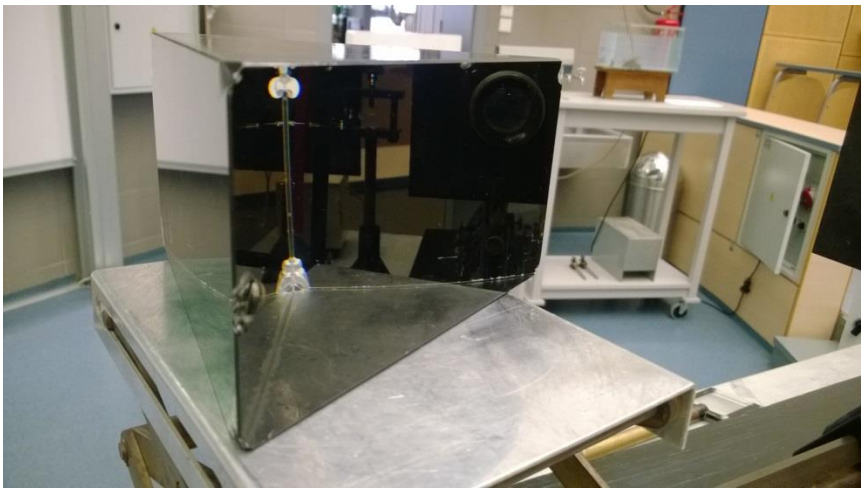
$$180^0 = 180^0 - \delta + \alpha - \beta + \alpha' - \beta' \Rightarrow$$

$$\boxed{\delta = \alpha - \beta + \alpha' - \beta' = \alpha + \alpha' - \gamma} \; .$$

Ker $n = n(\lambda)$ je odklon (δ) posameznih komponent bele svetlobe na prizmi različen:



SLIKA: LOM BELE SVETLOBE NA PRIZMI (DISPERZIJA) - svetlobni spekter vidne svetlobe

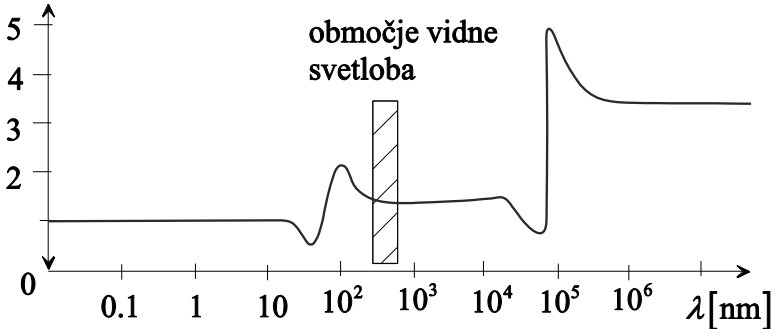


vzrok: lomni količnik je odvisen od valovne dolžine : $n = n(\lambda)$ (disperzija)

DODATEK: ODVISNOST LOMNEGA KOLIČNIKA OD VALOVNE DOLŽINE

(preprost teoretični model – samo za UNI)

kamena sol:



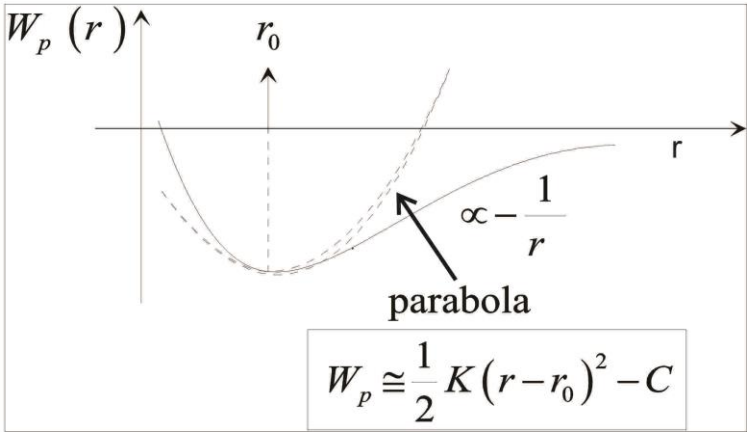
PRIMER: LOMNI KOLIČNIK RAZREDČENEGA VODIKOVEGA (H) PLINA

pozitivni del naboja: proton ($m_p \equiv$ masa protona)
negativni del naboja: elektron ($m_e \equiv$masa elektrona)

$m_e << m_p$

r $\equiv$ razdalja med protonom in elektronom

$W_p \equiv$ potencialna energija med protonom in elektronom: $W_e = \dfrac{e_{proton}\, e_{elektron}}{4\pi\, \varepsilon_0\, r} \propto -\dfrac{1}{r}$



sila v bližini minimuma W_p: $F = -\dfrac{\partial W_p}{\partial r} = -K(r - r_0)$, kjer vzamemo $W_p \cong \dfrac{1}{2} K (r - r_0)^2 - C$

če $r = r_0 + s$, kjer je s odstopanje od ravnovesne razdalje

$$W_p = \frac{1}{2} K s^2 - C$$

$$F_p = -\partial W_p / \partial s = -K s \qquad (1)$$

Sistem opišemo v težiščnem sistemu, kjer je:

$$r_T = \frac{m_p r_p + m_e r_e}{\left(m_p + m_e\right)} = 0 , \qquad (2)$$

torej

$$m_p r_p + m_e r_e = 0 . \qquad (3)$$

Enačbo (3) odvajamo po času:

$$m_p \frac{\mathrm{d}r_p}{\mathrm{d}t} + m_e \frac{\mathrm{d}r_e}{\mathrm{d}t} = 0 ,$$

oziroma

$$m_p v_p + m_e v_e = 0 . \qquad (4)$$

kjer je $v_p = \frac{\mathrm{d}r_p}{\mathrm{d}t}$ hitrost protona in $v_e = \frac{\mathrm{d}r_e}{\mathrm{d}t}$ hitrost elektrona. Iz enačbe (4) sledi:

$$v_p = -\frac{m_e}{m_p} v_e \quad . \qquad (5)$$

Ker je masa protona veliko večja od mase elektrona: $\frac{m_e}{m_p} << 1$ od tod sledi, da je v težiščnem sistemu protona in elektrona

$$\left|v_p\right| << \left|v_e\right| \qquad (6)$$

Zato v prvem približku vzamemo, da proton miruje in zapišemo **Newtonov zakon** samo za elektron:

$$m_e a = -Ks - e_0 E \qquad (7)$$

kjer smo upoštevali **sila** $F = -Ks$ (enačba (1)).

$a = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ $\equiv$ pospešek elektrona

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$ $\equiv$ hitrost elektrona

E $\equiv$ jakost zunanjega električnega polja

Predpostavimo, da je jakost zunanjega električnega polja na mestu atomov posledica EM valovanja s krožno frekvenco ω, kjer je

$$E = E_0 \sin(\omega t)\,, \tag{8}$$

električno polje na mestu vodikovega atoma. Zgornjo enačbo $E = E_0 \sin(\omega t)$ vstavimo v enačbo $m_e\, a = -Ks - e_0 E$ in dobimo:

$$\boxed{m_e \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + K\, s + e_0\, E_0 \sin(\omega t) = 0} \tag{9}$$

Rešitev enačbe (9) iščemo z nastavkom:

$$s = s_0 \sin(\omega t + \delta)\ . \tag{10}$$

Nastavek $s = s_0 \sin(\omega t + \delta)$ in

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -s_0 \omega^2 \sin(\omega t + \delta) \tag{11}$$

vstavimo v enačbo (9) in dobimo:

$$-m_e\, s_0\, \omega^2 \sin(\omega t + \delta) + K s_0 \sin(\omega t + \delta) = -e_0\, E_0 \sin(\omega t)\ . \tag{12}$$

V enačbo (12) vstavimo:

$$\sin(\omega t + \delta) = \sin(\omega t)\cos\delta + \cos(\omega t)\sin\delta$$

in dobimo:

$$\sin(\omega t)\left[\left(\omega_0^2 - \omega^2\right) s_0 \cos\delta + \frac{e_0\, E_0}{m_e}\right] + \cos(\omega t)\left[\left(\omega_0^2 - \omega^2\right) s_0 \sin\delta\right] = 0\,, \tag{13}$$

kjer je $\omega_0^2 = \dfrac{K}{m_e}$ lastna krožna frekvenca nihanja elektrona

Enačba (13) mora veljati posebej za člen s $\sin(\omega t)$ in člen s $\cos(\omega t)$:

$$\left(\omega_0^2 - \omega^2\right) s_0 \cos\delta + \frac{e_0\, E_0}{m_e} = 0\,, \tag{14}$$

$$\sin\delta = 0,\quad \delta = 0, \pm\pi, \pm 2\pi \ldots\,. \tag{15}$$

Iz enačbe (14): $\left(\omega_0^2-\omega^2\right)s_0\cos\delta+\dfrac{e_0\,E_0}{m_e}=0$, sledi vrednost amplitude s_0 v obliki :

$$s_0=\frac{e_0\,E_0}{m_e\left(\omega_0^2-\omega^2\right)}\,, \tag{16}$$

kjer smo upoštevali $\delta=-\pi$. Izračunajmo še **polarizacijo** $P_0=n_0\,p_e$:

$$P_0=n_0\,p_e=n_0\,e_0\,s_0=\frac{n_0\,e_0^2\,E_0}{m_e\left(\omega_0^2-\omega^2\right)}\cong\left(\varepsilon-1\right)\varepsilon_0\,E_0\,, \tag{17}$$

kjer je $p_e=e_0\,s_0$ inducirani dipolni moment atoma, n_0 pa volumska gostota atomov.
Upoštevali smo zvezo med susceptibilnostjo (χ) in dielektričnostjo (ε) : $\chi=\varepsilon-1$.
Iz enačbe (17) sledi:

$$\varepsilon=1+\frac{n_0\,e_0^2}{m_e\,\varepsilon_0\left(\omega_0^2-\omega^2\right)}$$

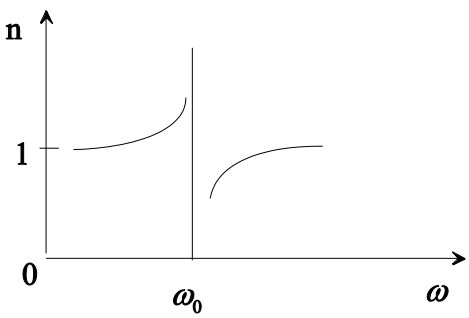
Iz zveze med lomnim količnikom in dielektrično konstanto ε dobimo:

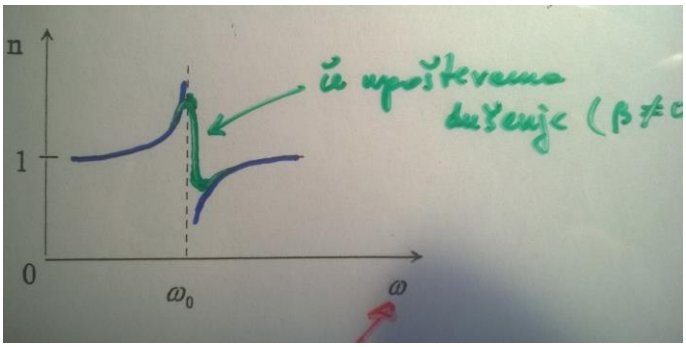
$$n=\sqrt{\varepsilon}=\left(1+\frac{n_0\,e_0^2}{m_e\,\varepsilon_0\left(\omega_0^2-\omega^2\right)}\right)^{1/2} \tag{19}$$

Ob uporabi razvoja $\left(1+x\right)^{1/2}=1+\dfrac{1}{2}x+....$ iz enačbe (19) sledi:

$$n\cong 1+\frac{1}{2}\frac{n_0\,e_0^2}{m_e\,\varepsilon_0\left(\omega_0^2-\omega^2\right)}$$

Opomba: ker v Newtonovi enačbi $m_e\,a=-Ks-e_0E$ (kjer je $E=E_0\sin\left(\omega t\right)$) nismo upoštevali člena zaradi **dušenja**, zgornji izraz za lomni količnik n pri $\omega=\omega_0$ divergira.





ČE upoštevamo še DUŠENJE:

Enačba (6.10.13) mora veljati posebej za člen s $\sin(\omega t)$ in člen s $\cos(\omega t)$. Torej

$$\left(\omega_0^2-\omega^2\right)s_0\cos\delta+\frac{e_0 E_0}{m_e}=0\ , \tag{6.10.14}$$

$$\sin\delta=0,\quad \delta=0,\pm\pi,\pm 2\pi\ldots\ . \tag{6.10.15}$$

Iz enačbe (6.10.14) sledi:

$$s_0=\frac{e_0 E_0}{m_e\left(\omega_0^2-\omega^2\right)}\ ,$$

dušenje

$$m_e\frac{d^2s}{dt^2}+Ks+\beta\frac{ds}{dt}+e_0E_0\sin(\omega t)=0$$

$$s_0=\frac{e_0E_0}{m_e\left[(\omega_0^2-\omega^2)^2+\left(\frac{\beta\omega}{m_e}\right)^2\right]^{1/2}} \tag{6.10.16}$$

kjer smo upoštevali $\delta=-\pi$. Izračunajmo še polarizacijo P_0:

$$P_0=\bar{n}e_0 s_0=\frac{n e_0^2 E_0}{m_e\left(\omega_0^2-\omega^2\right)}\cong(\varepsilon-1)\varepsilon_0 E_0\ , \tag{6.10.17}$$

kjer smo upoštevali $\chi=\varepsilon-1$, $\bar{n}$ pa je volumska gostota atomov. Iz enačbe (6.10.17) sledi:

$$\boxed{\varepsilon=1+\frac{n e_0^2}{m_e\varepsilon_0\left(\omega_0^2-\omega^2\right)}}\ .$$

$$\varepsilon=1+\frac{n e_0^2}{m_e\varepsilon_0\left[(\omega_0^2-\omega^2)+\left(\frac{\beta\omega}{m_e}\right)^2\right]^{1/2}} \tag{6.10.18}$$

Iz zveze med lomnim količnikom in dielektrično konstanto ε dobimo:

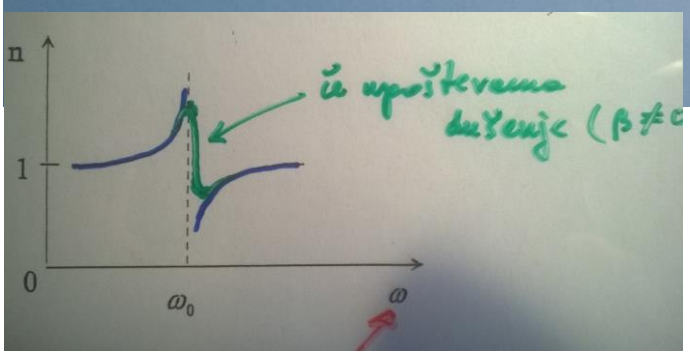
$$n=\sqrt{\varepsilon}=\left(1+\frac{n e_0^2}{m_e\varepsilon_0\left(\omega_0^2-\omega^2\right)}\right)^{1/2}\ . \tag{6.10.19}$$

S pomočjo razvoja $(1+x)^{1/2}=1+\frac{1}{2}x+\ldots$ iz enačbe (6.10.19) sledi:

$$\boxed{n\cong 1+\frac{1}{2}\frac{n e_0^2}{m_e\varepsilon_0\left(\omega_0^2-\omega^2\right)}}\ .$$

$$n=1+\frac{1}{2}\frac{n e_0^2}{m_e\varepsilon_0\left[(\omega_0^2-\omega^2)^2+\left(\frac{\beta\omega}{m_e}\right)^2\right]^{1/2}} \tag{6.10.20}$$

OD DUŠENJA



INTERFERENČNI POJAVI (VALOVNA NARAVA SVETLOBE)

KOHERENTNOST IZVOROV EM VALOVANJ IN INTERFERENCA

GOSTOTA ENERGIJSKEGA TOKA V ELEKTROMAGNETNEM VALOVANJU

$$\boxed{\bar{j} = \bar{w}\, c_0} \qquad \begin{array}{l} j \equiv \text{gostota energijskega toka EM valovanja} \\ c_0 \equiv \text{hitrost širjenja EM valovanja} \end{array} \qquad (1)$$

Gostota energije

$$\boxed{\begin{array}{l} w_E = \frac{1}{2}\varepsilon_0 E^2 \\ w_B = \frac{1}{2}\mu_0 H^2 = \frac{1}{2}\frac{B^2}{\mu_0} \end{array}}$$

$$\boxed{w = w_E + w_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}} \qquad (2)$$

$$\boxed{\vec{E} = \vec{B} \times \vec{c}_0}$$

$$\left.\begin{array}{l} \text{če:} \quad E = E_0 \cos(\omega t - kx) \\ \qquad B = B_0 \cos(\omega t - kx) \end{array}\right\} \Rightarrow \boxed{E_0 = B_0 c_0}$$

$$\bar{w} = \frac{1}{2}\varepsilon_0 E_0^2 \; \overline{\cos^2(\omega t - kx)} + \frac{1}{2\mu_0} B_0^2 \; \overline{\cos^2(\omega t - kx)} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 =$$

$$= \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0}\frac{E_0^2}{c_0^2} \quad \text{torej:}$$

$$\bar{w} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2, \qquad (3)$$

kjer smo upoštevali:

$$\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$$

$$\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$$

$$B_0 = \frac{E_0}{c_0}$$

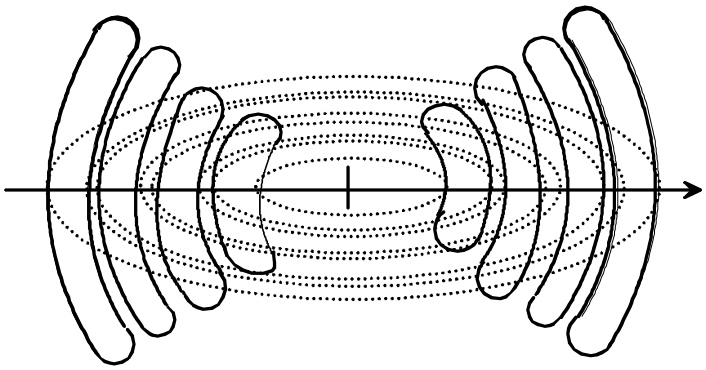
$$c_0^2 = \frac{1}{\varepsilon_0 \mu_0}$$

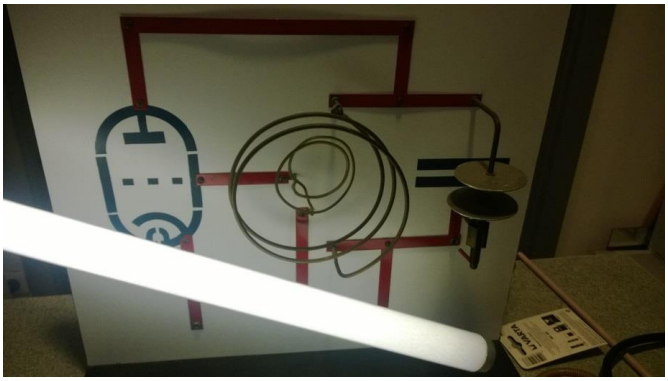
- torej:

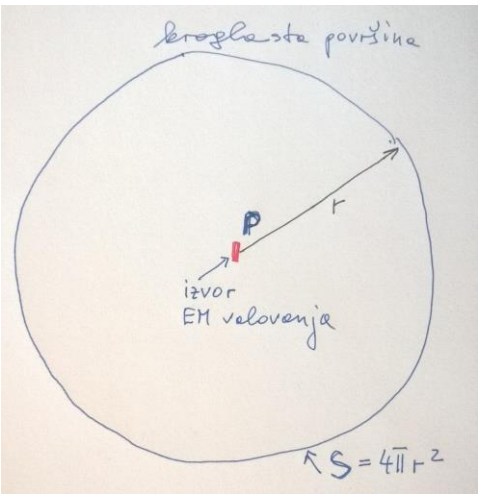
$$\boxed{\bar{j} = \bar{w} c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 = \frac{1}{2} \frac{B_0^2}{\mu_0} c_0} \tag{4}$$

- če antena oddaja izotropno:

$$\boxed{\bar{j} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}} \tag{5}$$







- **Poyntingov vektor**:

$$\boxed{\vec{P} = \vec{E} \times \vec{H}} \qquad (6)$$

podaja:

- velikost gostote energijskega toka
- smer razširjanja valovanja

Dokaz prve trditve:

če $E = E_0 \cos(\omega t - kx)$, $H = H_0 \cos(\omega t - kx)$ in $B_0 = \mu_0 H_0$ velja:

$$\overline{|\vec{P}|} = E_0 H_0 \overline{\cos^2(\omega t - kx)} = \frac{1}{2} E_0 H_0 = \frac{1}{2} E_0 \frac{B_0}{\mu_0} = \frac{1}{2} E_0 \frac{E_0}{\mu_0 c_0} \cdot \frac{c_0}{c_0} = \frac{1}{2} E_0 \frac{E_0}{\mu_0 c_0^2} c_0 =$$

$$= \frac{1}{2} E_0 \frac{\varepsilon_0 \mu_0}{\mu_0} E_0 c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 \ .$$

kjer smo upoštevali:

$$B_0 = \frac{E_0}{c_0} \quad \text{in} \quad c_0^2 = \frac{1}{\varepsilon_0 \mu_0} \ .$$

torej:

$$\boxed{\bar{j} = \frac{1}{2} \varepsilon_0 E_0^2 c_0}$$

INTERFERENCA DVEH KOHERENTNIH VALOVANJ

MATEMATIČNA FORMULACIJA

- povprečna gostota energijskega toka v vakuumu:

$$\bar{j} = \bar{w} c_0 = (\bar{w}_E + \bar{w}_B) c_0 = \varepsilon_0 \overline{E^2} c_0 \qquad (8)$$

- **eno** ravno EM valovanje: $E = E_0 \cos(\omega t - kx)$

$$\boxed{\bar{j} = \bar{w} c_0 = \varepsilon_0 \overline{E_0^2} c_0 = \varepsilon_0 E_0^2 \overline{\cos^2(\omega t - kx)} c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0 \ ,}$$

kjer smo upoštevali $\overline{\cos^2(\omega t - kx)} = \frac{1}{2}$.

