

FARADAYEV ZAKON (indukcija)

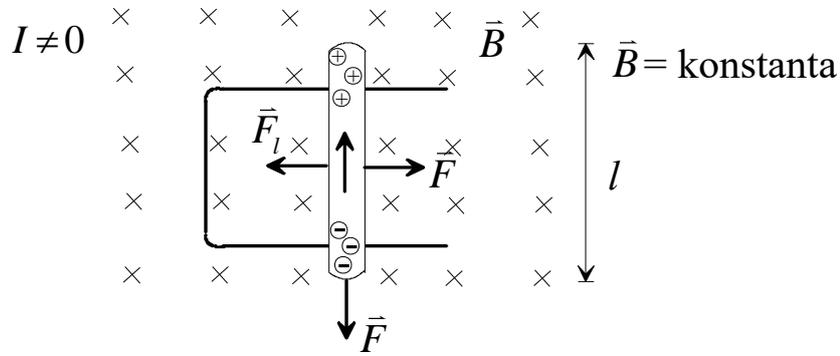


**Michael Faraday
(1791 – 1867)**

INDUKCIJA PRI PREMIKANJU RAVNEGA VODNIKA



vodnik moramo vleči s silo, čeprav $v = \text{konst.}$



\vec{F} = zunanja sila v desno

sila na prevodniške elektrone : $\vec{F}_- = -e_0 \vec{v} \times \vec{B}$

sila na vodnik (premično prečko) po katerem teče ind. tok I : $\vec{F}_l = I \vec{l} \times \vec{B}$

Če $\vec{v} = \text{konst.}$ velja:

$$\vec{F} = -\vec{F}_l = -I \vec{l} \times \vec{B}$$

Delo zunanje sile \vec{F} : $\vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{v} dt = U_i I dt$

$$\vec{F} \cdot \vec{v} = -I (\vec{l} \times \vec{B}) \cdot \vec{v} = U_i I$$

$$U_i = -(\vec{l} \times \vec{B}) \cdot \vec{v}$$

$$U_i = \vec{v} \cdot (\vec{B} \times \vec{l})$$

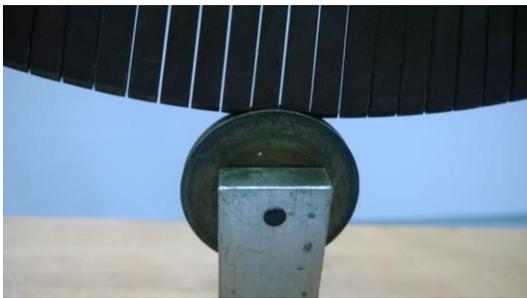
nehomogeno polje in neraven vodnik:

$$U_i = \int \vec{v} \cdot (\vec{B} \times d\vec{l})$$

Barlovo kolo



$$U_i = \vec{v} \cdot (\vec{B} \times \vec{l})$$



+

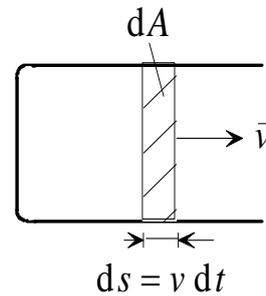


$$U_i = \vec{v} \cdot (\vec{B} \times \vec{l})$$

$$U_i dt = \vec{v} \cdot (\vec{B} \times \vec{l}) dt = - \vec{v} dt \cdot (\vec{l} \times \vec{B}) = - d\vec{s} \cdot (\vec{l} \times \vec{B}) =$$

$$= - \vec{B} \cdot (d\vec{s} \times \vec{l}) = - \vec{B} \cdot d\vec{S}$$

$$U_i = - \frac{\vec{B} \cdot d\vec{S}}{dt}$$



če $\vec{B} = \text{konst.}$: $\vec{B} \cdot d\vec{S} = d(\vec{B} \cdot \vec{S}) = d(\Phi_m)$

$$U_i = - \frac{d\Phi_m}{dt}$$

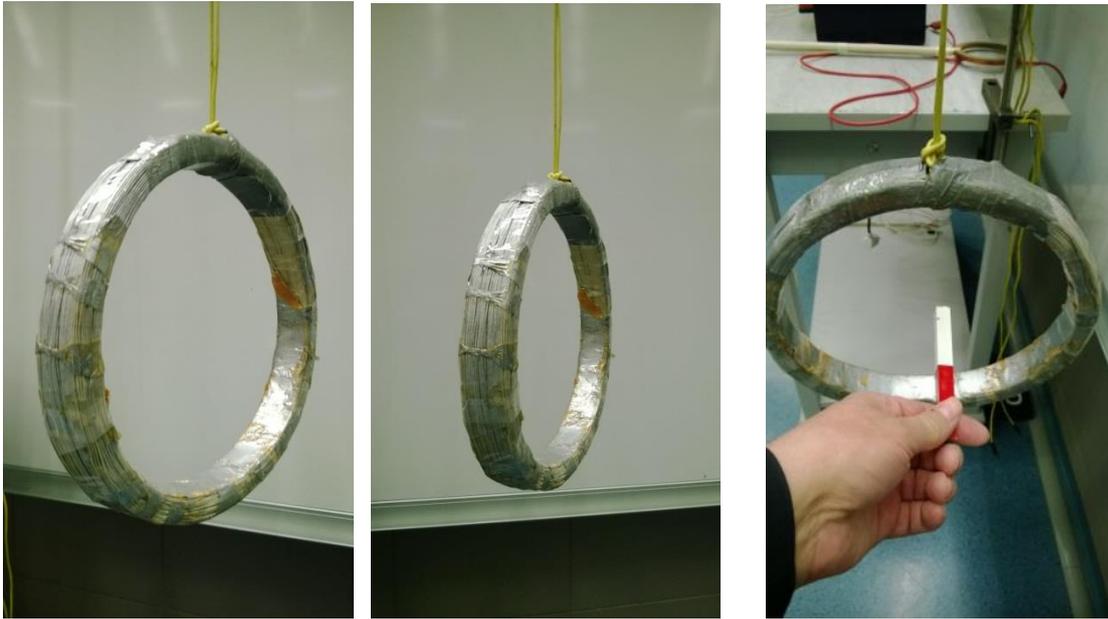
$$\Phi_m = \int \vec{B} \cdot d\vec{S}$$

velja posplošitev za primer nehomogeno magnetnega polje

FARADAYEV ali indukcijski zakon

$$\oint \vec{E} \cdot d\vec{s} = - \int \left(\frac{\partial \vec{B}}{\partial t} \right) \cdot d\vec{S}$$

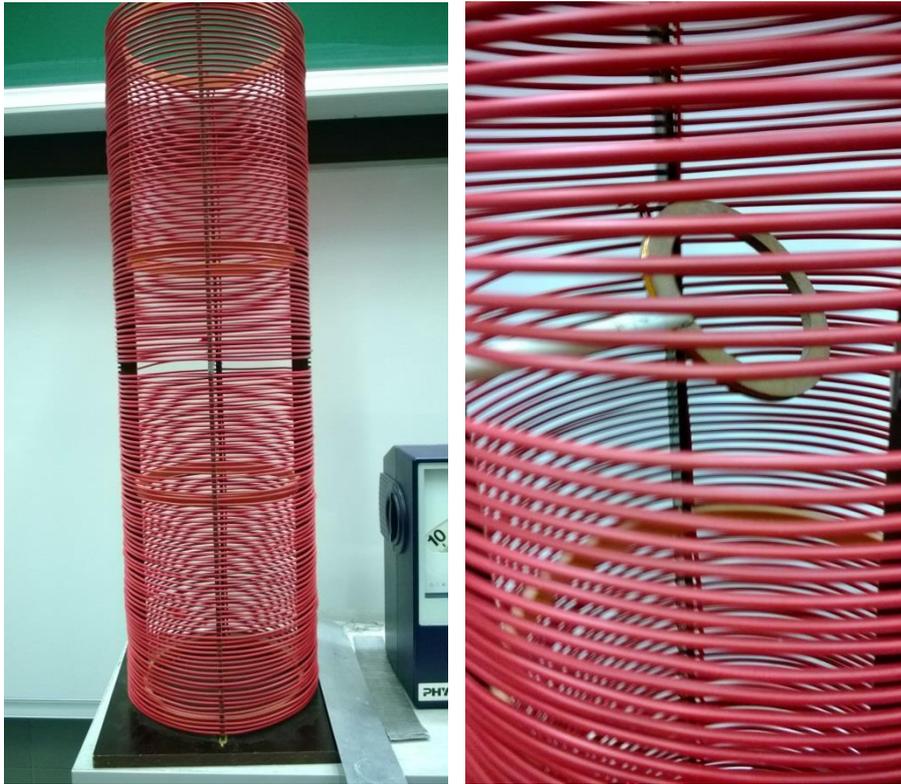
poskus: tuljavo zavrtimo v zemeljskem magnetnem polju ali pa potisnemo paličasti magnet v tuljavo (in potem ven)



$$U_i = - \frac{d\Phi_m}{dt}$$

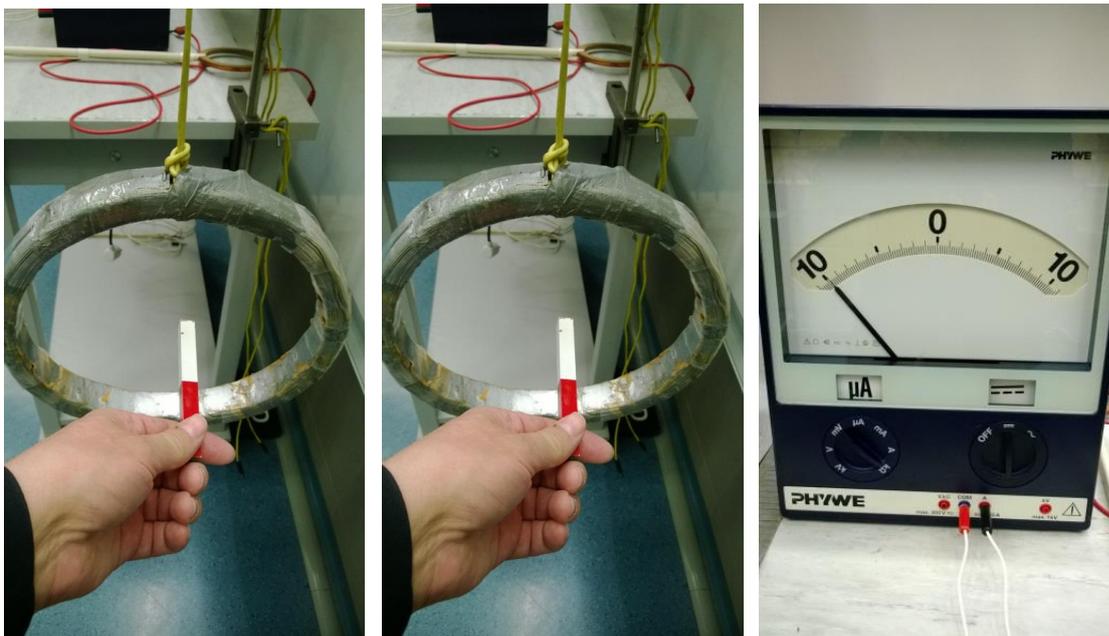
$\varphi = \omega t, \omega = \text{konst.}$
 $\Phi_m = N \vec{B} \cdot \vec{S} = NBS \cos(\omega t)$
 $U_i = - \frac{d\Phi_m}{dt} = \omega NBS \sin(\omega t)$

mala tuljava na ročaju se zavrti v dolgi pokončni tuljavi

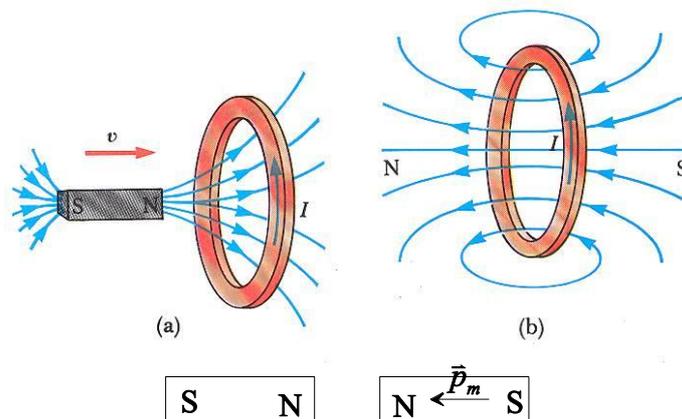


LENZOVO PRAVILO

SLIKA: paličasti magnet potisnemo v veliko tuljavo in iz nje



Inducirana napetost požene tok, ki se upira spremembi, ki je inducirano napetost povzročila:

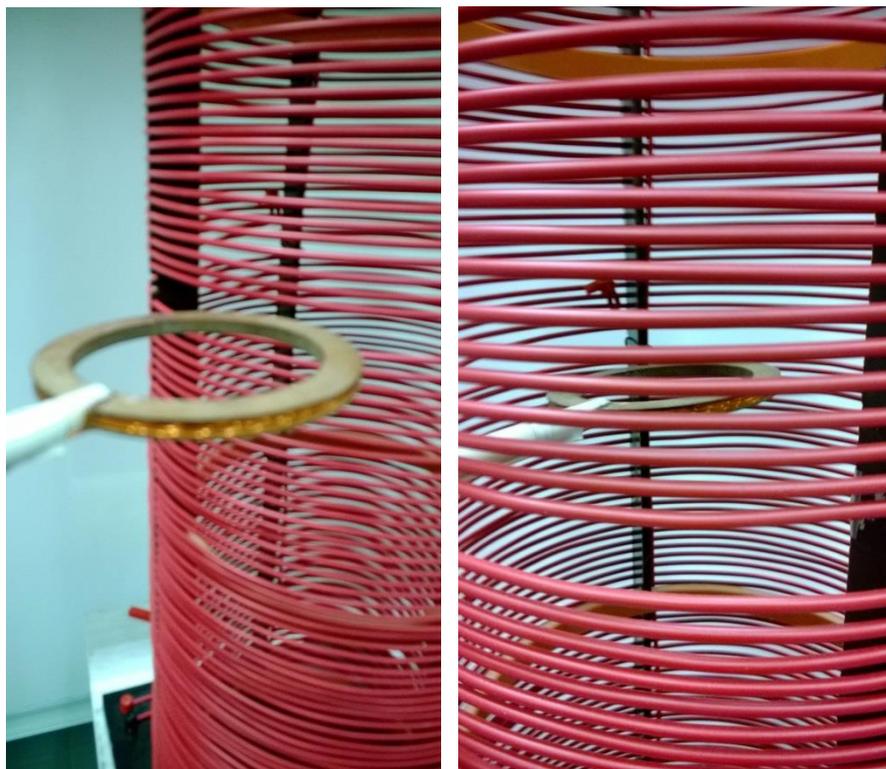


ODBOJNA SILA
 $S \equiv$ južni magnetni pol
 $N \equiv$ severni magnetni pol

SLIKA: obroček vrže v zrak (ilustracija Lenzovega pravila)



malo tuljavo vstavimo in potegnemo iz velike tuljave



MAXWELLOVE ENAČBE (osnovni zakoni elektrodinamike)



James Clerk Maxwell
(1831 – 1879)

$$\oint_S \vec{D} \, d\vec{S} = \int_V \rho_e \, dV$$
 zakon o električnem pretoku ([Gaussov zakon](#))

$$\oint_S \vec{B} \, d\vec{S} = 0$$
 zakon o magnetnem pretoku

$$\oint \vec{H} \, d\vec{s} = \int \left(\vec{j}_e + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$$
 zakon o magnetni napetosti ([Amperov zakon](#))

$$\oint \vec{E} \, d\vec{s} = - \int \left(\frac{\partial \vec{B}}{\partial t} \right) d\vec{S}$$
 indukcijski zakon ([Faradayev zakon](#))

Dodamo še zakon o ohranitvi naboja:

$$\oint_S \vec{j}_e \, d\vec{S} = - \int_V \left(\frac{\partial \rho_e}{\partial t} \right) dV$$

in [Lorentzova sila](#):

$$\vec{F} = e \vec{E} + e \vec{v} \times \vec{B}$$

Opozorilo: V Amperovem zakonu smo upoštevali **PREMIKALNI TOK**

- V krogih s kondenzatorjem **dosežemo veljavnost kontinuitetne enačbe**, če upoštevamo v kondenzatorju premikalni tok:

$$I = \frac{de}{dt} = C \frac{dU}{dt} = \varepsilon_0 \frac{S}{d} \frac{dU}{dt} = \varepsilon_0 S \frac{d\left(\frac{U}{d}\right)}{dt} = \varepsilon_0 S \frac{dE}{dt} = S \frac{d(\varepsilon_0 E)}{dt} = S \frac{dD}{dt}$$
$$I = S \frac{dD}{dt} \Rightarrow j_p = \frac{I}{S} = \frac{dD}{dt}$$

- vektorska oblika:

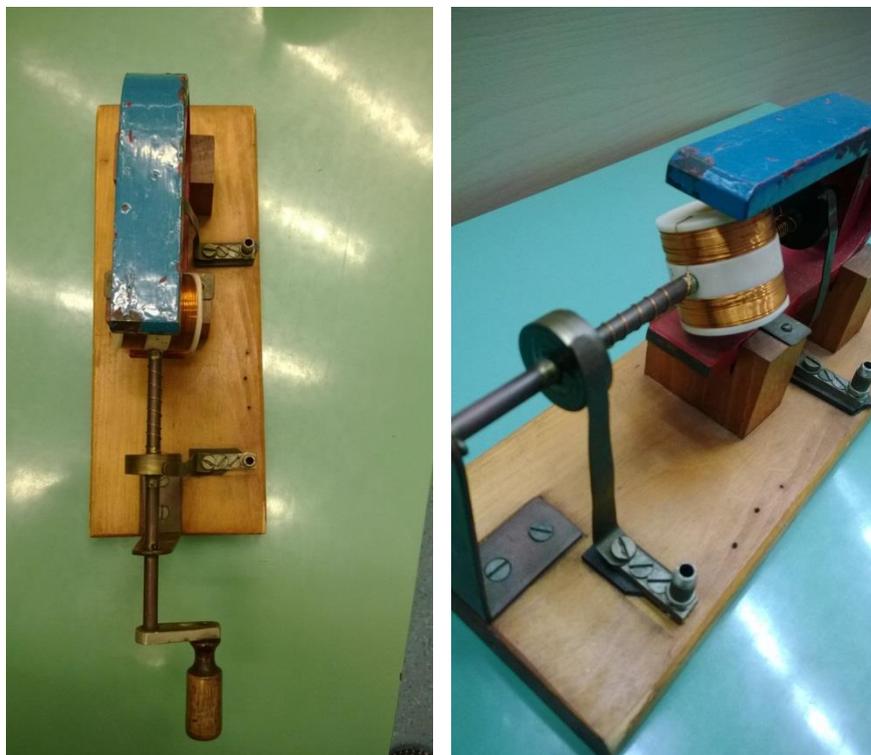
$$\vec{j}_p = \frac{d\vec{D}}{dt}.$$

- v nehomogenem polju velja:

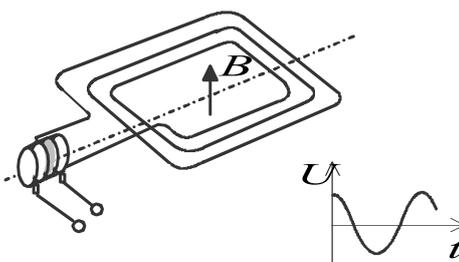
$$I_p = \int \vec{j}_p \cdot d\vec{S} = \int \left(\frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