- **dve** ravni EM valovanji (fazna razlika δ),

V izbrani točki prostora x_0 uporabimo načelo superpozicije:

$$E_1 = E_{01}\cos\left(\omega t - k\,x_0 - \frac{1}{2}\delta\right)$$

$$E_2 = E_{02}\cos\left(\omega t - k\,x_0 + \frac{1}{2}\delta\right)$$

$$E = E_1 + E_2$$

torej:

$$\begin{aligned}(E_1+E_2)^2 &= \left[E_{01}\cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right) + E_{02}\cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right)\right]^2 = \\ &= E_{01}^2\cos^2\left(\omega t - kx_0 - \frac{1}{2}\delta\right) + 2E_{01}E_{02}\cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right)\cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right) + \\ &\quad + E_{02}^2\cos^2\left(\omega t - kx_0 + \frac{1}{2}\delta\right)\end{aligned} \tag{9}$$

Poglejmo posebej mešani člen v enačbi (6.3.9):

$$\begin{aligned}&2E_{01}E_{02}\cos\left(\omega t - kx_0 - \frac{1}{2}\delta\right)\cos\left(\omega t - kx_0 + \frac{1}{2}\delta\right) = \\ &= 2E_{01}E_{02}\left[\cos(\omega t - kx_0)\cos\left(\frac{1}{2}\delta\right) + \sin(\omega t - kx_0)\sin\left(\frac{1}{2}\delta\right)\right]\cdot \\ &\qquad \cdot\left[\cos(\omega t - kx_0)\cos\left(\frac{1}{2}\delta\right) - \sin(\omega t - kx_0)\sin\left(\frac{1}{2}\delta\right)\right] = \\ &= 2E_{01}E_{02}\left[\cos^2(\omega t - kx_0)\cos^2\left(\frac{1}{2}\delta\right) - \sin^2(\omega t - kx_0)\sin^2\left(\frac{1}{2}\delta\right)\right].\end{aligned} \tag{10}$$

Iz enačbe (6.3.9) in (6.3.10) sledi (črta nad simboli označuje časovno povprečje):

$$\begin{aligned}\overline{(E_1+E_2)^2} &= E_{01}^2\,\overline{\cos^2\left(\omega t - k\,x_0 - \frac{1}{2}\delta\right)} + E_{02}^2\,\overline{\cos^2\left(\omega t - k\,x_0 + \frac{1}{2}\delta\right)} + \\ &+ 2E_{01}E_{02}\left[\overline{\cos^2(\omega t - kx_0)}\;\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2(\omega t - kx_0)}\;\overline{\sin^2\left(\frac{1}{2}\delta\right)}\right].\end{aligned} \tag{11}$$

Če upoštevamo

$$\overline{\cos^2 z} = \frac{1}{2} \quad \text{in} \quad \overline{\sin^2 z} = \frac{1}{2}$$

iz enačbe (6.3.11) :

$$\overline{(E_1+E_2)^2} = E_{01}^2\,\overline{\cos^2\left(\omega t - k\,x_0 - \frac{1}{2}\delta\right)} + E_{02}^2\,\overline{\cos^2\left(\omega t - k\,x_0 + \frac{1}{2}\delta\right)} +$$
$$+2\,E_{01}E_{02}\left[\overline{\cos^2(\omega t - kx_0)}\;\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2(\omega t - kx_0)}\,\overline{\sin^2\left(\frac{1}{2}\delta\right)}\right]. \tag{11}$$

sledi:

$$\overline{(E_1+E_2)^2} = \frac{1}{2}E_{01}^2 + \frac{1}{2}E_{02}^2 + E_{01}E_{02}\left[\overline{\cos^2\left(\frac{1}{2}\delta\right)} - \overline{\sin^2\left(\frac{1}{2}\delta\right)}\right],$$

oziroma

$$\overline{(E_1+E_2)^2} = \frac{1}{2}E_{01}^2 + \frac{1}{2}E_{02}^2 + E_{01}E_{02}\,\overline{\cos\delta}\,, \tag{12}$$

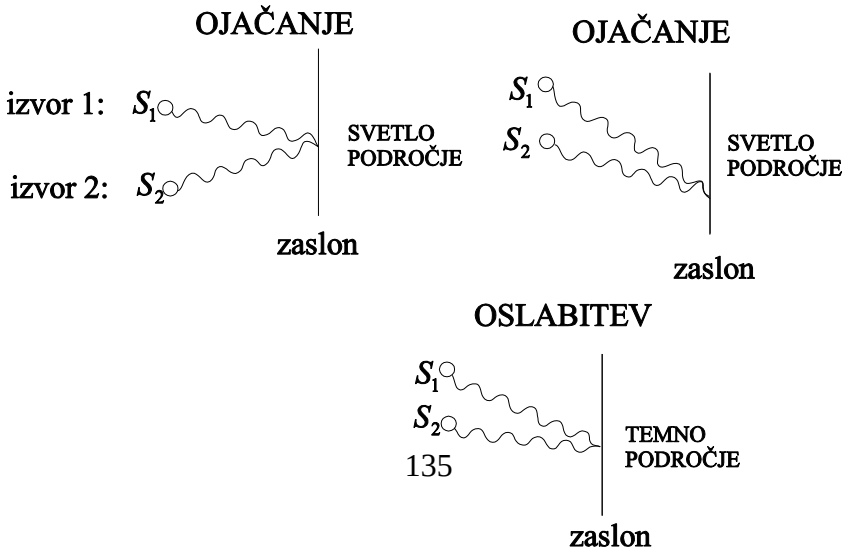
Torej:

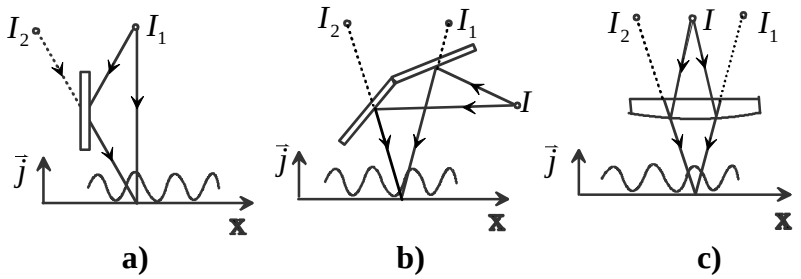
$$\boxed{\bar{j} = \overline{w}c_0 = \varepsilon_0\,\overline{(E_1+E_2)^2}\,c_0 = \frac{1}{2}\varepsilon_0 E_{01}^2 c_0 + \frac{1}{2}\varepsilon_0 E_{02}^2 c_0 + \varepsilon_0 E_{01} E_{02}\, c_0\,\overline{\cos\delta}\;.} \tag{13}$$

Zaključek:

- nekoherentna izvora, če $\delta = \delta(t)$ **naključen** $\Rightarrow$ $\overline{\cos\delta(t)} = 0$ $\Rightarrow$ ni interference
- koherentna izvora, če δ stalna $\Rightarrow$ $\overline{\cos\delta(t)} \neq 0$ $\Rightarrow$ je interferenca (glejte slike na naslednji strani)

- **Shematski prikaz interference dveh EM valovanj** (2 koherentna izvora)

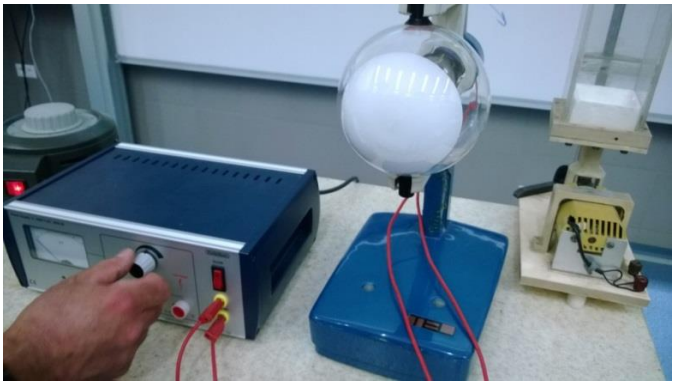


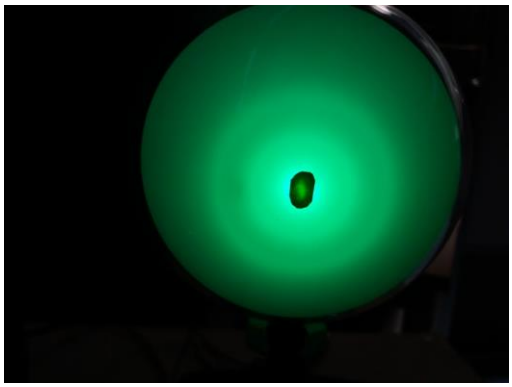


Nekaj naprav s katerimi dobimo dva navidezna izvora **koherentnih** valovanj.

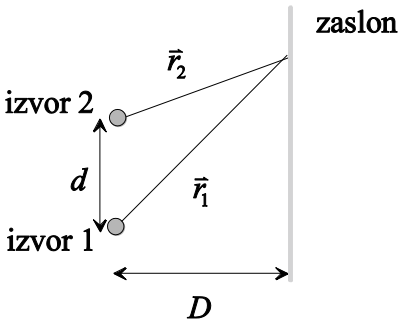
UKLONSKA SLIKA LASERSKE **SVETLOBE** : okrogla odprtina

DELCI: UKLONSKE SLIKE CURKA ELEKTRONOV NA KRISTALIH GRAFITA





Interferenca dveh točkastih koherentnih izvorov EM valovanja na zaslonu :



$\delta = 0$

$$E_1 = E_0\, e^{i(kr_1 - \omega t)}$$
$$E_2 = E_0\, e^{i(kr_2 - \omega t)}$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$
$$e^{-i\alpha} = \cos\alpha - i\sin\alpha$$

$$E_1 = E_0 \cos\left(kr_1 - \omega t\right)$$

$$E_2 = E_0 \cos\left(kr_2 - \omega t\right)$$

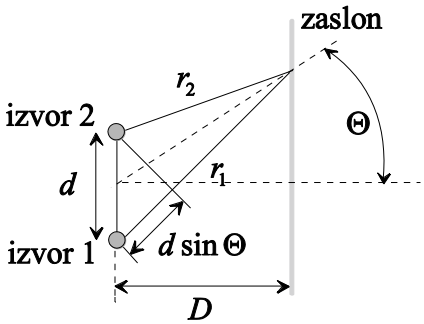
$$\left|E_1 + E_2\right|^2 = \left(E_1 + E_2\right)\left(E_1 + E_2\right)^* = \left(E_1 + E_2\right)\left(E_1^* + E_2^*\right) =$$
$$= E_1 E_1^* + E_2 E_2^* + E_1 E_2^* + E_1^* E_2 = E_0^2\left(2 + e^{ik(r_1 - r_2)} + e^{-ik(r_1 - r_2)}\right) =$$
$$= 2E_0^2 + E_0^2\, 2\cos\left(k\left(r_1 - r_2\right)\right) = 2E_0^2\left(1 + \cos\left(k\left(r_1 - r_2\right)\right)\right) =$$
$$= 4E_0^2 \cos^2\left[\frac{k\left(r_1 - r_2\right)}{2}\right]$$

$1 + \cos 2x = 2\cos^2 x$

Pogoj za maksimum (svetle interferenčne proge):

$$\frac{k\left(r_1 - r_2\right)}{2} = n\pi \Rightarrow r_1 - r_2 = \frac{n\,2\pi}{k}, n = 0,\ 1,\ 2$$

Definiramo kot Θ :



d ≡ je razdalja med izvoroma

Če upoštevamo $r_1 - r_2 \cong d\ \sin\Theta$ in $k = \frac{2\pi}{\lambda}$ velja:

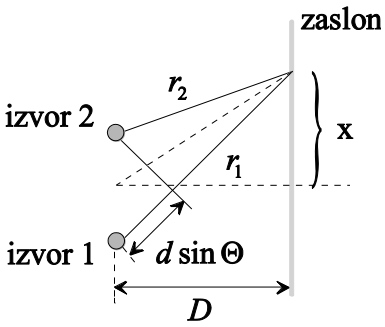
$$\left|E_1 + E_2\right|^2 = 4E_0^2\ \cos^2\left[\frac{k\left(r_1 - r_2\right)}{2}\right] = 4E_0^2\ \cos^2\left[\frac{\pi\, d\ \sin\Theta}{\lambda}\right].$$

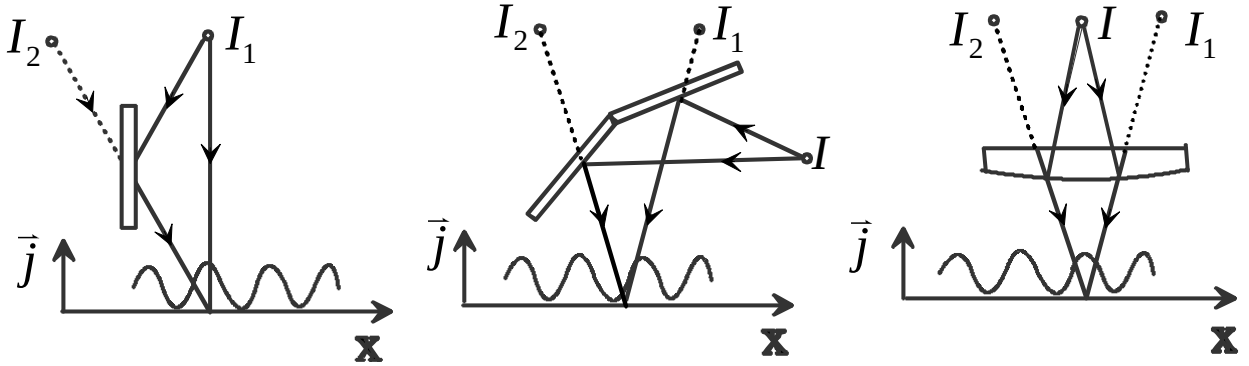
Povprečna gostota energijskega toka $\bar{j}$ je sorazmerna $\left|E_1 + E_2\right|^2$, torej:

$$\boxed{\bar{j} \propto \left|E_1 + E_2\right|^2 = 4E_0^2\ \cos^2\left[\frac{\pi\, d \sin\Theta}{\lambda}\right]}$$

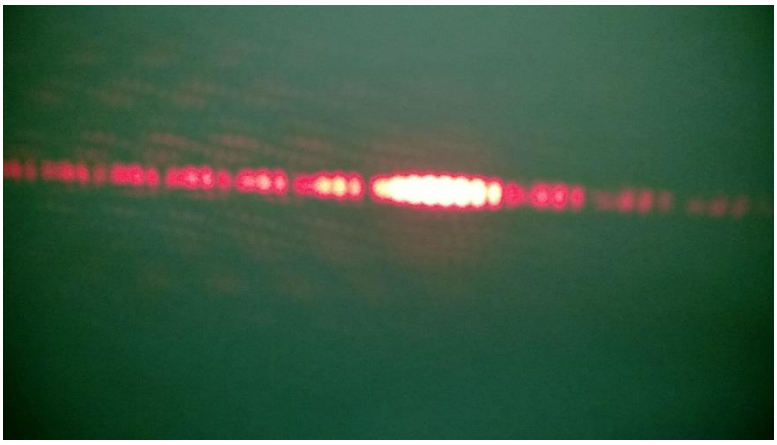
$$\boxed{\bar{j} \propto \left|E_1 + E_2\right|^2 = 4E_0^2 \cos^2\left[\frac{\pi d \sin\Theta}{\lambda}\right]}$$

Če D >> d: $\sin\Theta \approx tg\,\Theta = \frac{x}{D}$, torej: $\boxed{\bar{j} \propto \cos^2\left[\frac{\pi d x}{\lambda D}\right]}$



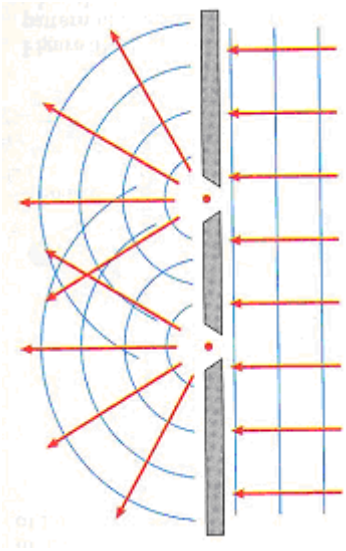


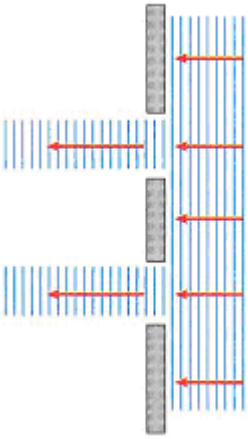
UKLON svetlobe :



6.4 UKLON SVETLOBE NA REŽAH IN INTERFERENCA

6.4.1 UKLON





sliki: Serway, 1992

Huygensovo načelo: vsako točko valovne fronte obravnavamo kot izvor krogelnih (krožnih) valov)

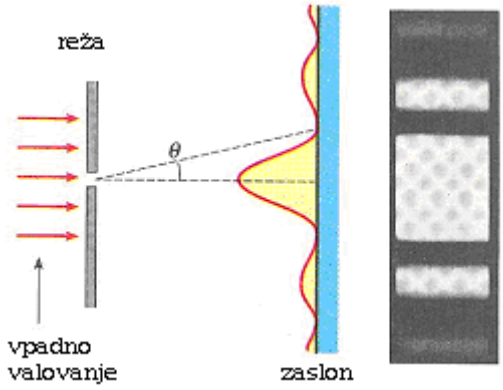
PRIMERJAVA: velika predavalnica na Jadranski: Prikaz Huygensovega načela na **vodni gladini** (valovna kad)

ena reža

dve reži

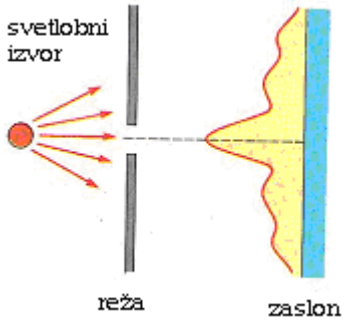
ravni valovi :

Fraunhofer-jev uklon



Zaslon je zelo oddaljen.

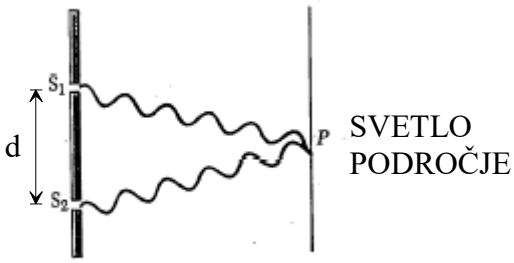
Fresnel-ov uklon

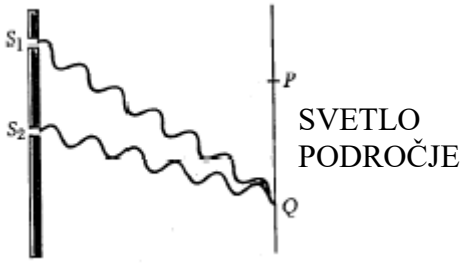


Zaslon je na končni razdalji (delna valovanja, ki interferirajo na izbranem delu zaslona ne smemo imeti za vzporedno).

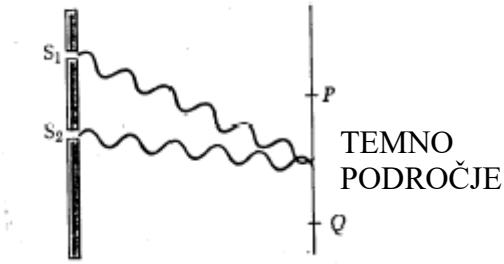
sliki: Serway, 1992

KONSTRUKTIVNA INTERFERENCA



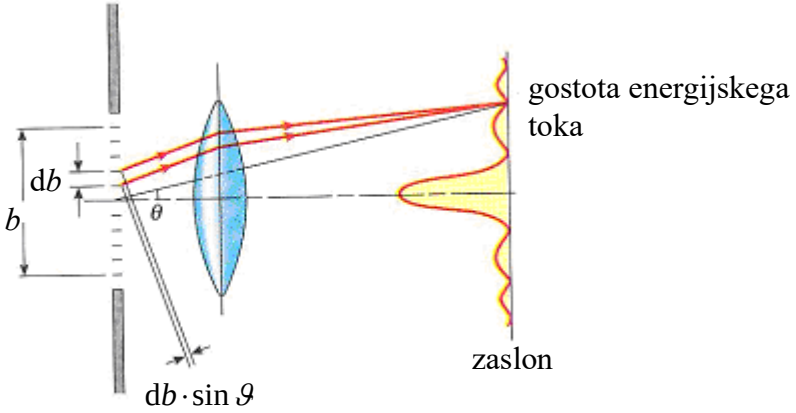


DESTRUKTIVNA INTERFERENCA

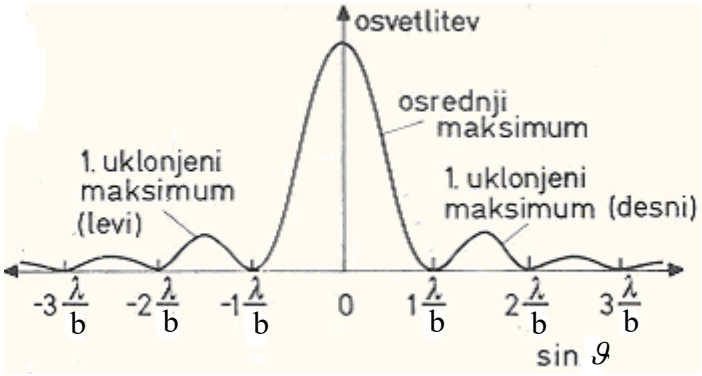


6.4.3 UKLON IN INTERFERENCA NA REŽI

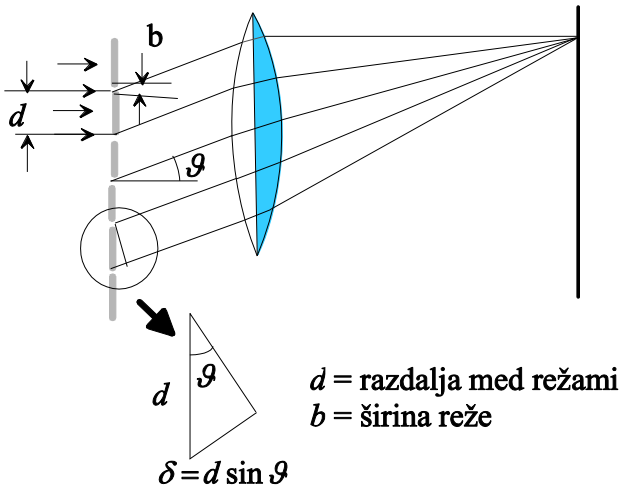
$$j = j_0 \frac{\sin^2(\pi b \sin\vartheta/\lambda)}{(\pi b \sin\vartheta/\lambda)^2}$$

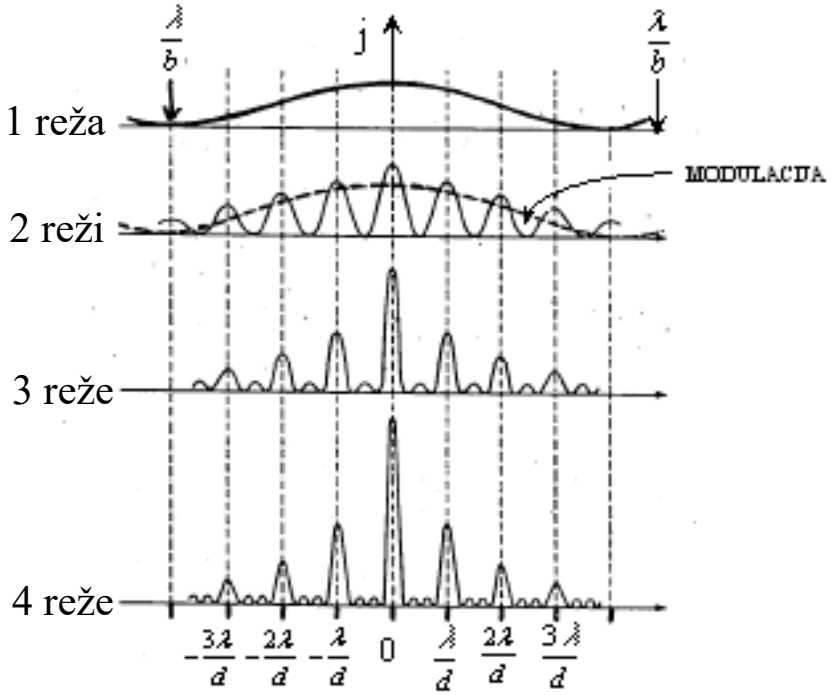


b ≡ širina reže $db \cdot \sin\vartheta$

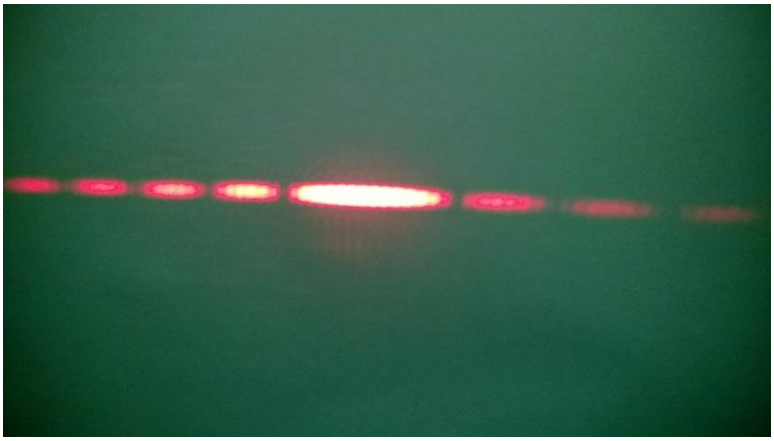


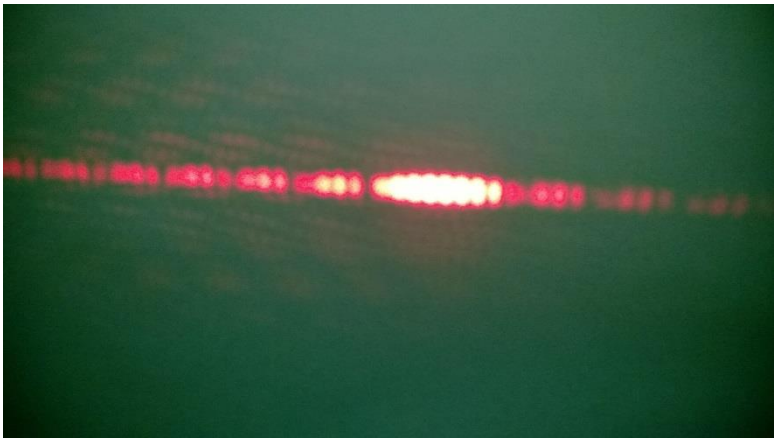
UKLON IN INTERFERENCA NA SISTEMU REŽ V ENAKIH RAZMIKIH

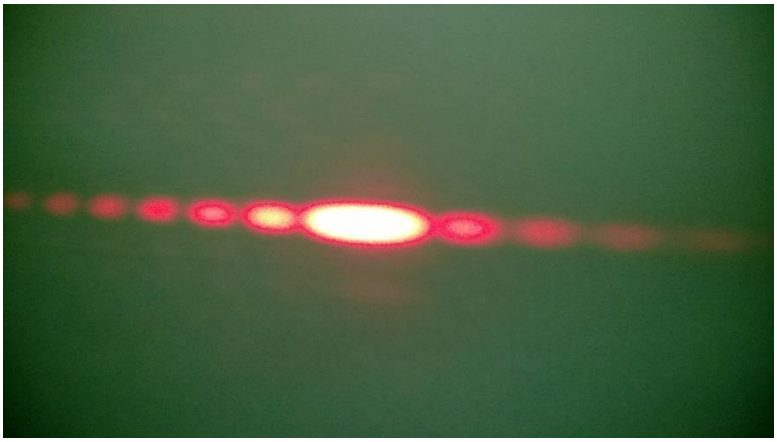




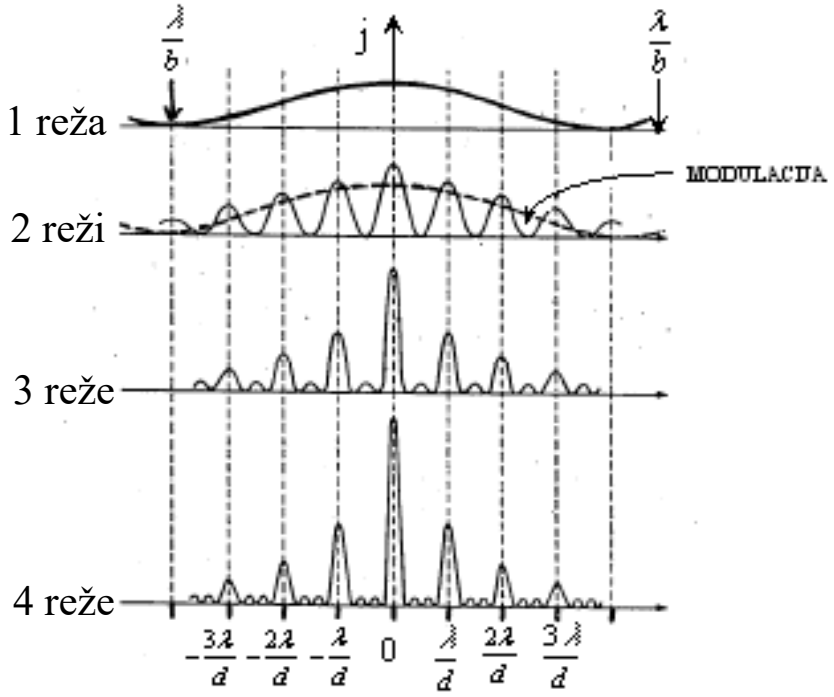
EKSPERIMENTI na režah: ena reža, dve reži, več rež



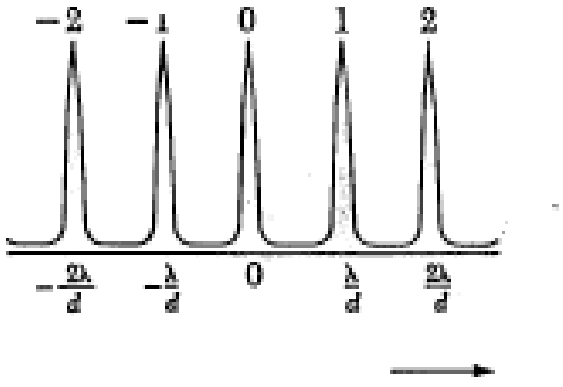


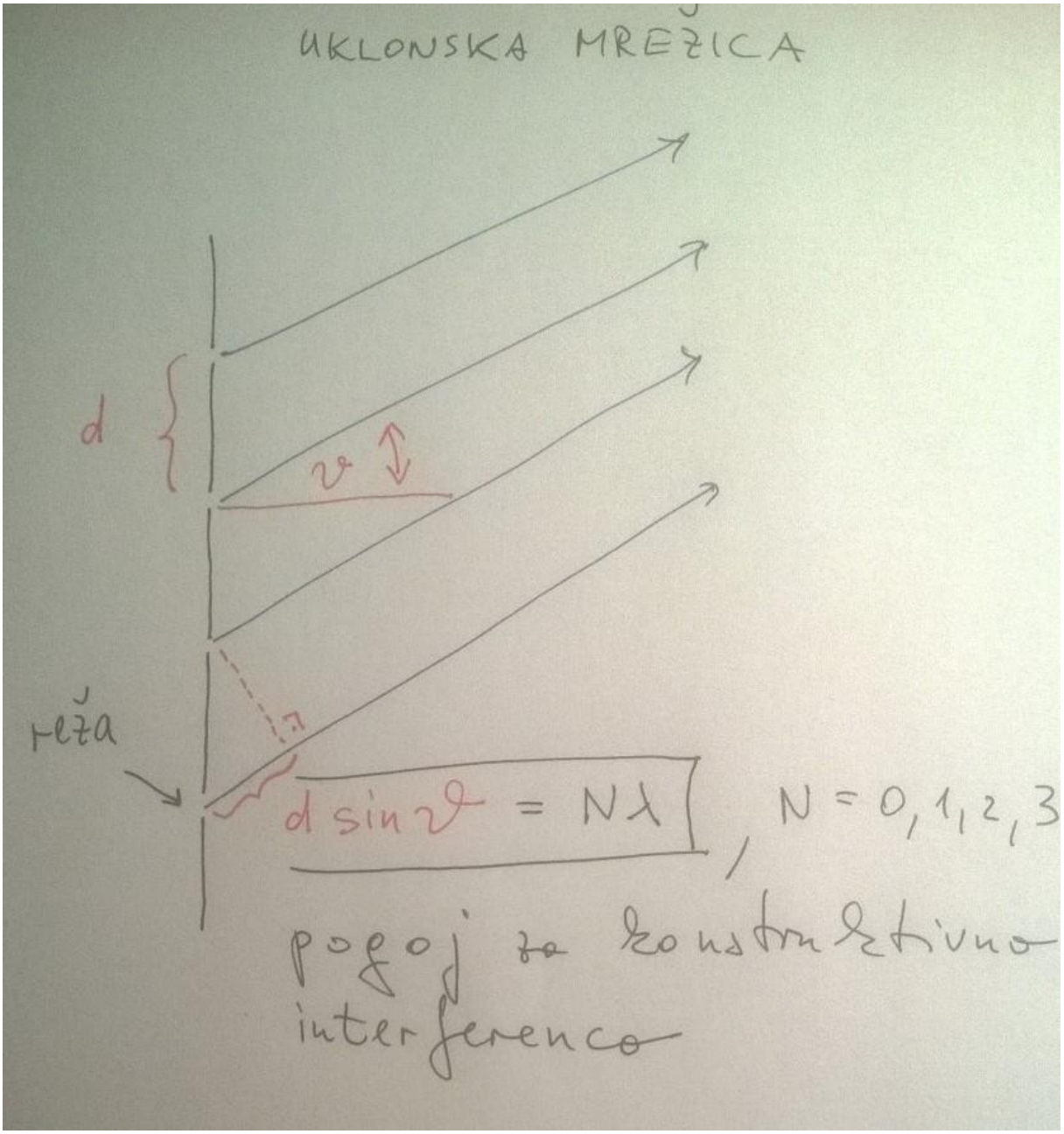


okrogla odprtina :



Gostota energijskega toka **uklonske mrežice**:

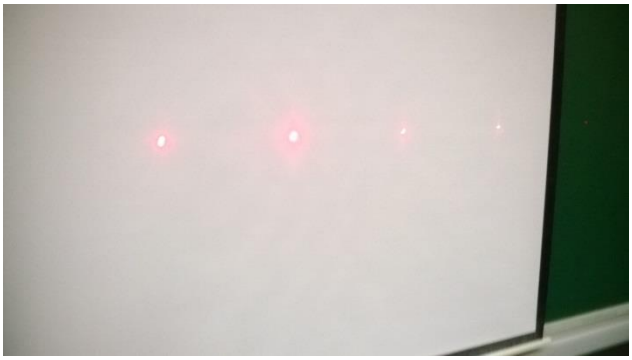


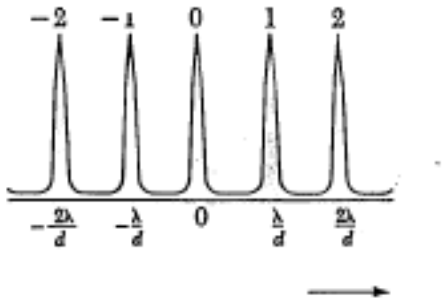


Torej, pogoj za ojačanje pri uklonski mrežici :

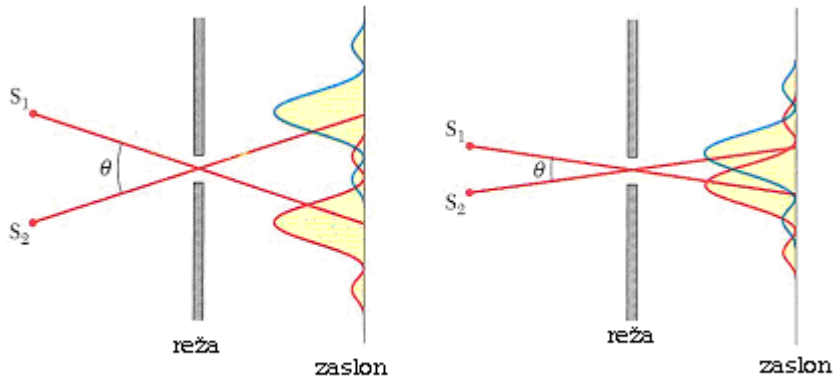
$$d \sin \vartheta = \mathrm{N}\lambda, \quad \mathrm{N} = 0, 1, 2, 3, \ldots$$

uklonska mrežica: pogoj za ojačanje: $d \sin \vartheta = \mathrm{N}\lambda, \quad \mathrm{N} = 0, 1, 2, 3, \ldots$