GENERATORJI ELEKTRIČNEGA TOKA

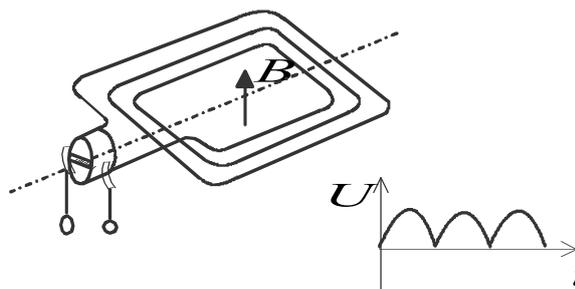
SLIKA: generator izmeničnega toka (vrteča tuljava med poloma podkvastega magneta, komutator)



brez komutatorja:



s komutatorjem:



• Delo, ki je potrebno za vrtenje generatorja
izmeničnega toka (žanke v mag. polju)

1

$N = \text{število ovijev}$

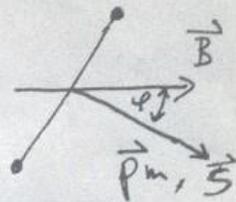
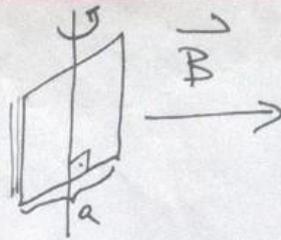
$$\Phi_m = N \vec{B} \cdot \vec{S} = NBS \cos(\omega t)$$

$$U_i = - \frac{d\Phi_m}{dt} = NBS\omega \sin(\omega t)$$

$$I = \frac{U_i}{R} = \frac{NBS\omega}{R} \sin(\omega t)$$

$$R = \frac{\rho \cdot 4a}{S_0}$$

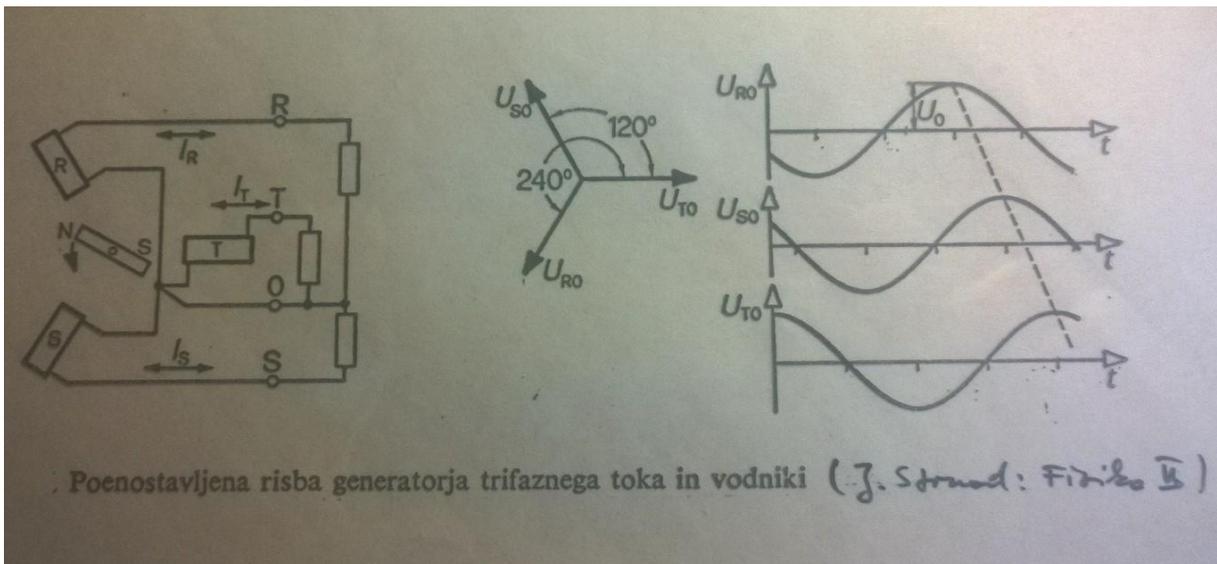
$\varphi = \omega t$
 $S_0 \equiv \text{presek žice}$

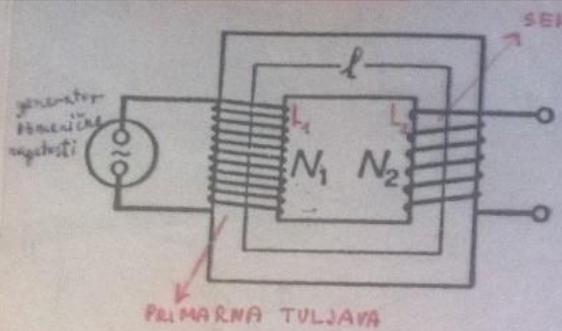


$$\underline{M} = P_m B \sin(\omega t) = \widetilde{N I S} B \sin(\omega t) =$$

$$= \frac{N^2 B S \omega}{R} S B \sin^2(\omega t) = \underline{\underline{\frac{N^2 B^2 S^2 \omega}{R} \sin^2(\omega t)}}$$

$$\underline{A} = \int_0^{2\pi} M d\varphi = \frac{N^2 B^2 S^2 \omega}{R} \int_0^{2\pi} \sin^2 \varphi d\varphi = \underline{\underline{\pi \frac{N^2 B^2 S^2 \omega}{R}}}$$





\approx jedrom povečamo magnetni pretok in dosežemo, da je magnetni pretok skozi obe tuljavi sklenjen
 $l \equiv$ srednji obseg jedra

IDEALNI TRANSFORMATOR:

1. magnetna pretoka skozi obe tuljavi sta enaka
2. upor obeh tuljav za enosmernu napetost je zanemarljiv

neobremenjen transformator

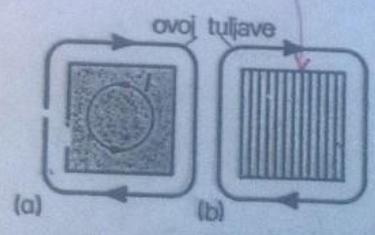
zaradi spremenljivega magnetnega pretoka se inducira napetost na primarni in sekundarni tuljavi:

$$\frac{(U_1)_0}{(U_2)_0} = \frac{N_1}{N_2}$$

tok ob uporabi na sekundarni tuljavi sklenemo skozi ohmski upor:

$$\frac{(I_1)_0}{(I_2)_0} = \frac{N_2}{N_1}$$

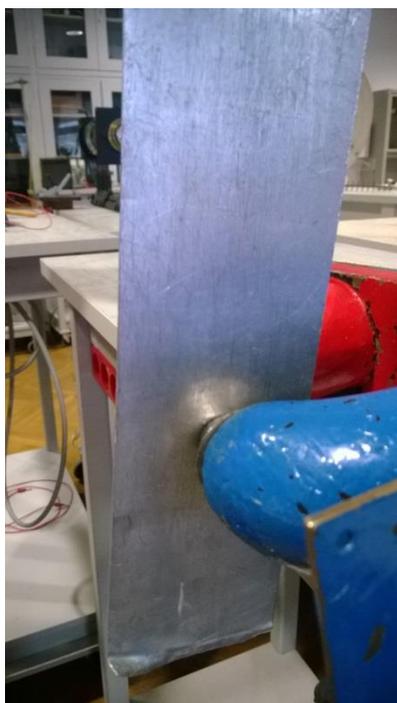
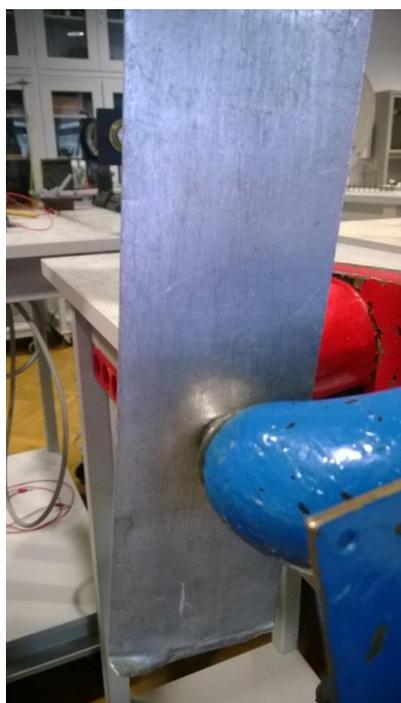
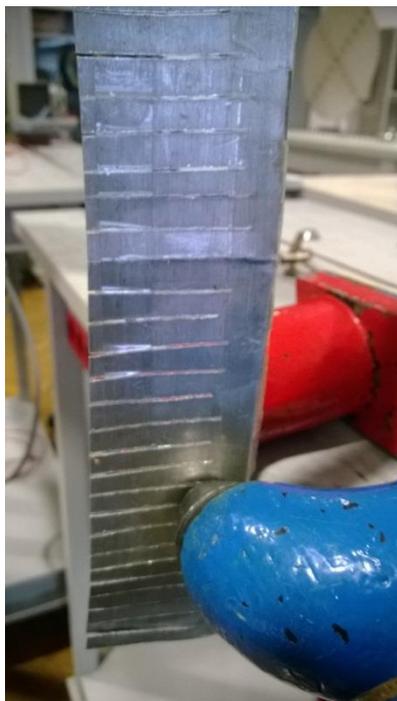
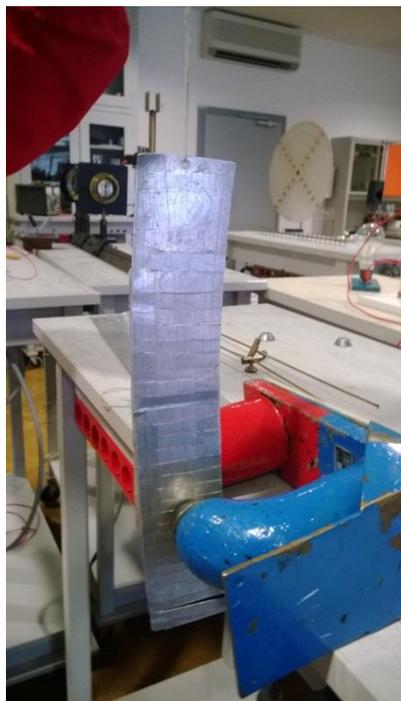
lamel prevlečene z lakom \Rightarrow med seboj so izolirane



Po železnem transformatorskem jedru iz enega kosa bi tekla velik inducirani tok (a), po jedru iz izoliranih tankih lamel pa takega toka ni (b)

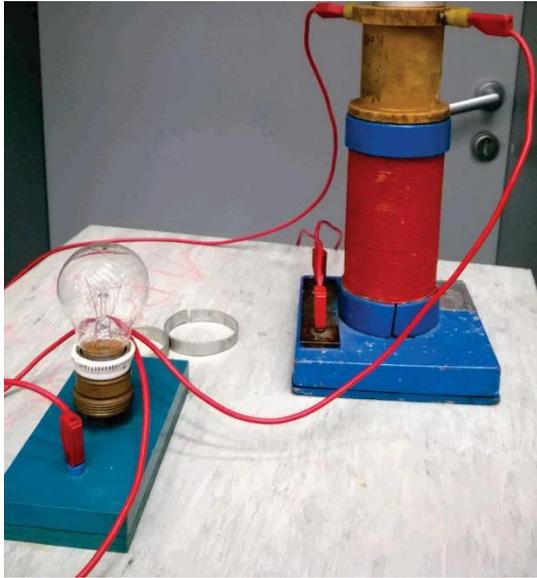
J. Strnad: Fizika II

SLIKA: padanje pločevinastega traku v magnetnem polju - trak narežemo, da zmanjšamo hitrost padanja ...

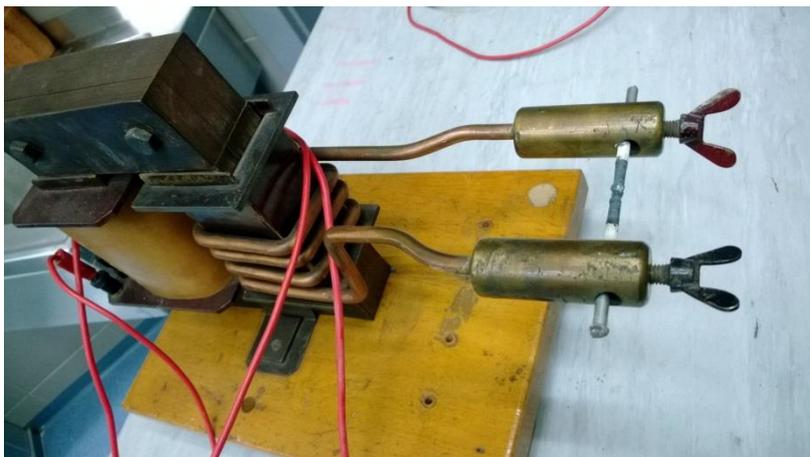


SLIKA: TRANSFORMATOR

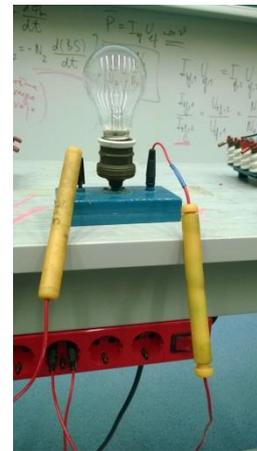
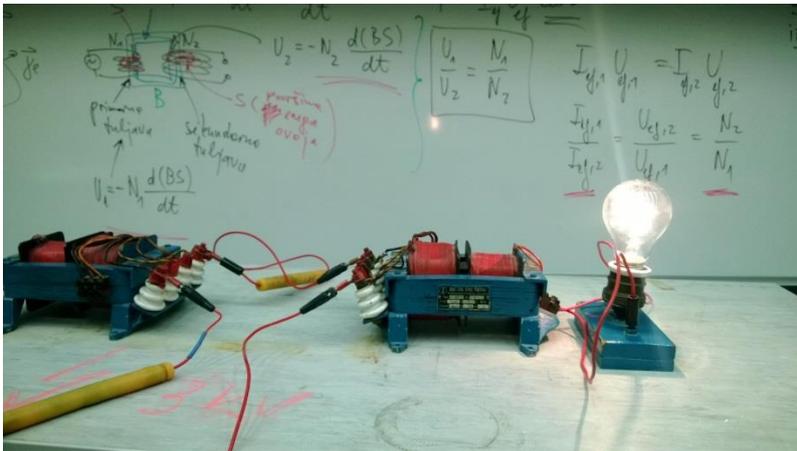
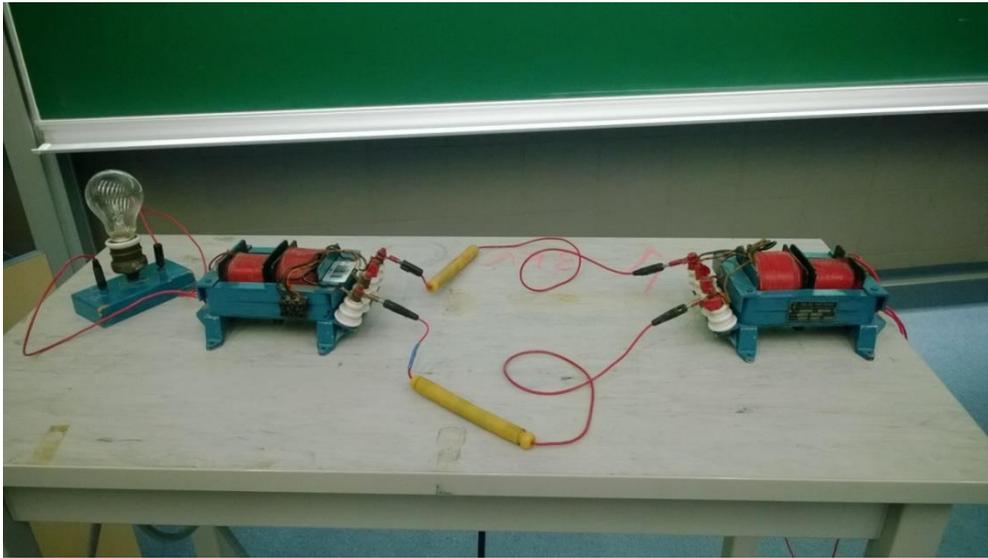
a) jedro, ki povezuje dve valjasti tuljavi



b) model varilnega aparata (pretalitev žebnja)

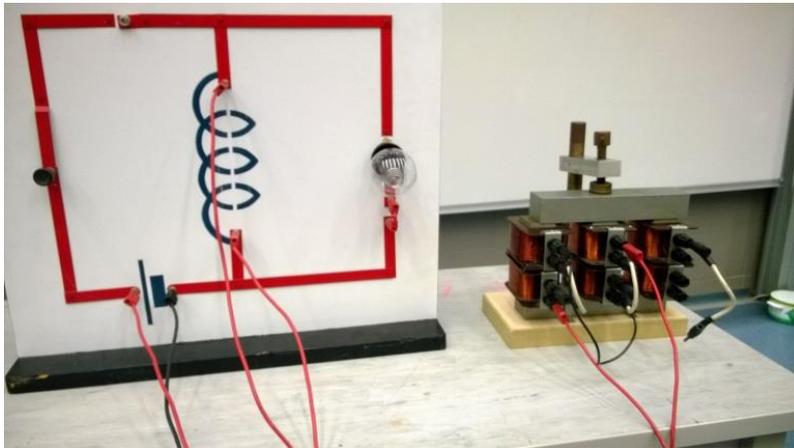


SLIKA: model daljnovoda (dva transformatorja in dva upora)



LASTNA INDUKTIVNOST

SLIKA: inducirana napetost pri poganjanju (žarnica) in prekinitvi toka skozi tuljavo (tlivka)



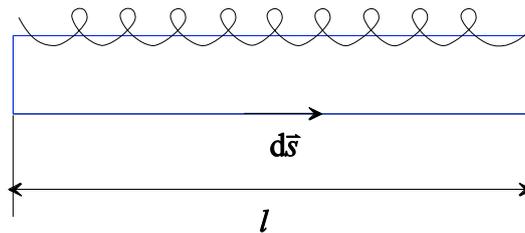
DEFINICIJA LASTNE INDUKTIVNOSTI

lastni magnetni pretok

$$\Phi_m = LI$$

$L \equiv$ induktivnost [Vs/A]

Primer: dolga tuljava z gostimi navoji po kateri teče električni tok I

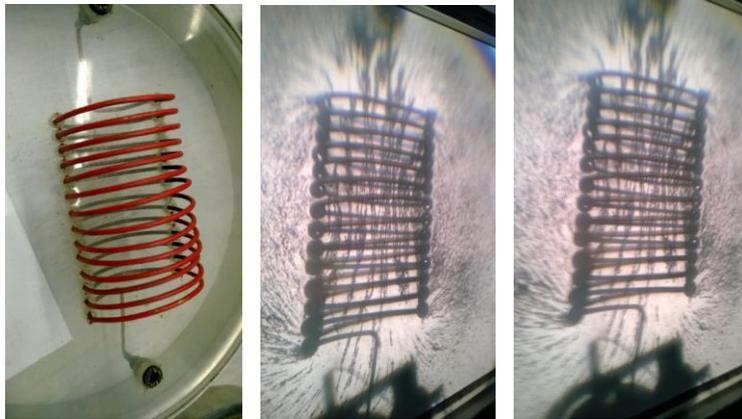


$l \equiv$ dolžina tuljave

$N \equiv$ število ovojev tuljave

$S \equiv$ površina enega ovoja

predpostavka: magnetno polje je samo znotraj tuljave



Uporabimo Amperov zakon

$$\oint \vec{B} \cdot d\vec{s} \cong Bl = \mu_0 N I$$

$$B = \mu_0 \frac{NI}{l}$$



Magnetni pretok skozi tuljavo je zato:

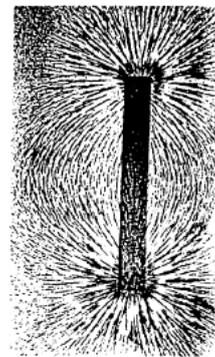
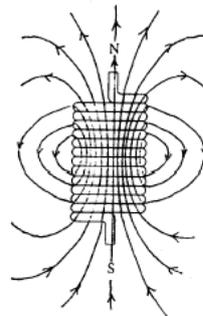
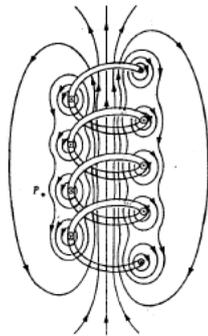
$$B = \mu_0 \frac{NI}{l}$$

$$\Phi_m = NBS = N \mu_0 \frac{NI}{l} S = \mu_0 \frac{N^2 S}{l} I = LI$$

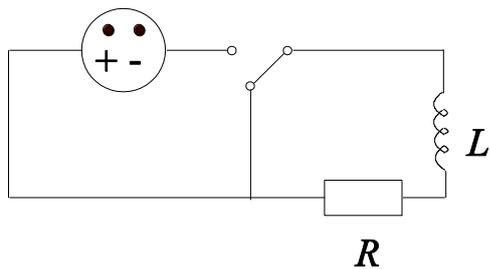
$$L = \mu_0 \frac{N^2 S}{l}$$



dolga tuljava



ČASOVNA ODVISNOST TOKA, KI GA POŽENE TULJAVA PO UPORNIKU OB IZKLOPU NAPETOSTNEGA IZVORA



$$\Phi_m = LI$$

$$U_L = -\frac{d\Phi_m}{dt} = -L \frac{dI}{dt}$$

$t < 0$: zunanji izvir (U_0) poganja stacionaren tok $I_0 = U_0/R$

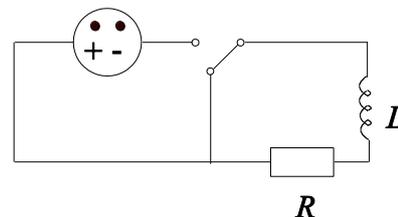
$t \geq 0$: preklopimo pretikalo

$$U_R + U_L = 0$$

$$-IR - L \frac{dI}{dt} = 0$$

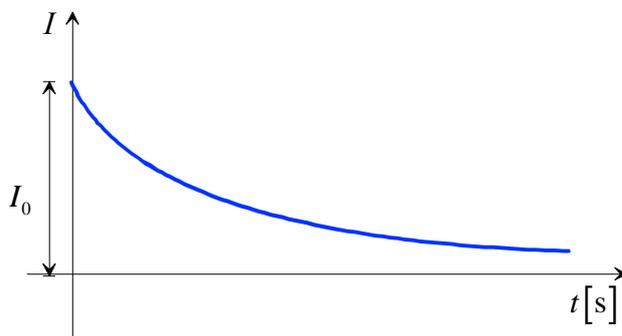
$$\frac{dI}{I} = -\frac{R}{L} dt = -\frac{dt}{\tau}$$

$$\tau = \frac{L}{R}$$



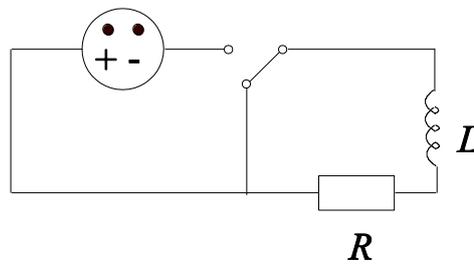
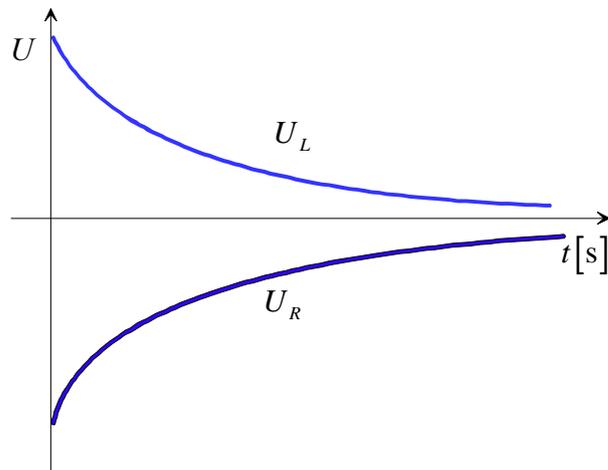
$$I = I_0 e^{-t/\tau}$$

tok:



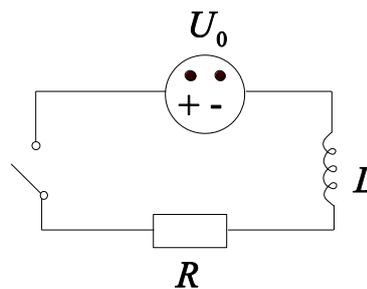
$$U_R + U_L = 0$$

$$U_L = -U_R = IR = RI_0 e^{-t/\tau}$$



$$\tau = \frac{L}{R}$$

POGANJANJE TOKA SKOZI TULJAVO



$$U_0 + U_R + U_L = 0$$

$$U_0 - IR - L \frac{dI}{dt} = 0$$

$$U_0 - IR - L \frac{dI}{dt} = 0$$

nova spremenljivka: $U_0 - IR = x$

$$-dI R = dx$$

$$-R \frac{dI}{dt} = \frac{dx}{dt}$$

$$\frac{dI}{dt} = -\frac{1}{R} \frac{dx}{dt}$$

$$-L \frac{dI}{dt} = \frac{L}{R} \frac{dx}{dt}$$

Torej:

$$t=0, I=0, x=x_0=U_0$$

$$x + \frac{L}{R} \frac{dx}{dt} = 0$$

$$\int_{U_0}^{U_0-IR} \frac{dx}{x} = - \int_0^t \frac{R}{L} dt$$

$$\ln \frac{U_0 - IR}{U_0} = -\frac{t}{\tau} \Rightarrow (U_0 - IR) = U_0 e^{-t/\tau}$$

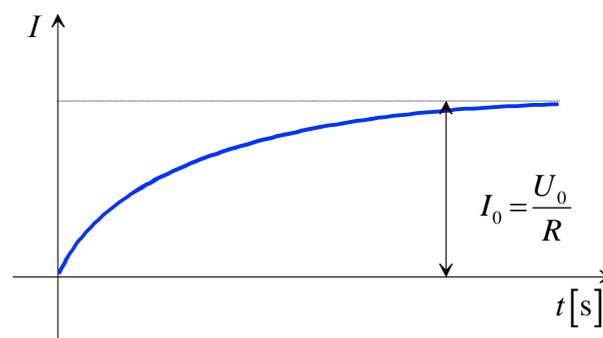
$$RI = U_0 (1 - e^{-t/\tau})$$

$$I = \frac{U_0}{R} (1 - e^{-t/\tau})$$

$$I_0 = \frac{U_0}{R}$$

$$I = I_0 (1 - e^{-t/\tau})$$

tok:



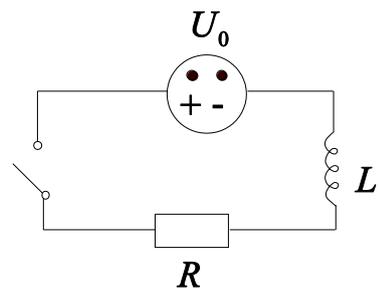
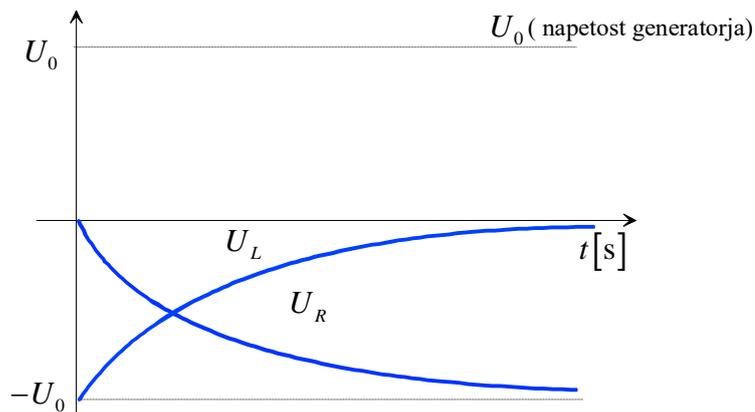
$$U_0 + U_R + U_L = 0$$

$$I = I_0 (1 - e^{-t/\tau})$$

$$U_R = -IR = -RI_0 (1 - e^{-t/\tau}) = -U_0 (1 - e^{-t/\tau})$$

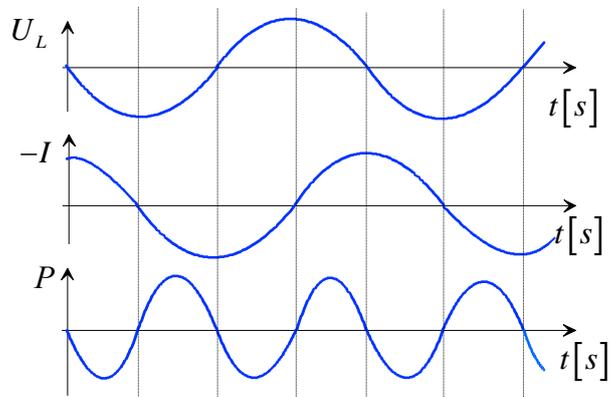
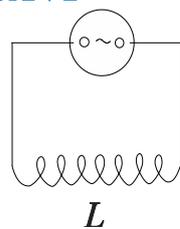
$$U_L = -U_0 - U_R = -U_0 + U_0 - U_0 e^{-t/\tau} = -U_0 e^{-t/\tau}$$

napetost:



IZMENIČNI TOK PO IDEALNI TULJAVI

gonilna napetost: $U_0 \sin(\omega t)$



$$U_0 \sin(\omega t) + U_L = 0$$

$$U_0 \sin(\omega t) - L \frac{dI}{dt} = 0$$

$$U_0 \sin(\omega t) = L \frac{dI}{dt}$$

↓

$$dI = \frac{U_0}{L} \sin(\omega t) dt$$

$$dI = \frac{U_0}{\omega L} \sin(\omega t) d(\omega t)$$

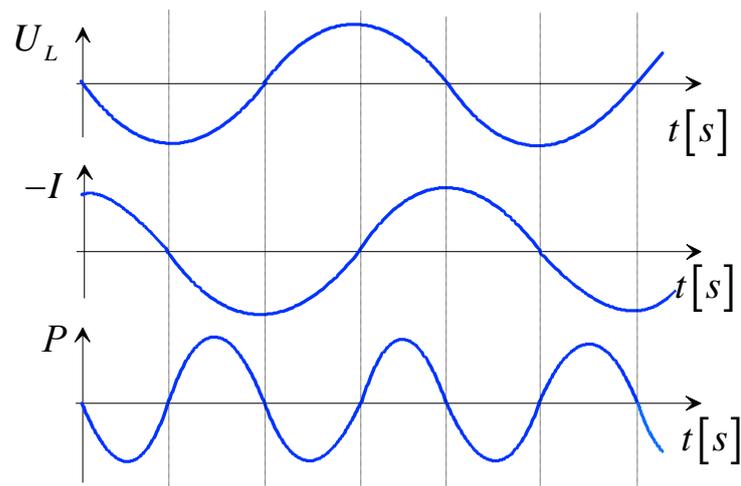
$$I = \frac{U_0}{\omega L} \int \sin(\omega t) d(\omega t) = -\frac{U_0}{\omega L} \cos(\omega t) = \frac{U_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I(t) = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I_0 = \frac{U_0}{\omega L}$$

$$U_L = -L \frac{dI}{dt} \Rightarrow U_L = -U_0 \sin(\omega t) = U_0 \sin(\omega t - \pi)$$

Tuljava rabi **povprečno moč** $\bar{P} = \frac{1}{2} I_0 U_0 \cos \delta = 0$



$$P = I u = I_0 U_0 \cos \omega t \cos(\omega t + \delta) =$$

$$= I_0 U_0 \cos \omega t [\cos \omega t \cos \delta - \sin \omega t \sin \delta] =$$

$$= I_0 U_0 \left[\underbrace{\cos^2 \omega t}_{\frac{1}{2}} \cos \delta - \underbrace{\sin \omega t \cos \omega t}_{\frac{1}{2} \sin(2\omega t)} \sin \delta \right] \Rightarrow$$

$$\bar{P} = I_0 U_0 \left[\underbrace{\overline{\cos^2 \omega t}}_{\frac{1}{2}} \cos \delta - \underbrace{\overline{\frac{1}{2} \sin(2\omega t)}}_{0} \sin \delta \right]$$

$$\boxed{\bar{P} = \frac{1}{2} I_0 U_0 \cos \delta}$$

$$\bar{P} = I_y U_y \cos \delta, \text{ kjer}$$

$$I_y = \frac{I_0}{\sqrt{2}}$$

$$U_y = \frac{U_0}{\sqrt{2}}$$

$$\begin{aligned} \sin(2\omega t) &= 2 \sin(\omega t) \cos(\omega t) \\ \sin(\omega t) \cos(\omega t) &= \frac{1}{2} \sin(2\omega t) \end{aligned}$$