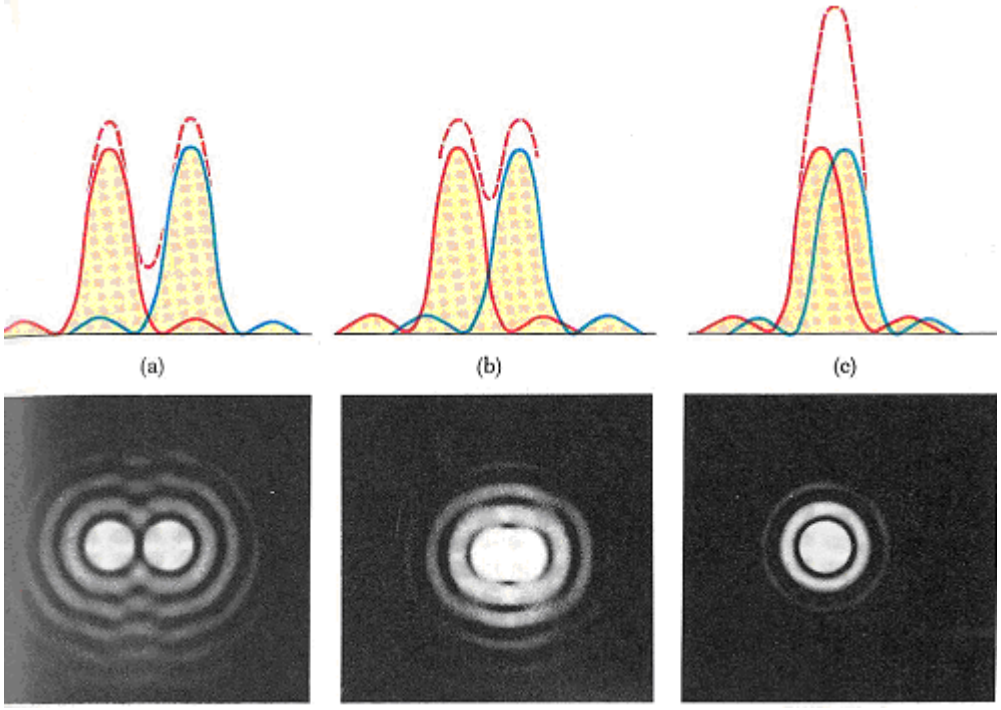


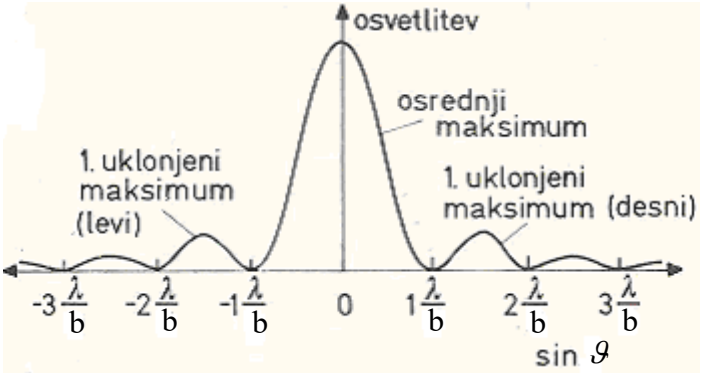


LOČLJIVOST zaradi uklona, pomembna pri MIKROSKOPIH :

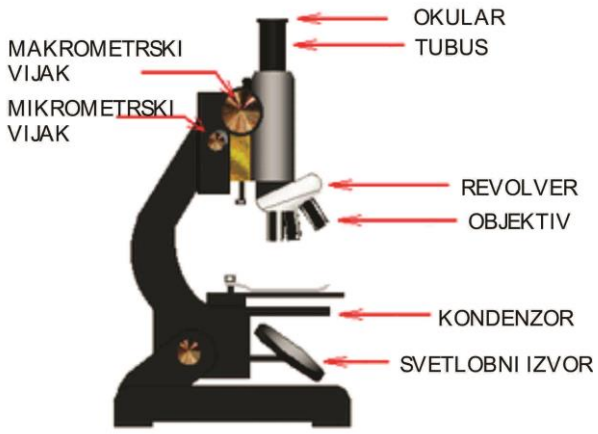


(Serway, 1992)

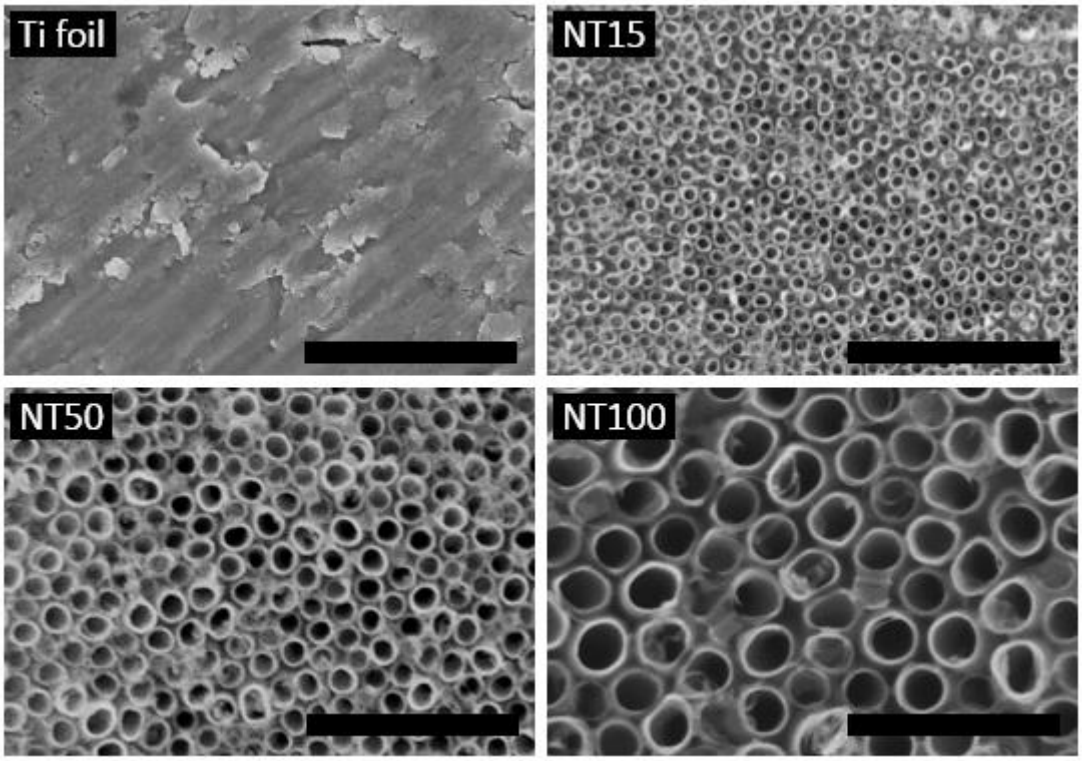




Valovne dolžine vidne svetlobe so velike (od 380 nm do 780 nm), zato imajo **optični mikroskopi slabo ločljivost**.



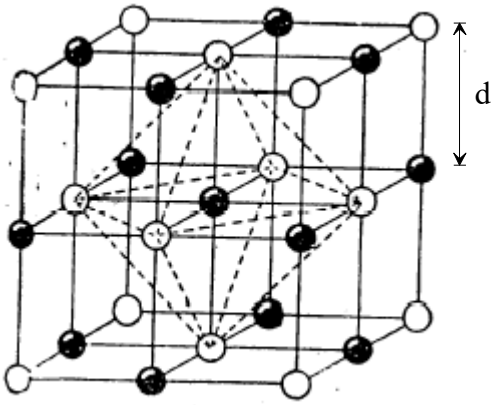
Rešitev : **elektronski mikroskopi**

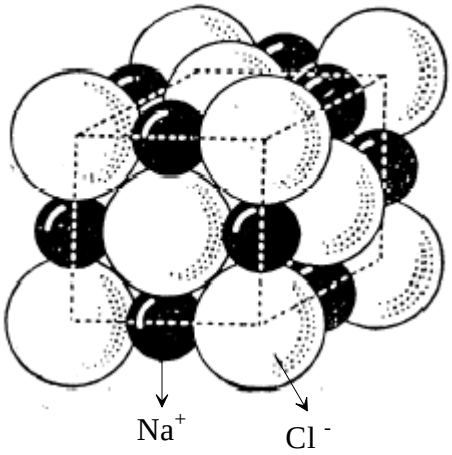


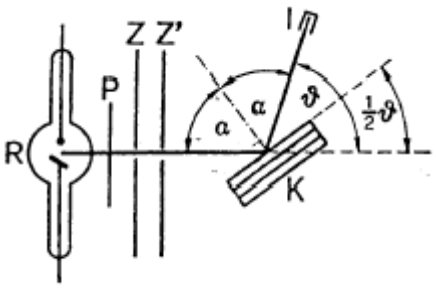
Laboratorij za fiziko FE in IJS : dolžina merila (črna črtica) = 500 nm

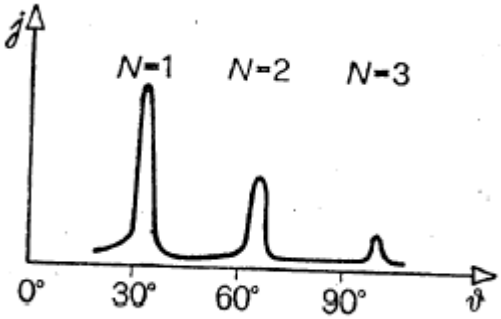
UKLONSKA SLIKA KRISTALA NaCl, BRAGGOVA ENAČBA

Ionska kristalna mreža NaCl

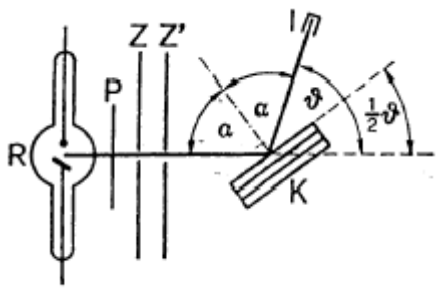


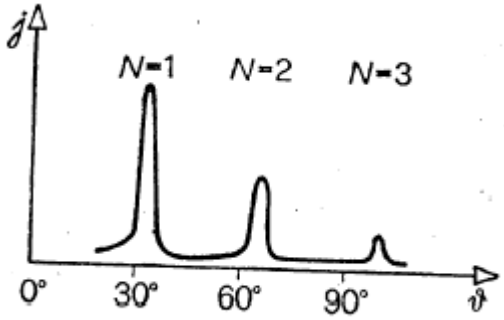


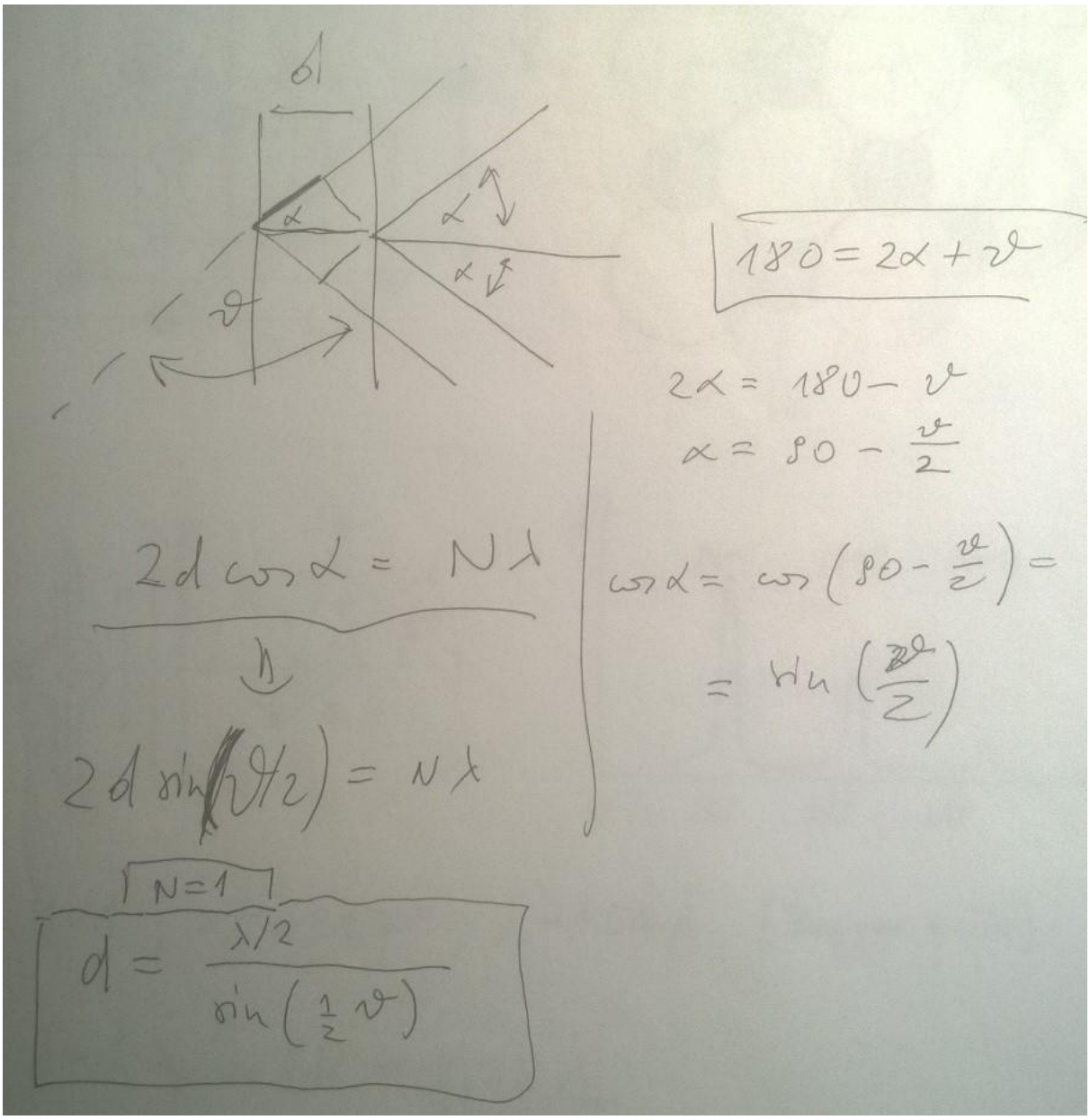




slike: J Strnad, Fizika II







$$\boxed{180 = 2\alpha + \vartheta}$$

$$2\alpha = 180 - \vartheta$$

$$\alpha = 90 - \frac{\vartheta}{2}$$

$$\cos\alpha = \cos\left(90 - \frac{\vartheta}{2}\right) = \sin\left(\frac{\vartheta}{2}\right)$$

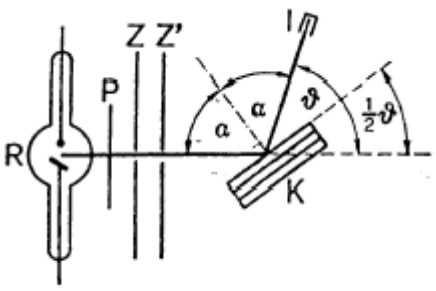
$$2d\cos\alpha = N\lambda$$

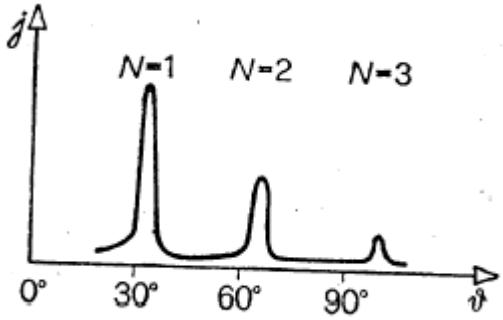
$$\Downarrow$$

$$2d\sin(\vartheta/2) = N\lambda$$

$$\boxed{N=1}$$

$$\boxed{d = \frac{\lambda/2}{\sin\left(\frac{1}{2}\vartheta\right)}}$$





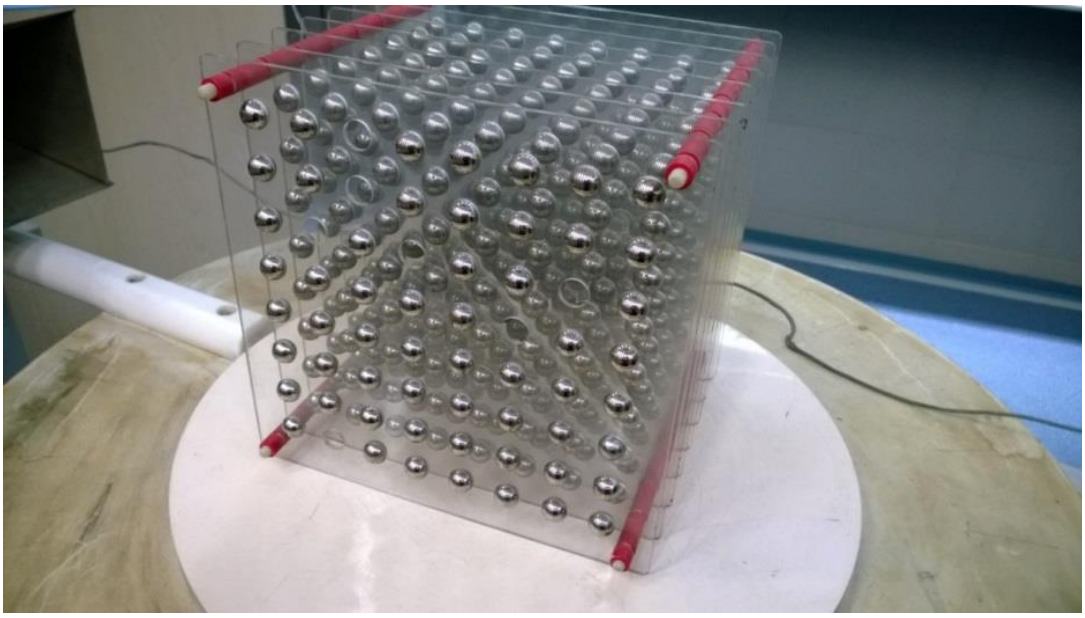
Braggova enačba:

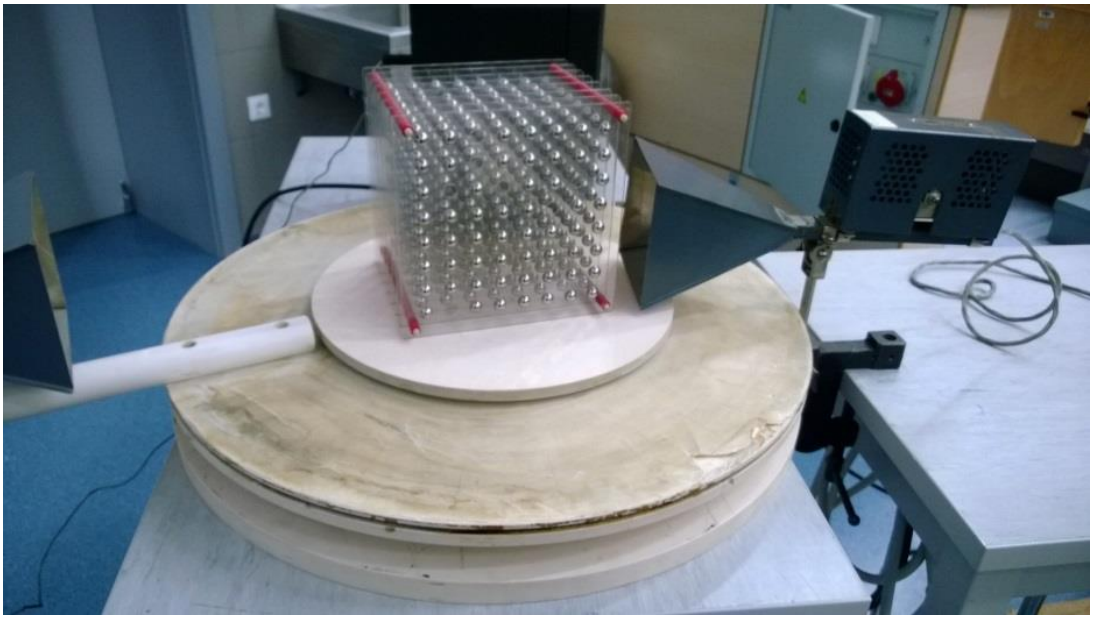
$$\boxed{2d\sin\left(\frac{1}{2}\vartheta\right)=N\lambda}$$

Primer kristala NaCl:

$$N=1,\ \vartheta=31.7^0,\lambda=1.54\,\mathring{A}\quad d=\frac{\frac{1}{2}\lambda}{\sin\frac{1}{2}\vartheta}=2.82\,\mathring{A}$$

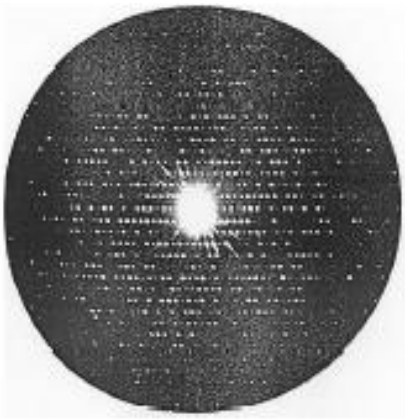
EKSPERIMENT: **UKLONSKE SLIKE KRISTALOV** Braggovo sipanje na modelu kristala (mikrovalovi)



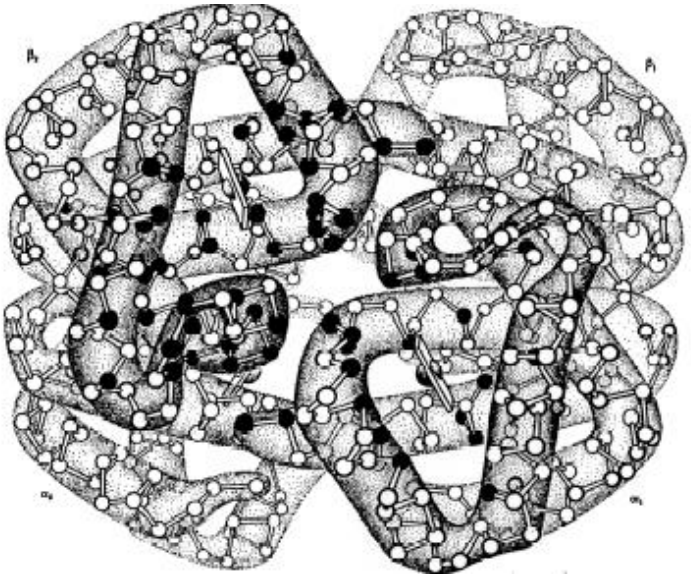


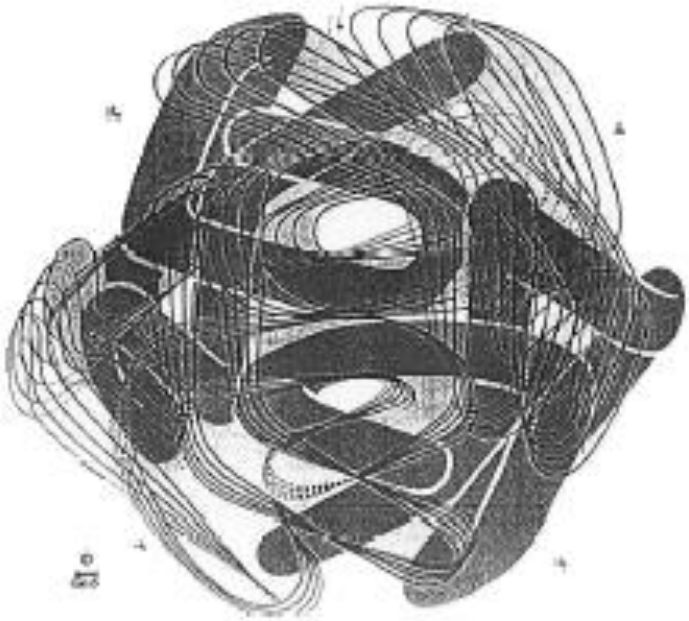
UKLONSKE SLIKE IN STRUKTURE PROTEINOV:

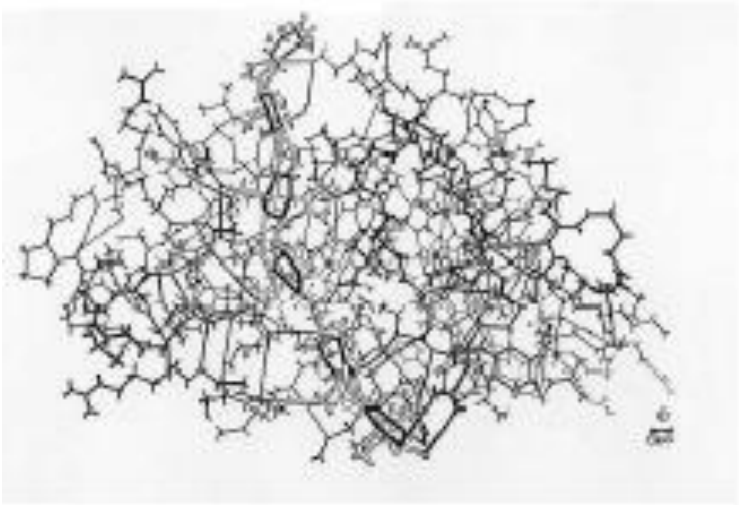
Uklonska slika:

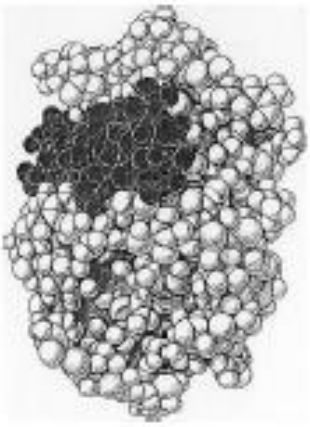


Strukture proteinov, ki so določene s pomočjo uklonskih slik:



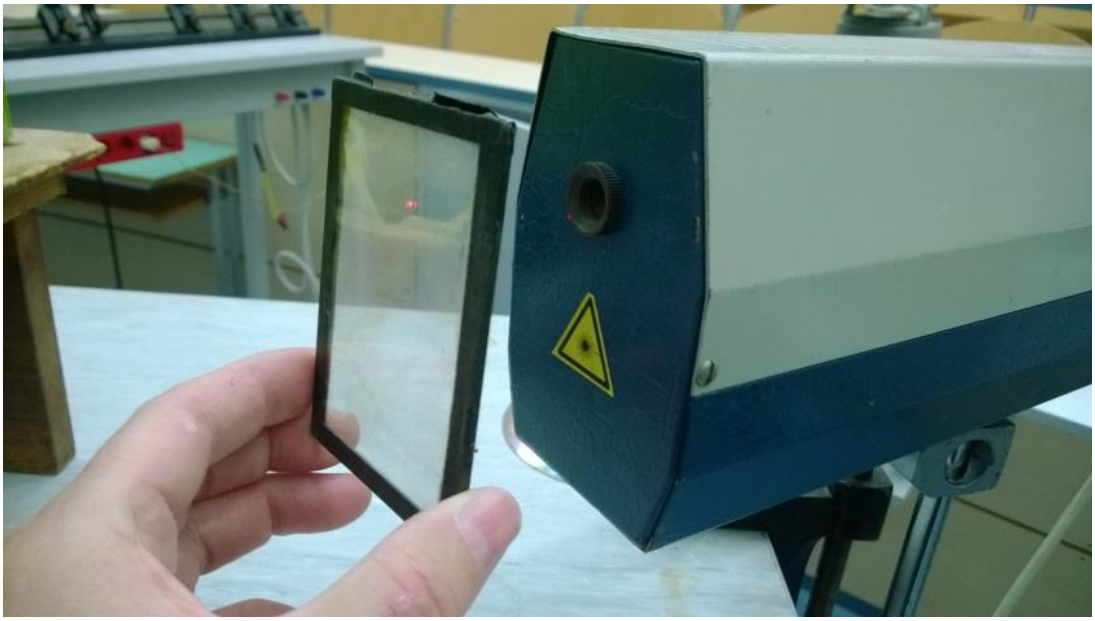


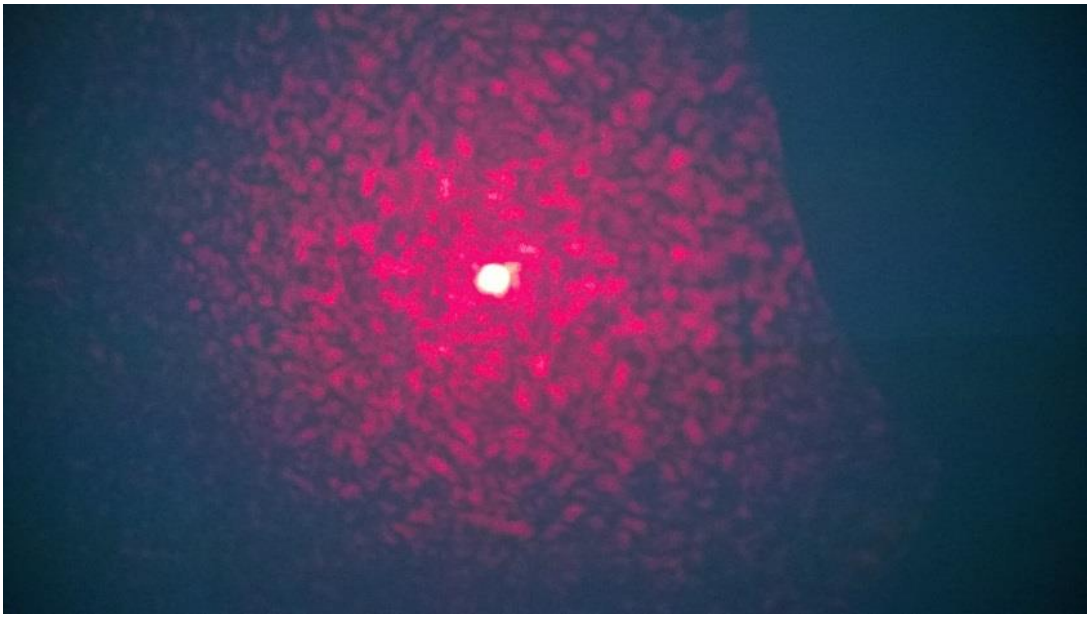




slike: A.L.Lehniger, Principles of Biochemistry

EKSPERIMENT: SIPANJE SVETLOBE (na cvetnem prahu)





Glejte še :

http://physics.fe.uni-lj.si/publications/pdf/nydl2017.pdf

ELEKTROTEHNIŠKI VESTNIK 84(3): 69-75, 2017

My first electron density map: A beginner's guide to small angle X-ray diffraction

Ngai Ying Denise Li, Šárka Perutková, Aleš Iglič and Michael Rappolt

DODATEK: SPREMEMBA FAZE EM VALOVANJA PRI ODBOJU

Študirali bomo odboj EM valovanja na optično redkejši snovi (n' < n) in odboj EM valovanja na optično gostejši snovi (n' > n), kjer sta:

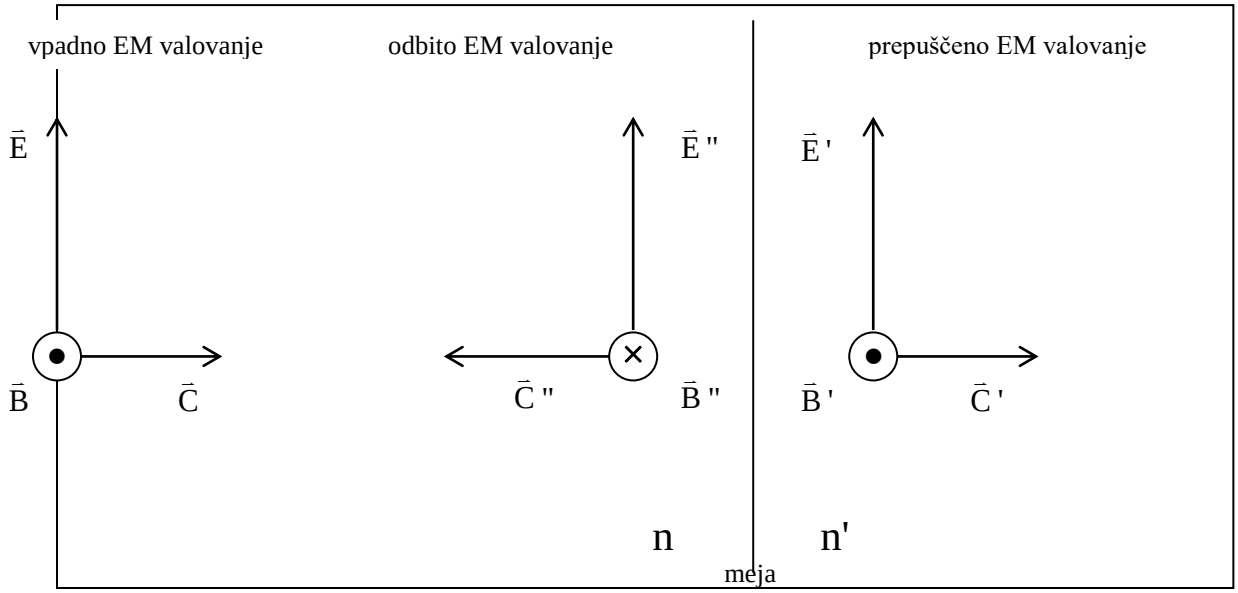
$$n = \sqrt{\varepsilon} \qquad (1)$$

in

$$n' = \sqrt{\varepsilon'} \qquad (2)$$

lomna količnika, ε in ε' pa ustrezni dielektrični konstanti. Upoštevamo μ ≅ 1.

E , B → vpadno EM valovanje
E", B" → odbito EM valovanje
E' , B' → prepuščeno EM valovanje



Slika: Pravokotni vpad EM valovanja na mejo dveh sredstev, $|\vec{C}| = C = |\vec{C}''| = C''$.

Hitrosti EM valovanja v snovi na levi strani meje (C, C") in v snovi na desni strani meje (C') sta:

$$C' = \frac{C_0}{n'}, \qquad (3)$$

$$C = C'' = \frac{C_0}{n}, \qquad (4)$$

kjer je C_0 hitrost EM valovanja v vakuumu. Upoštevamo robna pogoja na meji dveh sredstev:

$$E + E'' = E' , \qquad (5)$$

$$H - H'' = H' . \qquad (6)$$

Iz zvez:

$$B = \mu_0 H, \qquad (7)$$
$$B' = \mu_0 H',$$
$$B'' = \mu_0 H'',$$

kjer smo upoštevali $\mu = \mu' = \mu'' \cong 1$, in iz enačbe (6) sledi

$$B - B'' = B' \qquad (8)$$

Za EM valovanje velja $\vec{E} = \vec{B} \times \vec{C}$, torej

$$E = B\,C, \qquad (9)$$
$$E' = B'\,C',$$
$$E'' = B''C''.$$

Iz enačb (8) in (9) tako sledi

$$\frac{E}{C} - \frac{E''}{C} = \frac{E'}{C'}, \qquad (10)$$

od tod pa

$$E - E'' = E'\frac{C}{C'}. \qquad (11)$$

Ob upoštevanju enačb (3) in (4) iz enačbe (11) sledi

$$E - E'' = E'\left(\frac{n'}{n}\right), \qquad (12)$$

Enačbi (5) in (12) predstavljata sistem dveh enačb za dve neznanki E' in E'' (pri znani vrednosti E, n in n'). Iz enačbe (5) izrazimo E'':

$$E'' = E' - E \qquad (13)$$

in vstavimo v enačbo (12). Tako dobimo:

$$E - E' + E = E'\left(\frac{n'}{n}\right),$$

od tod pa sledi:

$$\boxed{E' = \frac{2n}{(n+n')}E\ .} \qquad (14)$$

Iz enačb (13) in (14) pa dobimo:

$$\boxed{E'' = \left(\frac{n-n'}{n+n'}\right)E\ .} \qquad (15)$$

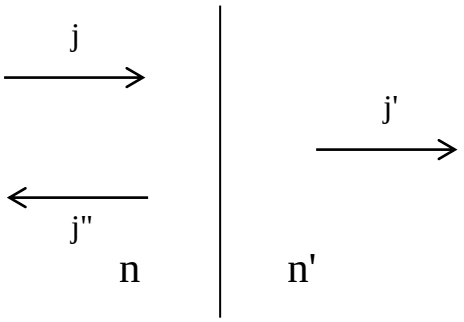
Na osnovi enačbe (15) torej sklepamo, da*:

1. **se EM valovanje na optično redkejši snovi (n' < n) odbije z enako fazo (če E > 0 ⇒ E" > 0)**

in

2. **se EM valovanje na optično gostejši snovi (n' > n) odbije z nasprotno fazo (če E > 0 ⇒ E" < 0).**

Izračunajmo še odbojnost (a) in prepustnost (b).



odbojnost (albedo) $$a = \frac{j''}{j} \qquad (16)$$

prepustnost $$b = \frac{j'}{j}\,. \qquad (17)$$

Ob upoštevanju $\bar{j} = \bar{w}c = \frac{1}{2}\varepsilon\varepsilon_0 E_0^2 c$ iz enačb (14), (15), (16) in (17) sledi:

$$b = \frac{j'}{j} = \frac{\frac{1}{2}\varepsilon'\varepsilon_0 E'^2_0 c'}{\frac{1}{2}\varepsilon\varepsilon_0 E_0^2 c} = \frac{\varepsilon'}{\varepsilon}\cdot\frac{c'}{c}\left(\frac{E'_0}{E_0}\right)^2 = \frac{n'^2}{n^2}\cdot\frac{n}{n'}\left(\frac{E'_0}{E_0}\right)^2 = \frac{n'}{n}\left(\frac{E'_0}{E_0}\right)^2 = \frac{4nn'}{(n+n')^2}\,, \qquad (18)$$

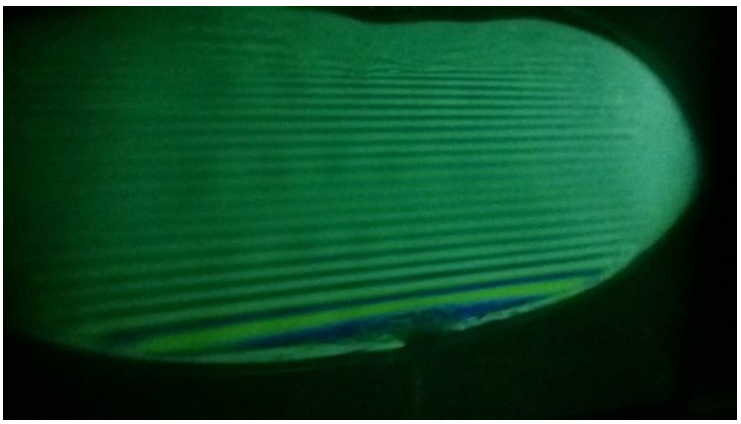
$$a = \frac{j''}{j} = \frac{(n-n')^2}{(n+n')^2}\,, \qquad (19)$$

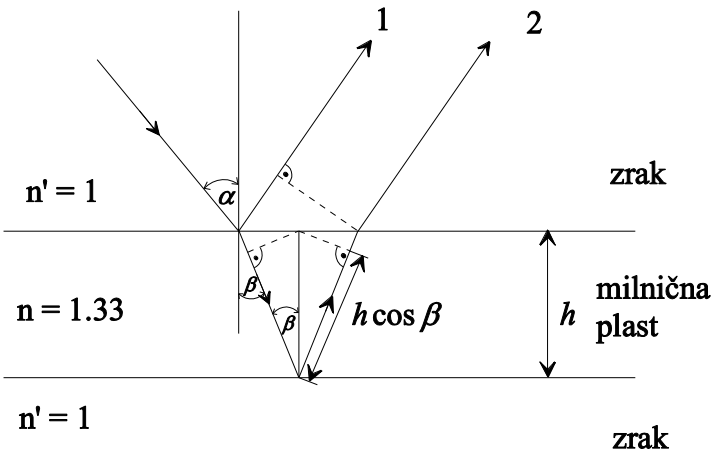
kjer smo upoštevali tudi enačbe (1) - (4) .

* glejte še knjgo J. Strnad, *Fizika II*

INTERFERENCA (ODBOJ) EM VALOVANJA NA TANKIH PLASTEH

INTERFERENCA NA MILNIČNI OPNI





V milnični plasti je valovna dolžina svetlobe enaka $\lambda = \lambda_0 / n$. Razlika v poti (δ) med žarkom 1 in 2 je približno (glejte sliko):

$$\delta = 2h\cos\beta + \frac{\lambda}{2},$$

kjer smo z $\lambda/2$ upoštevali, da se žarek 1 odbije na optično gostejši snovi (n'< n). Pogoj za ojačanje zapišemo v obliki:

$$2h\cos\beta + \frac{\lambda}{2} = N\lambda,$$

kjer je N = 1, 2, … Upoštevamo $\lambda = \lambda_0 / n$ in dobimo:

$$2h\cos\beta + \frac{\lambda_0}{2n} = N\frac{\lambda_0}{n},$$

od koder sledi:

$$\boxed{2nh\cos\beta = \frac{\lambda_0}{2}(2N-1).} \qquad (1)$$

Ob upoštevanju lomnega zakona:

$$\frac{\sin\alpha}{\sin\beta} = n,$$

dobimo:

$$\frac{\sin^2\alpha}{\sin^2\beta} = \frac{\sin^2\alpha}{1-\cos^2\beta} = n^2 \ ,$$

od tod pa sledi:

$$\frac{1}{n^2}\sin^2\alpha = 1-\cos^2\beta \ ,$$

oziroma:

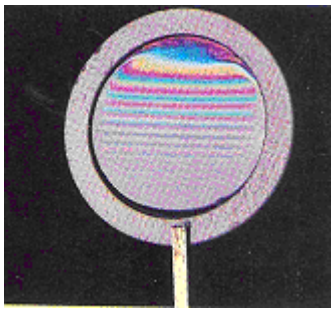
$$\cos\beta = \left(1-\frac{\sin^2\alpha}{n^2}\right)^{1/2}. \qquad (2)$$

Za N = 1 in ob upoštevanju enačbe (2) iz enačbe (1) sledi:

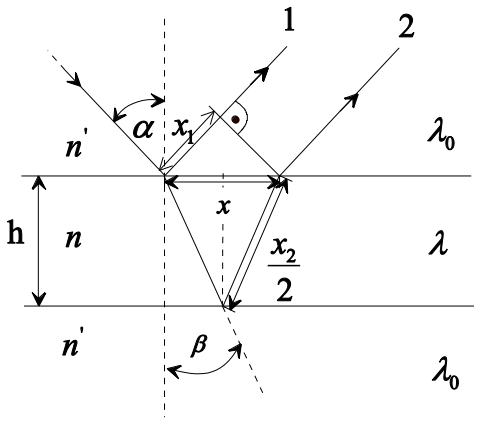
$$2nh\left(1-\frac{\sin^2\alpha}{n^2}\right)^{1/2} = \frac{\lambda_0}{2} \ , \qquad (3)$$

S pomočjo enačbe (6.7.3) izračunamo debelino milnične plasti:

$$h = \frac{\lambda_0}{4}\left(n^2 - \sin^2\alpha\right)^{-1/2} \qquad (4)$$



slika: Serway, 1992



Daljša izpeljava:

$$E = E_0 \cos\left(\omega t - k\,x\right)$$

Pogoj za konstruktivno interferenco žarkov 1 in 2:

$$\pi + \omega t - k_1 x_1 = \omega t - k_2 x_2 + N\,2\pi\,, \tag{D.1}$$

kjer smo upoštevali dodatno fazno razliko $\delta = \pi$, ker je $n > n'$.

Velja:

$$k_2 = \frac{2\pi}{\lambda} = \frac{2\pi\,n}{\lambda_0}\,;\quad k_1 = \frac{2\pi}{\lambda_0}\ . \tag{D.2}$$

Iz enačbe (D.1) sledi:

$$k_2\,x_2 - k_1\,x_1 = N\,2\pi - \pi = \left(2\,N - 1\right)\pi\ . \tag{D.3}$$

Ob upoštevanju enačb (D.2) iz enačbe (D.3) sledi:

$$\frac{2\pi n}{\lambda_0}\,x_2 - \frac{2\pi}{\lambda_0}\,x_1 = \left(2N - 1\right)\pi\ , \tag{D.4}$$

oziroma:

$$n x_2 - x_1 = \frac{\left(2N - 1\right)\lambda_0}{2}\ . \tag{D.5}$$

Velja (glejte sliko):

$$x_1 = x \sin\alpha \tag{D.6}$$

$$x_2 = \frac{x}{\sin\beta} \tag{D.7}$$

Ob upoštevanju enačb (D.6) in (D.7) predelamo levo stran enačbe (D.5) v obliko:

$$n\,x_2 - x_1 = n\frac{x}{\sin\beta} - x\sin\alpha = n\frac{x}{\sin\beta} - x\,n\sin\beta =$$

(kjer smo upoštevali lomni zakon $\sin\alpha = n\sin\beta$)

$$= n\,2h\,tg\,\beta\left(\frac{1}{\sin\beta} - \sin\beta\right) =$$

(kjer smo upoštevali $x = 2h\,tg\,\beta$)

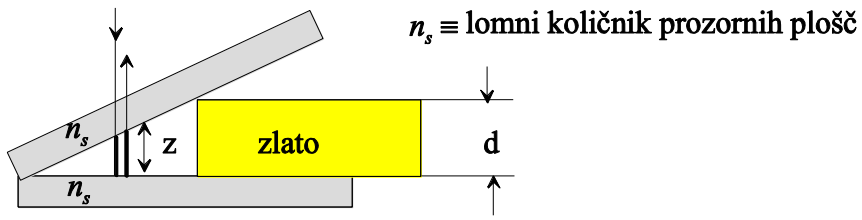
$$= n\,2h\left(\frac{1}{\cos\beta} - \frac{\sin^2\beta}{\cos\beta}\right) = n\,2h\frac{\left(1-\sin^2\beta\right)}{\cos\beta} = 2\,n\,h\cos\beta \quad . \qquad \text{(D.8)}$$

Ob upoštevanju enačbe (D.8) iz enačbe (D.5) sledi:

$$n\,x_2 - x_1 = 2n\,h\cos\beta = \frac{(2N-1)\lambda_0}{2} \quad ,$$

kar se ujema z rezultatom v enačbi (1).

MERJENJE DEBELINE (ZLATEGA) LISTIČA



Pogoj za ojačanje:

$$\delta = 2\,z + \frac{\lambda_0}{2} = N\,\lambda_0 \;,\quad N = 1, 2, 3, \ldots \qquad \text{(5)}$$

Iz (5) sledi:

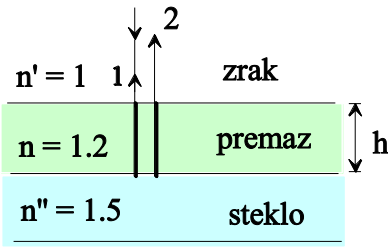
$$z + \frac{\lambda_0}{4} = N\frac{\lambda_0}{2} \;, \qquad \text{(6)}$$

torej:

$$\boxed{z = \frac{1}{4}(2N-1)\lambda_0}$$

INTERFERENČNI FILTER

(žarek gre skozi vse tri plasti, t.j. zrak, premaz, steklena ploščica)



$$\boxed{n'' > n > n'}$$

Pogoj za oslabitev odbitih žarkov 1 in 2:

$$2h = N\frac{\lambda}{2}, \qquad N = 1, 3, 5 \tag{7}$$

kjer je

$$\lambda = \frac{\lambda_0}{n} \tag{8}$$

Torej:

$$\boxed{2h = N\frac{\lambda_0}{2n},} \qquad N = 1, 3, 5 \tag{9}$$

Oziroma:

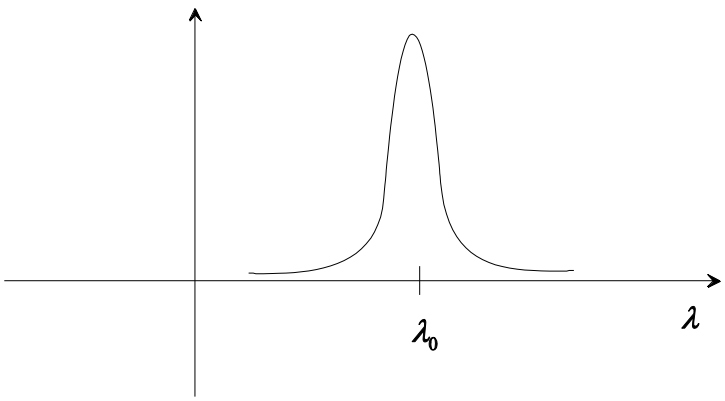
$$2h = (2N+1)\frac{\lambda_0}{2n}, \quad N = 0, 1, 2. \tag{10}$$

Debeline vrhnje plasti z lomnim količnikom n, ki ustrezajo oslabitvam odbite svetlobe:

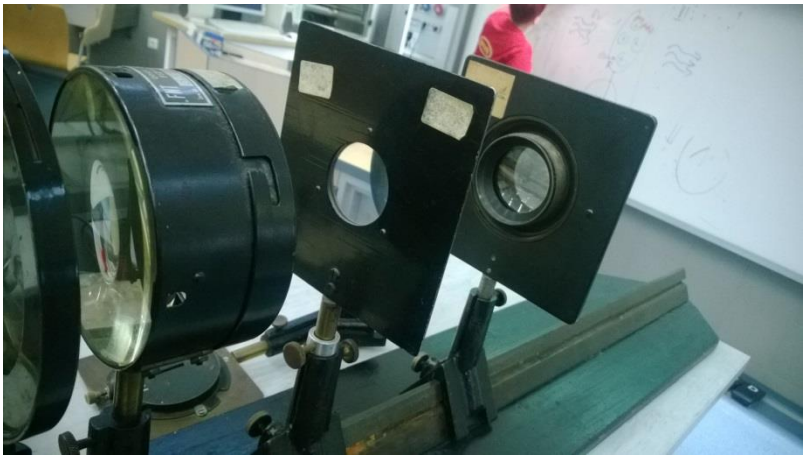
$$\boxed{h = (2N+1)\frac{\lambda_0}{4n},} \quad N = 0, 1, 2. \tag{11}$$

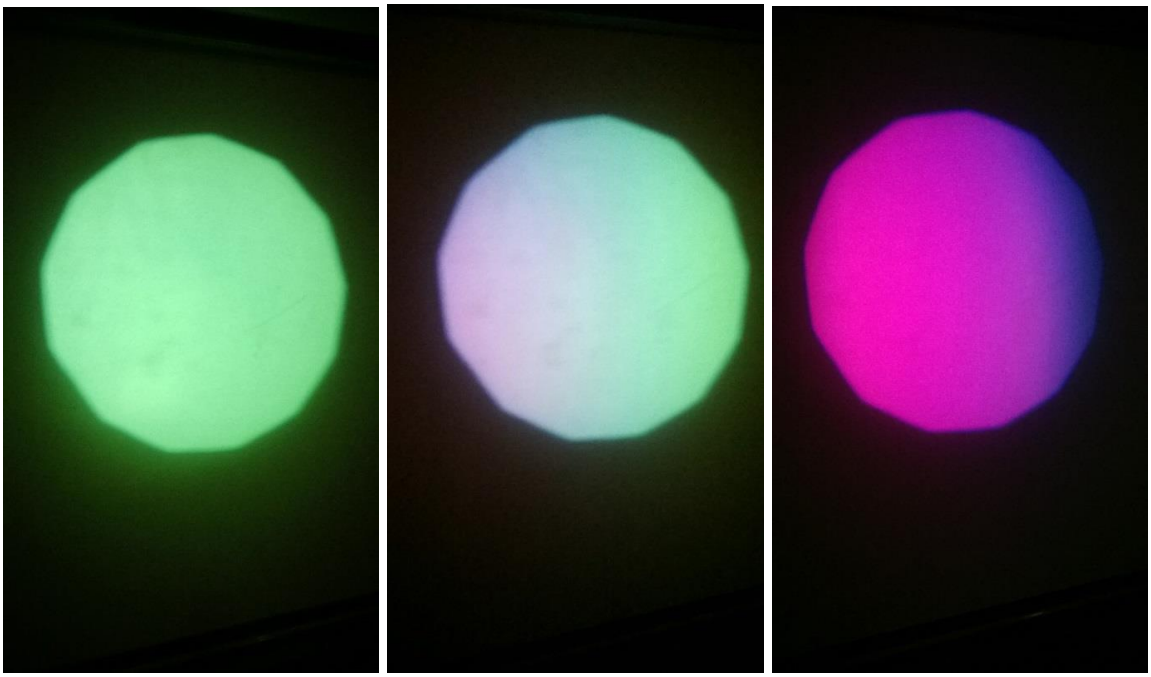
Najmanjša debelina je za N = 0:

$$\boxed{h_{\min} = \frac{\lambda_0}{4n},} \tag{12}$$

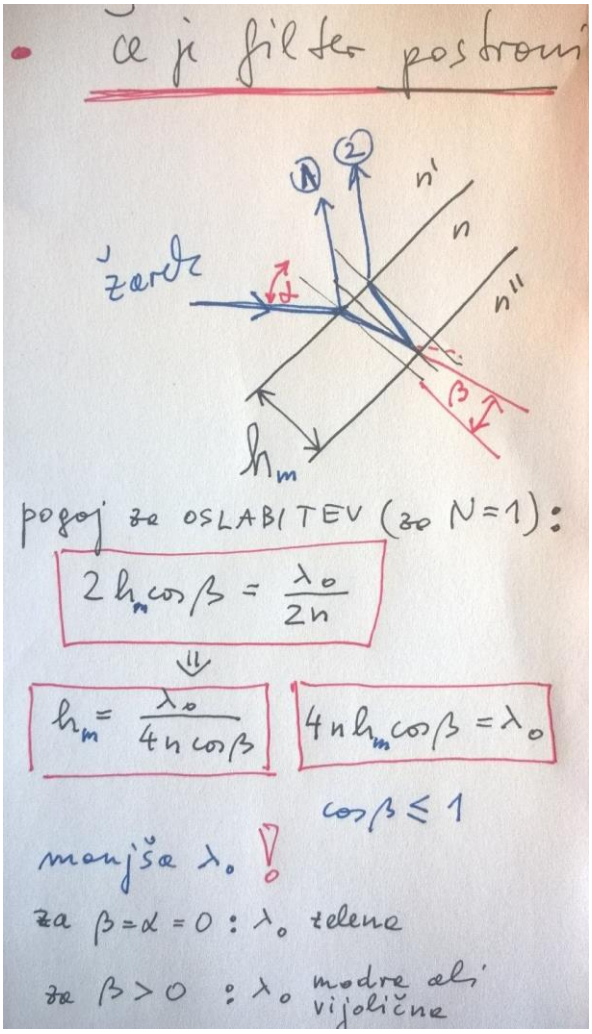


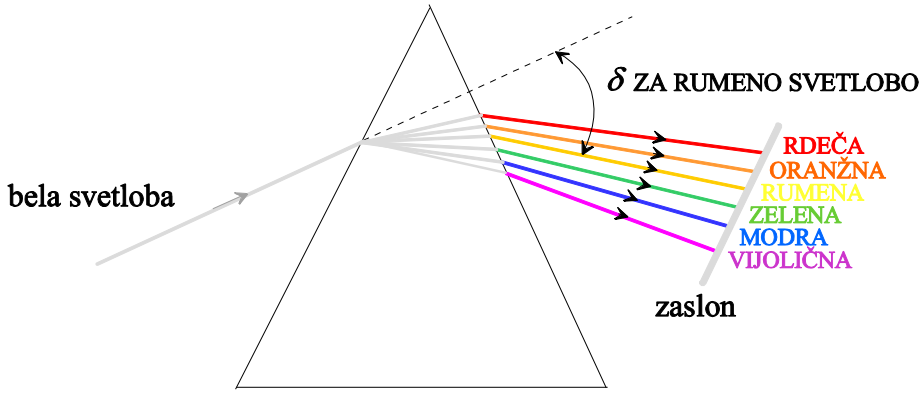
EKSPERIMENT: INTERFERENČNI FILTER



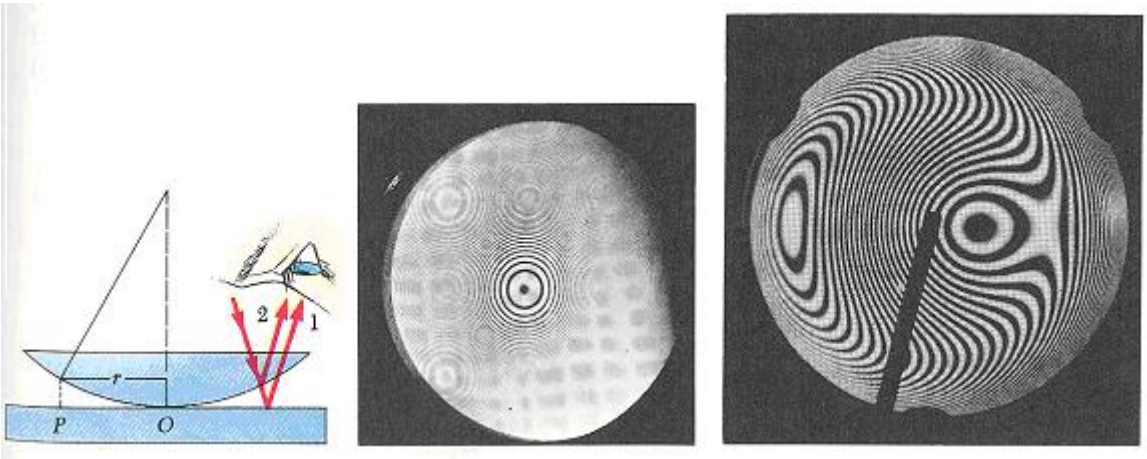


Opombe: n'' > n > n' , žarek gre skozi vse tri plasti (zrak, premaz, steklena ploščica)

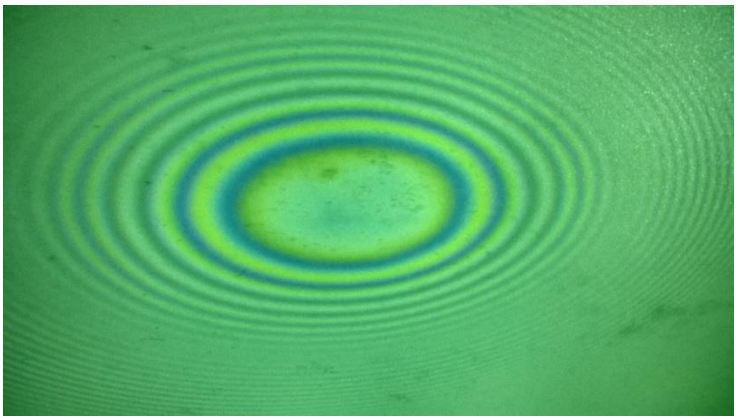




NEWTONOVI KOLOBARJI



slike: Serway (knjiga)

naša slika: 

• NEWTONOVI KOLOBARJI:

Plankonveksno leco postavimo s konveksno stranjo navzdol na ravno stekleno ploščo. Pravokotno na površino leče posvetimo z enobarvno svetlobo valovne dolžine 670 nm. Na ravni ploskvi leče opazujemo interferenčne kolobarje odbite svetlobe. Koliksen je krivinski polmer leče, če je polmer dvajsetega temnega interferenčnega kolobarja 1,1 cm ?

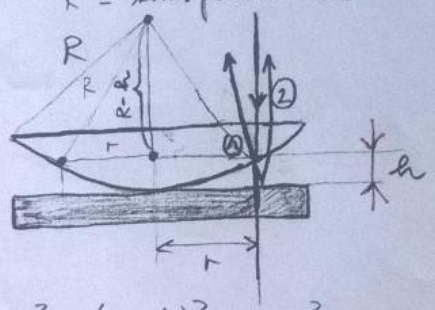
$\lambda_0 = 670$ nm

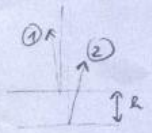
$r = 1{,}1$ cm

$\boxed{N = 20}$

$r \equiv$ radij kolobarja

$R \equiv$ kriv. polmer leče





oslabitev: $\left(\frac{\lambda_0}{2}, \frac{3}{2}\lambda_0, \frac{5}{2}\lambda_0 \ldots\right)$

$\delta_{total} = (2N+1)\frac{\lambda_0}{2}$, $N = 0, 1, 2,$

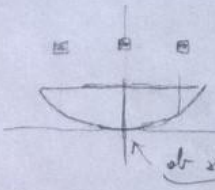
$\delta = \delta_{total} - \delta_r = (2N+1)\frac{\lambda_0}{2} - \frac{\lambda_0}{2} =$

$= N\lambda_0 + \frac{\lambda_0}{2} - \frac{\lambda_0}{2} = N\lambda_0$

$r^2 + (R-h)^2 = R^2$

$r^2 + R^2 - 2Rh + h^2 = R^2$, $h^2 \ll 1$

$r^2 - 2Rh = 0 \Rightarrow \boxed{h = \frac{1}{2} r^2/R}$



ob stiku temna lisa $\left(\delta_r = \frac{\lambda}{2}\right)$

oslabitev:

$\boxed{\delta_r = \frac{\lambda_0}{2}}$ $\boxed{\delta_{total} = \delta_r + \delta}$ $\Rightarrow$ $\boxed{\delta = 2h = N\lambda_0}$ $N = 0, 1, \ldots 20, \ldots$

$\Downarrow$

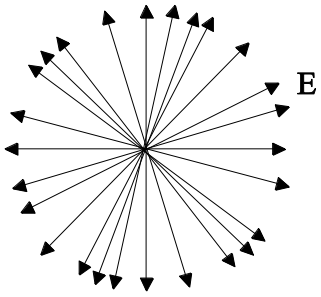
$\boxed{h = N\frac{\lambda_0}{2}}$

$N=20$: $h = \frac{1}{2} r^2/R = N\frac{\lambda_0}{2} = 10\lambda_0$ $\rightarrow$ $R = \frac{r^2}{N\lambda_0}$

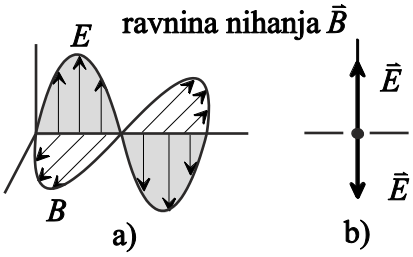
$R = \frac{r^2}{2 \cdot 10 \cdot \lambda_0} = \frac{(1{,}1)^2 \cdot 10^{-4}}{2 \cdot 670 \cdot 10^{-8}} = 90{,}3$ cm ✓

POLARIZACIJA SVETLOBE
POLARIZIRANA IN NEPOLARIZIRANA SVETLOBA

- **Nepolarizirana svetloba (EM valovanje)**

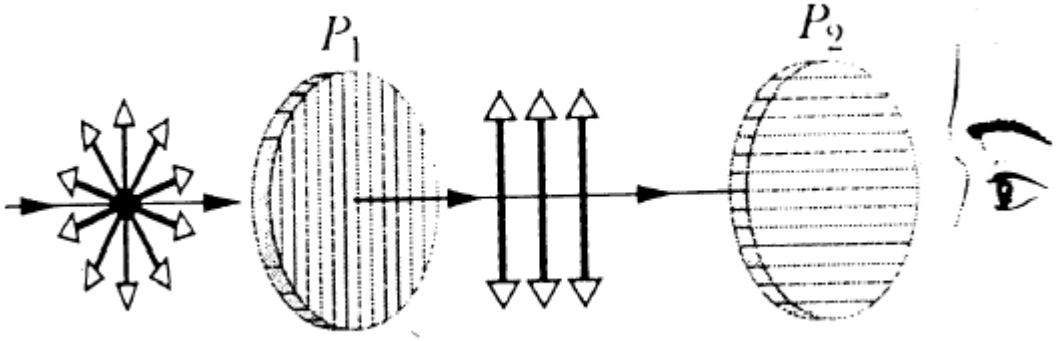


- **Linearno polarizirano elektromagnetno valovanje (svetloba)**



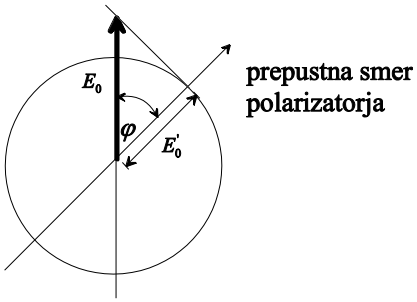
Polarizacija v mehaniki: Nepolarizirano (np) in linearno polarizirano (lp) valovanje na prožni vrvici. P polarizator, A analizator. Za analizatorjem ni valovanja. Daljice kažejo smer gibanja delov vrvice.

PREPUSTNOST POLARIZATORJA



popolnoma nepolarizirano
EM valovanje

linearno polarizirano
EM valovanje



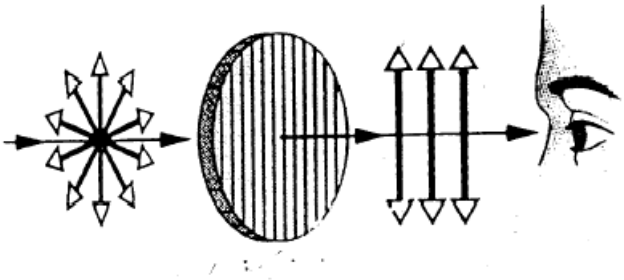
$$E_0' = E_0 \cos\varphi$$

$$j_{prep} = \frac{1}{2}\varepsilon_0 E_0'^2 c_0 = \frac{1}{2}\varepsilon_0 E_0^2 \left(\cos^2 \varphi\right) c_0$$

$$j_{vpad} = \frac{1}{2}\varepsilon_0 E_0^2 c_0$$

prepustnost polarizatorja $= \dfrac{j_{prep}}{j_{vpad}} = \cos^2 \varphi$

Primer: vpadno valovanje je popolnoma nepolarizirano

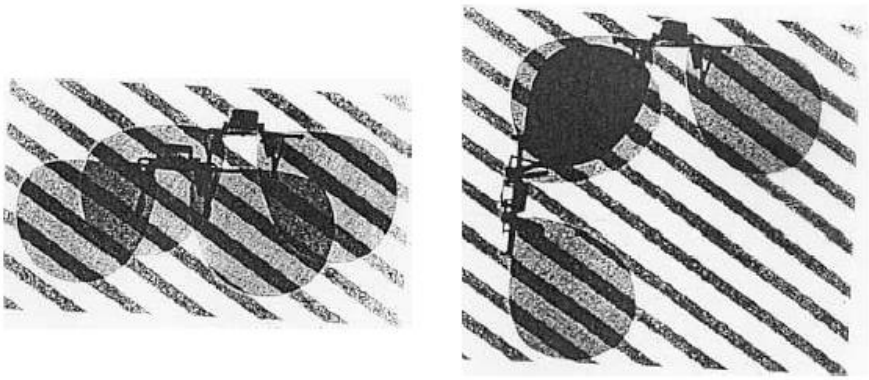


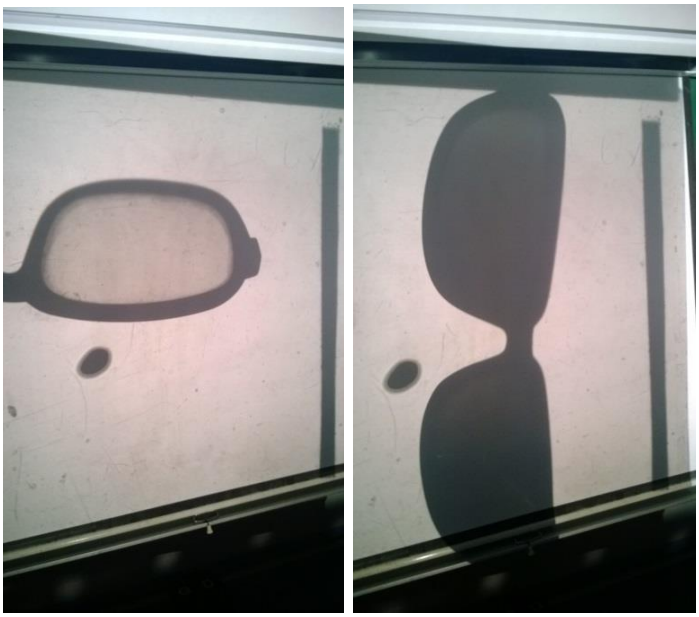
$$\bar{j}_{prep} = \frac{1}{2}\varepsilon_0 E_0^2 \overline{\cos^2\varphi} c_0 = \frac{1}{2}\varepsilon_0 E_0^2 \frac{1}{2} c_0 ,$$

ker $\overline{\cos^2\varphi} = \frac{1}{2}$

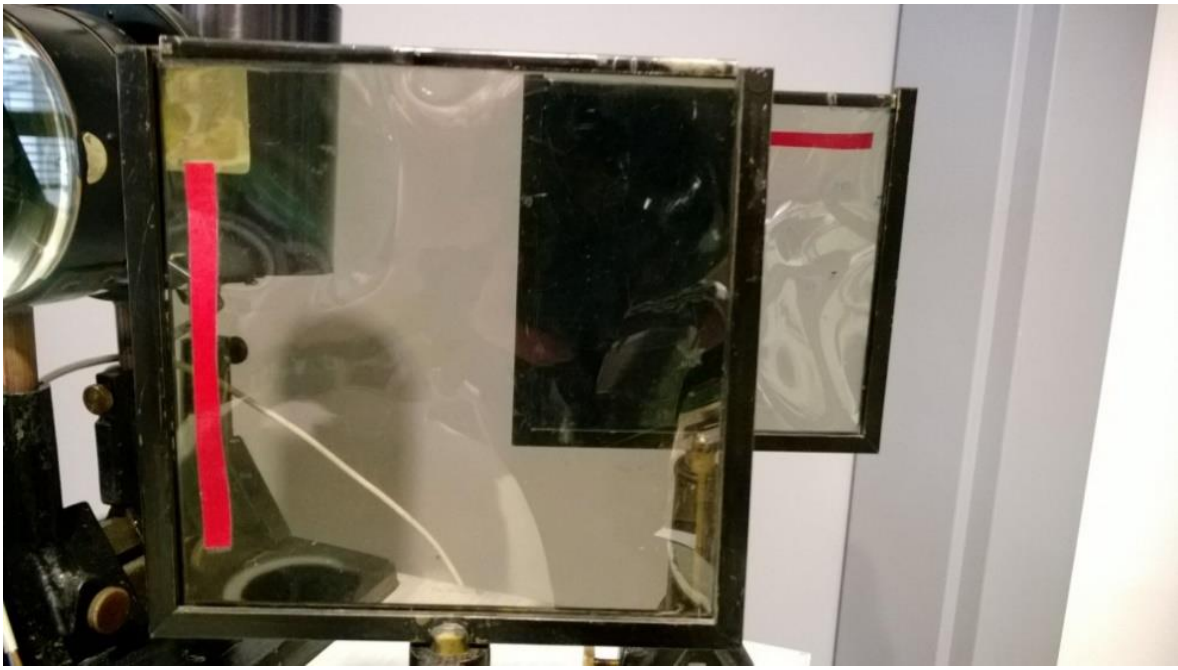
Torej: prepustnost je $\frac{1}{2}$.

PRIMER: POLARIZACIJSKA OČALA





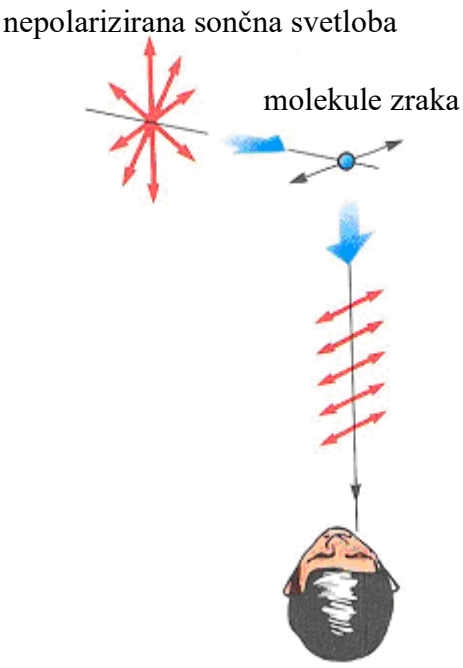
EKSPERIMENT: prepustnost kombinacije polarizatorjev (analizator in polarizator na optični klopi)

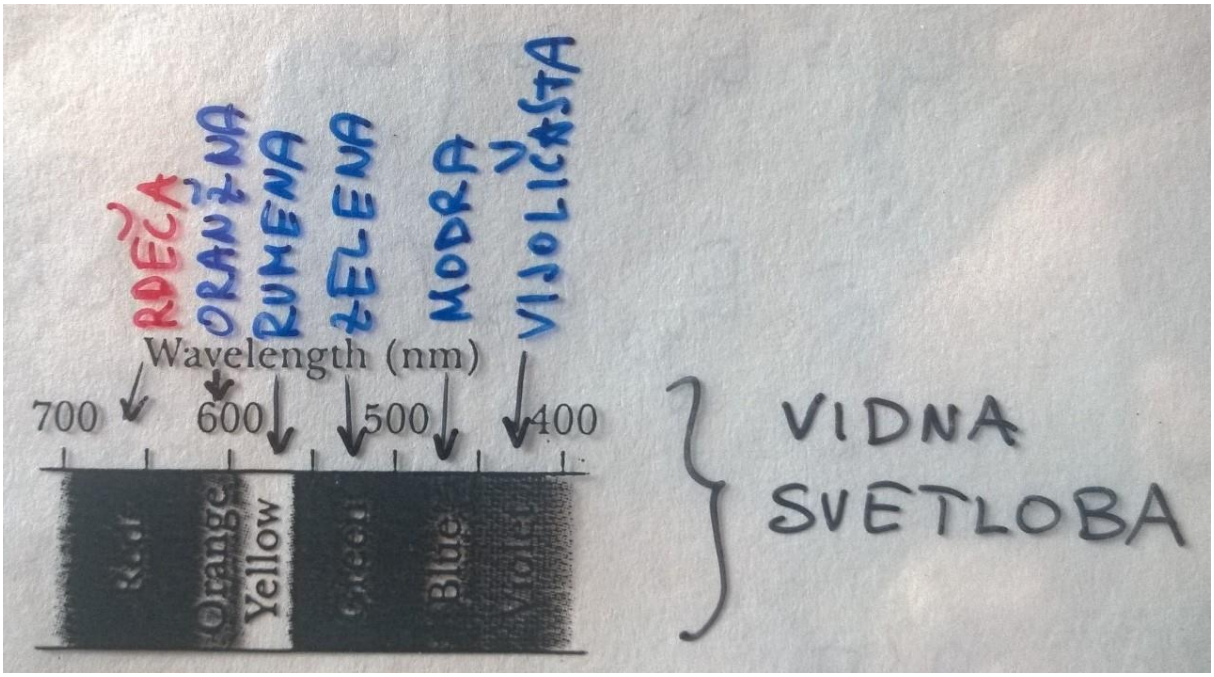


SIPANJE SONČNE SVETLOBE

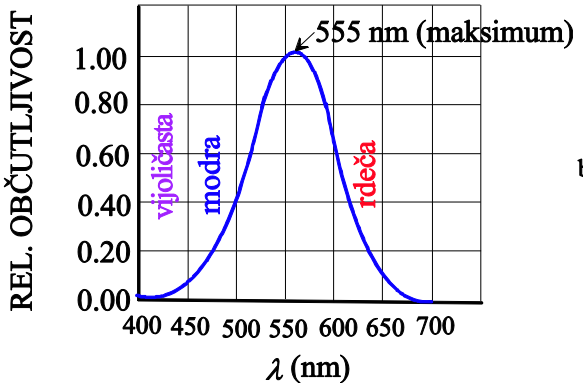
Sipana svetloba (sevanje molekul kot dipolov)

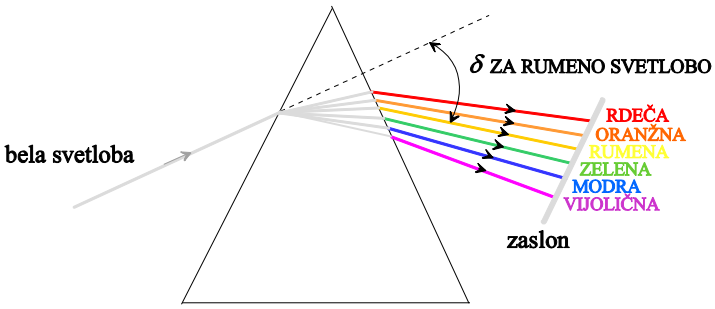
$$\left.\begin{aligned} j &\propto \omega^4 \propto \frac{1}{\lambda^4} \\ j &\propto \sin^2 \vartheta \end{aligned}\right\} \text{ gostota energijskega toka}$$



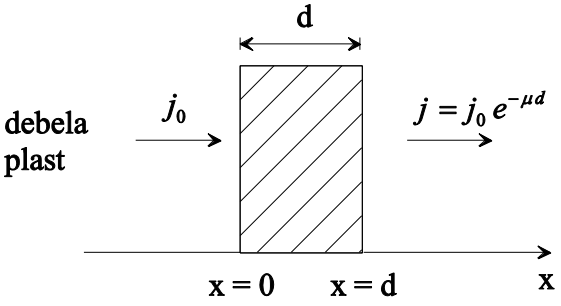


ČLOVEŠKO OKO



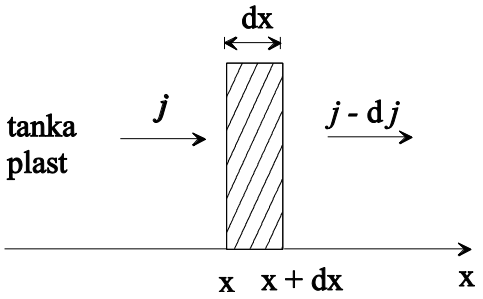


ABSORPCIJA EM VALOVANJA (svetlobe)



$\mu \equiv$ absorpcijski koeficient

Izpeljava:



$$\mathrm{d}j = -\mu\, j\, \mathrm{d}x, \tag{1}$$

$$\frac{\mathrm{d}j}{j} = -\mu\, \mathrm{d}x\,, \tag{2}$$

$$\int_{j_0}^{j(x)} \frac{\mathrm{d}j}{j} = -\mu \int_0^x \mathrm{d}x\,, \tag{3}$$

$$\ln j \Big|_{j_0}^{j(x)} = -\mu\, x\,, \tag{4}$$

$$\ln j(x) - \ln j_0 = -\mu\, x\,, \tag{5}$$

$$\ln\left(\frac{j(x)}{j_0}\right) = -\mu\, x\,, \tag{6}$$

$$\boxed{j(x) = j_0\, e^{-\mu x}} \tag{7}$$

Razpolovna debelina:

$$\frac{j_0}{2} = j_0\, e^{-\mu x_{1/2}} \ ,$$

$$\frac{1}{2} = e^{-\mu x_{1/2}} \ ,$$

$$\ln\left(\frac{1}{2}\right) = -\mu\, x_{1/2} \ ,$$

$$\boxed{x_{1/2} = \frac{\ln 2}{\mu}}$$

Najmočnejša absorpcija EM valovanja je pri frekvenci EM valovanja (ν), ki se ujema z eno izmed lastnih frekvenc:

EKSPERIMENT: ABSORPCIJA (kiveta s črnilom, senzor)

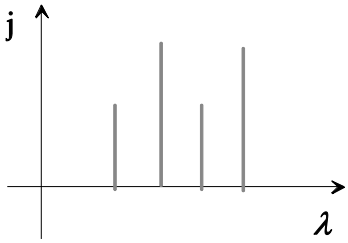
0
1
2
3
PHYWE
0,3mA
PHYWE

FOTOMETRIJA

VRSTE SPEKTROV

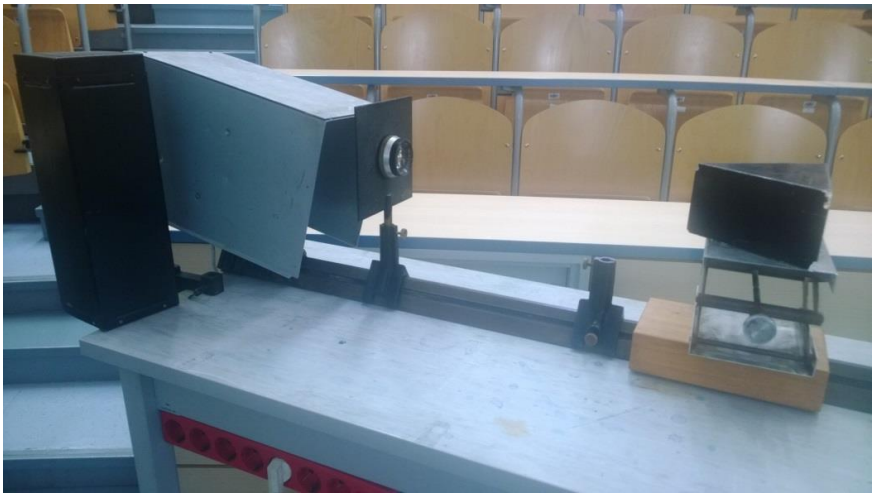
- **absorpcijski** spekter
- **emisijski** spekter: spekter, ki ga svetila (izvori EM valovanja) sevajo
- **odbojni** spekter

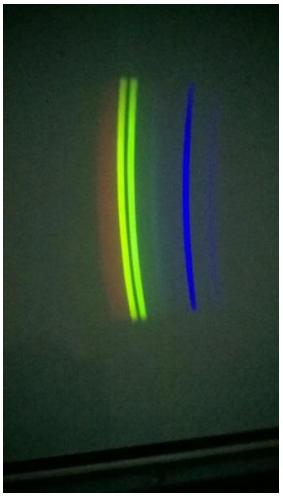
- **črtasti** emisijski spektri (v plinih)



$j \equiv$ gostota energijskega toka
$\lambda \equiv$ valovna dolžina

Primer: EMISIJSKI spekter živega srebra- Hg luč

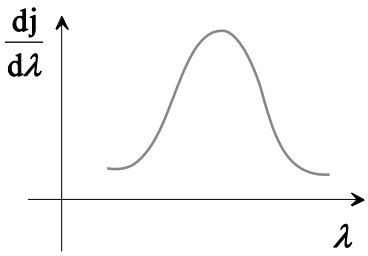




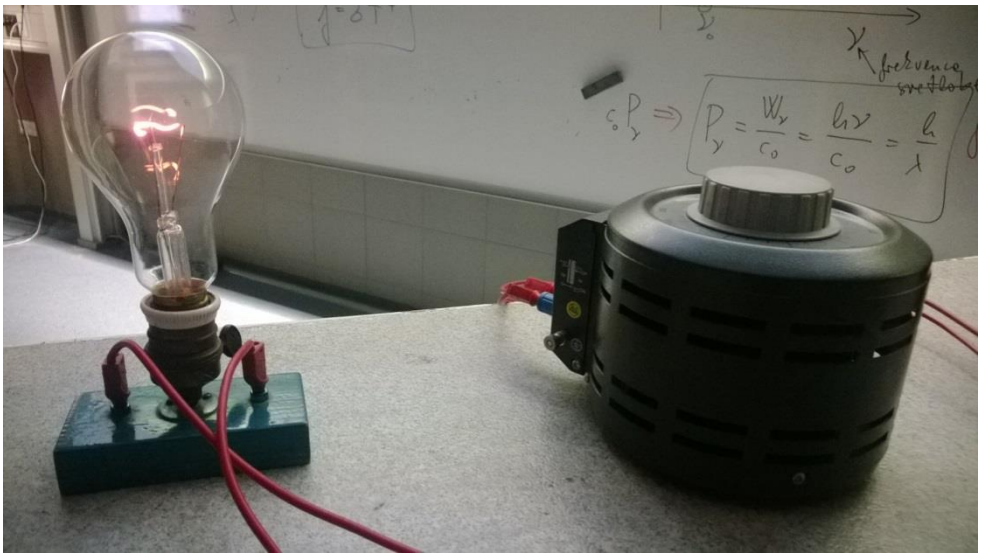
Primer: EMISIJSKI spekter natrija - črtasti spekter (Na + primesi, žarnica, prizma in uklonska mrežica)

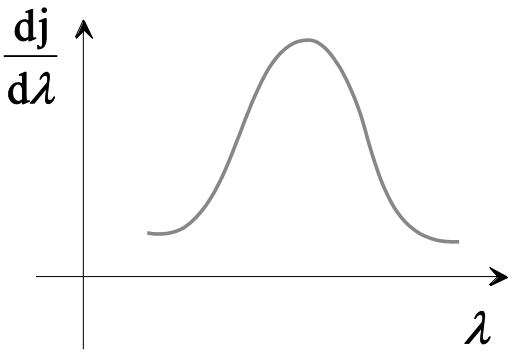
Primer: FLUORESCENCA (natrijeva para absorbira in izseva svetlobo)

- **zvezni** spektri



Primer: žarnica + spremenljivi upor (zvezni spekter)





$\mathrm{d}j$ je delež gostote energijskega toka j v intervalu $\left[\lambda - \frac{1}{2}\mathrm{d}\lambda\ ,\ \lambda + \frac{1}{2}\mathrm{d}\lambda\right]$

Skupna gostota energijskega (svetlobnega) toka:

$$j = \int \left(\frac{\mathrm{d}j}{\mathrm{d}\lambda}\right) \mathrm{d}\lambda$$

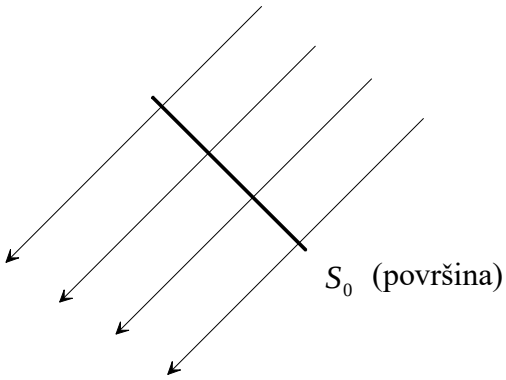
OSVETLJENOST (E)

EKSPERIMENT: ročna baterijska svetilka sveti na belo tablo v predavalnici

Gostota energijskega (svetlobnega) toka:

$$j = \frac{P}{S_0} \qquad (1)$$

kjer je P svetlobni (energijski) tok

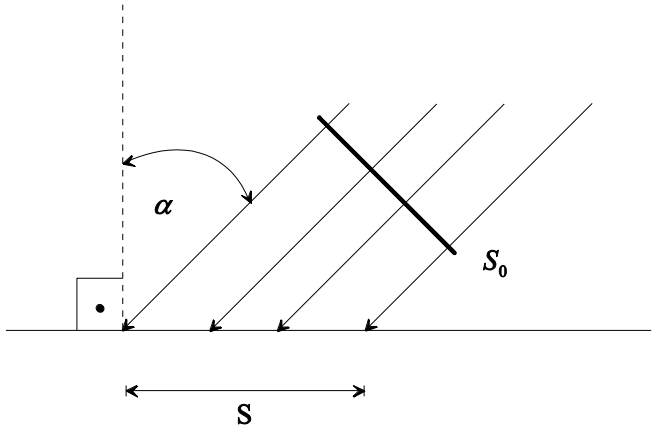


Osvetljenost dela ravne ploskve s površino S je (glejte sliko):

$$E = \frac{P}{S} = \frac{P}{(S_0/\cos\alpha)} = \left(\frac{P}{S_0}\right)\cos\alpha \ . \qquad (2)$$

Ob upoštevanju enačbe (1) iz enačbe (2) sledi:

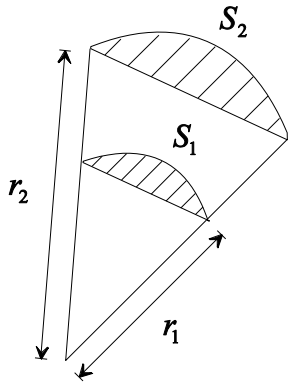
$$\boxed{E = j\cos\alpha} \qquad (3)$$



$$\cos\alpha = \frac{S_0}{S} \Rightarrow S = \frac{S_0}{\cos\alpha}$$

SVETILNOST TOČKASTEGA SVETILA

Definicija prostorskega kota

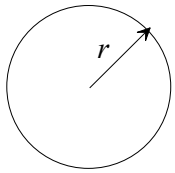


$$\frac{S_1}{r_1^2} = \frac{S_2}{r_2^2}$$

prostorski kot $\Omega = \frac{S}{r^2}$

Poln prostorski kot je 4π :

$$\Omega_{\text{poln}} = \frac{S_{\text{krogle}}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$



Infinitezimalni prostorski kot:

$$\boxed{\mathrm{d}\Omega = \frac{\mathrm{d}S}{r^2}} \quad . \tag{4}$$

Točkasto svetilo, ki seva enakomerno na vse strani:

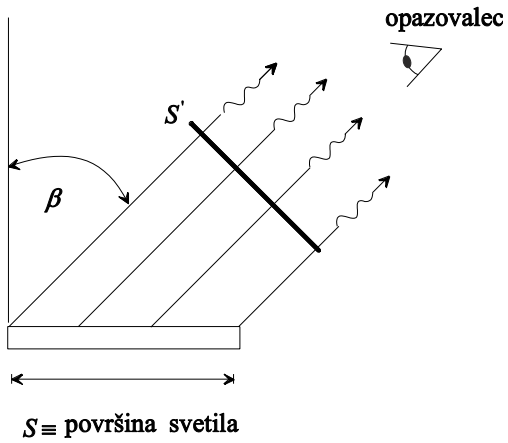
$$\boxed{j = \frac{P}{S} = \frac{P}{4\pi r^2} = \frac{I}{r^2}} \quad , \tag{5}$$

kjer je svetilnost točkastega svetila:

$$\boxed{I = \frac{P}{4\pi}} \quad . \tag{6}$$

SVETILNOST IN SVETLOST SVETILA

- **Ploskovno svetilo**



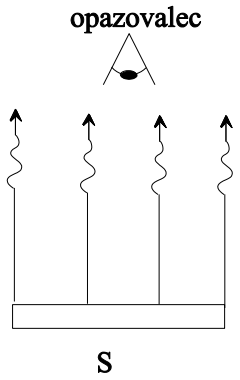
$$\cos\beta = \frac{S'}{S} \Rightarrow S' = S\cos\beta$$

svetilnost je enaka: $$\boxed{I = B(\beta)S' = B(\beta)S\cos\beta} \quad , \tag{7}$$

kjer B definiramo kot **svetlost** svetila

poseben primer: opazovanje ploskovnega svetila pod pravim kotom:

$$\boxed{I = B(\beta = 0)\, S} \tag{8}$$



posplošitev:

$$\boxed{\mathrm{d}I = B(\beta)\cos\beta\,\mathrm{d}S}$$ če svetilo ni ravna ploskev in sevanje ni izotropno (9)

Lambertov zakon:

$$B(\beta) = B_0 = \text{konst.} \tag{10}$$

$$\mathrm{d}I = B_0 \cos\beta\,\mathrm{d}S \tag{11}$$

Meritev: PRIMERJALNI FOTOMETER (model)

Zaključek :

$$\boxed{\mathrm{d}j = \frac{\mathrm{d}I}{r^2}}$$

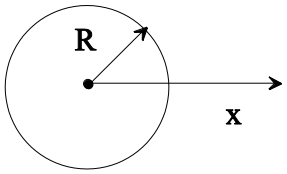
$$\boxed{\mathrm{d}I = \frac{\mathrm{d}P}{\mathrm{d}\Omega}}$$

$$\boxed{\mathrm{d}E = \frac{\mathrm{d}I}{r^2}\cos\alpha}$$

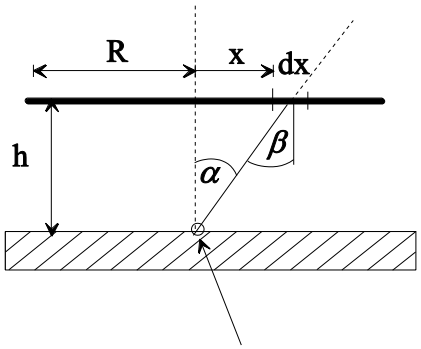
$$\boxed{\mathrm{d}I = B\,\mathrm{d}S\cos\beta}$$

PRIMER: Okroglo ploskovno svetilo s polmerom R seva po Lambertovem zakonu na spodaj ležečo površino.

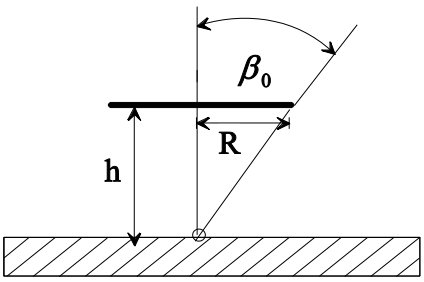
Tloris:



stranski ris:



računamo osvetljenost E v točki



$$\cos\beta = \frac{h}{r} \quad \Rightarrow \quad r = \frac{h}{\cos\beta}$$

$$tg\,\beta = \frac{x}{h}$$

$$\frac{1}{\cos^2\beta}\,\mathrm{d}\beta = \frac{\mathrm{d}x}{h}$$

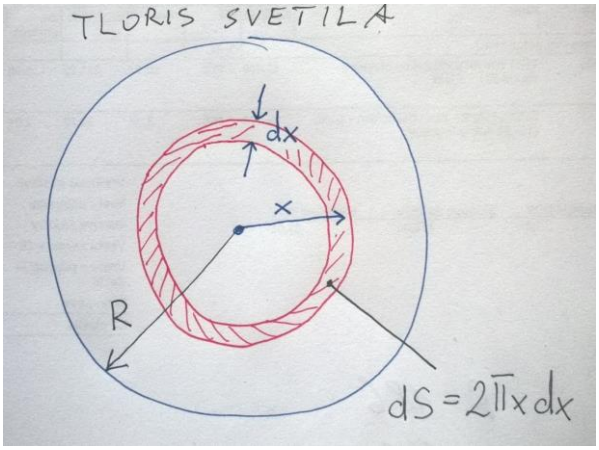
$$\mathrm{d}x = \frac{h}{\cos^2\beta}\,\mathrm{d}\beta$$

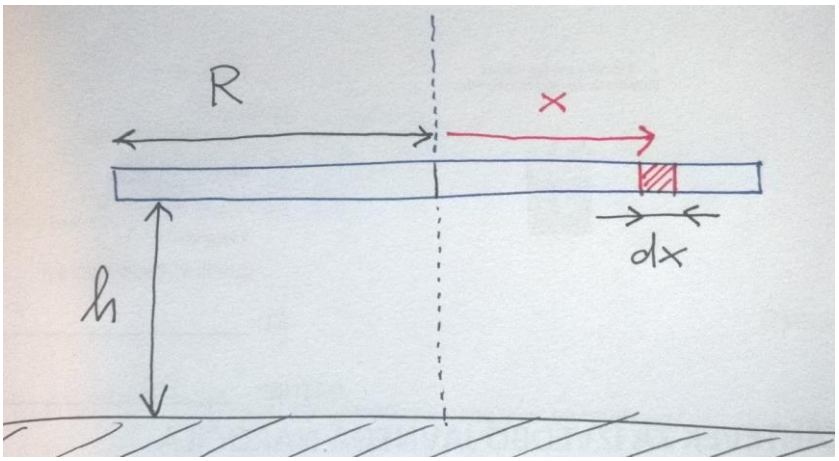
Velja:

$$\mathrm{d}I = B_0\cos\beta\;2\pi\,x\,\mathrm{d}x\,, \qquad (12)$$

$$\mathrm{d}E = \frac{\mathrm{d}I}{r^2} \cos\alpha = \frac{B_0}{r^2} \cos^2\beta \; 2\pi x \mathrm{d}x \quad . \tag{13}$$

V zgornjih dveh enačbah smo upoštevali $\mathrm{dS} = 2\pi x \mathrm{d}x$ ter $\alpha = \beta$ (glejte slike) :





Torej:

$$E = \int \mathrm{d}E = \int \frac{B_o \cos^4\beta 2\pi h \sin\beta h \mathrm{d}\beta}{\mathrm{h}^2 \cos\beta \cos^2\beta} = \int 2\pi B_0 \sin\beta \cos\beta \mathrm{d}\beta = \pi B_0 \int \sin(2\beta)\mathrm{d}\beta =$$

$$= \frac{\pi B_0}{2} \int_0^{2\beta_0} \sin\omega \mathrm{d}\omega = \frac{\pi B_0}{2}(-\cos\omega)\Big|_0^{2\beta} = \frac{\pi B_0}{2}(-\cos 2\beta_0 + 1) = \frac{\pi B_0}{2}(1 - \cos 2\beta_0). \tag{14}$$

Če je ploščato svetilo **zelo veliko** gre $\beta_0 \to \frac{\pi}{2}$ in iz enačbe (6.11.14) sledi:

$$\boxed{E \to \pi B_0} \tag{15}$$