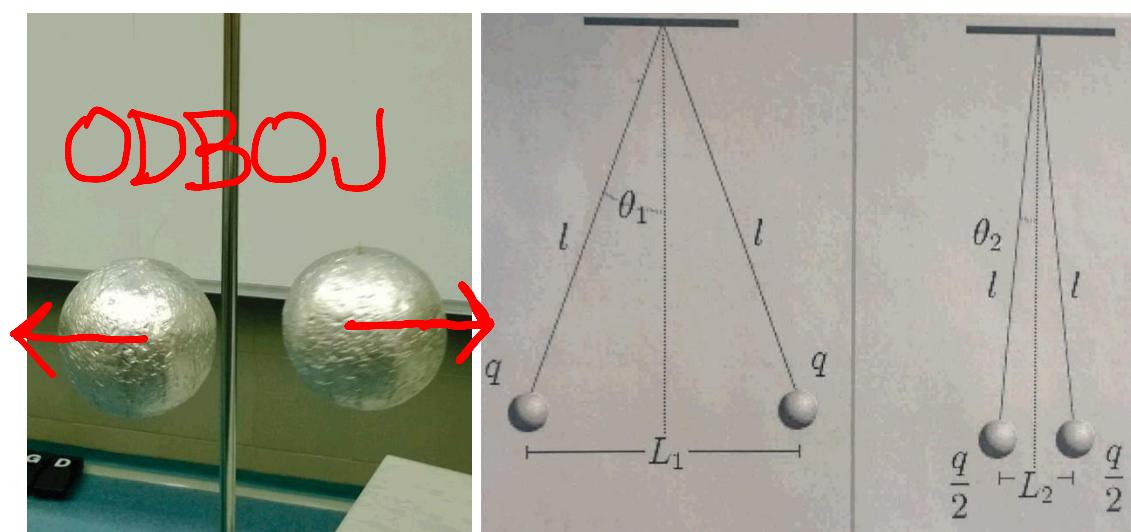


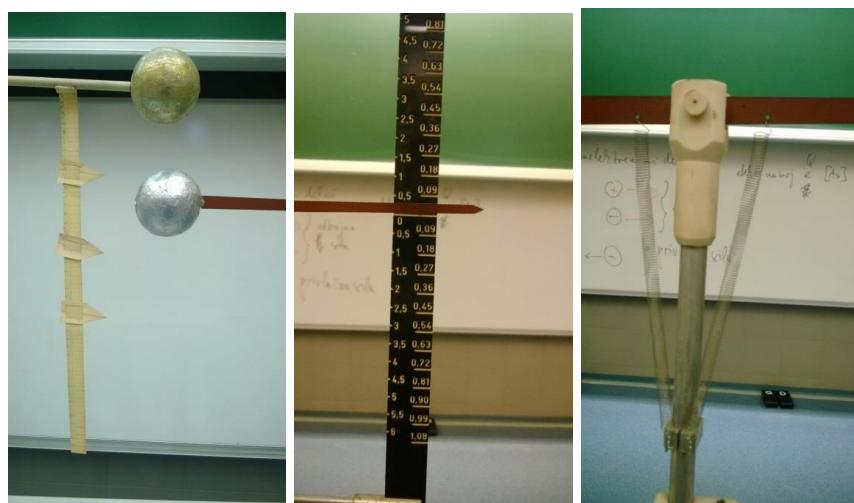
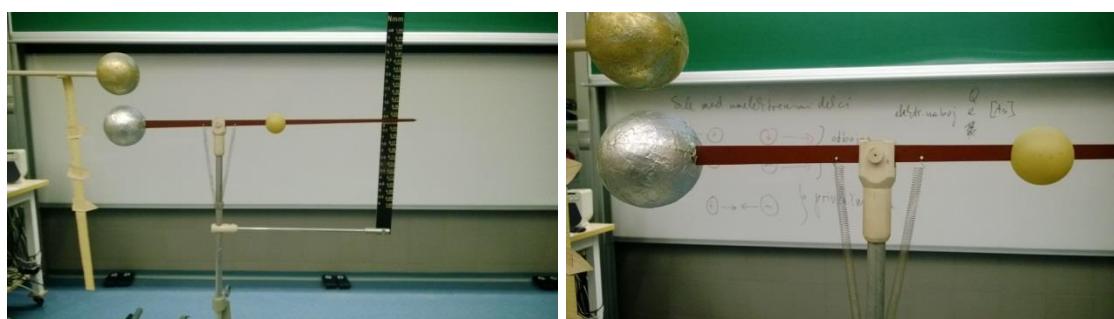
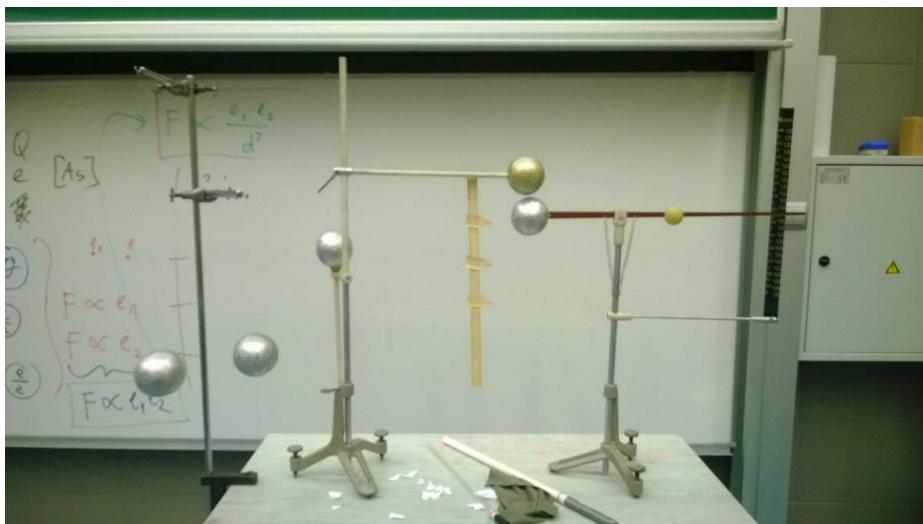
ATOMIKA IN OPTIKA

Coulombov zakon

steklena in polivinilna palica ter volnena krpa za naelektritev



sila med naelektrennima kroglama v odvisnosti od razdalje med kroglama



delitev naboja med naelekreno in nevtralno kroglo ter vpliv naboja na posamezni krogli na silo med kroglama (sila je sorazmerna naboju na kroglah)

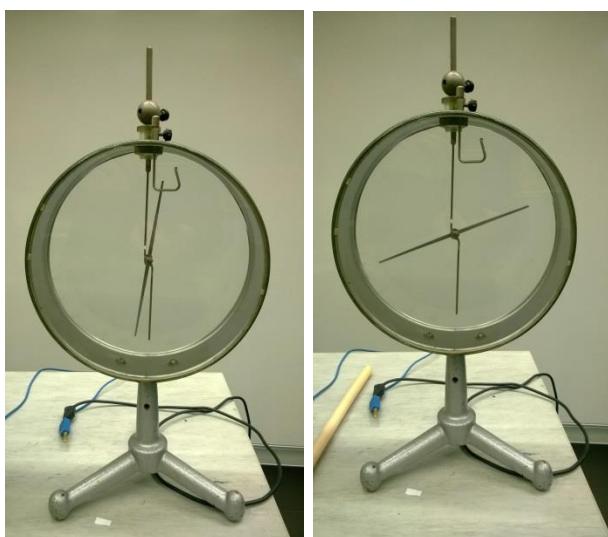


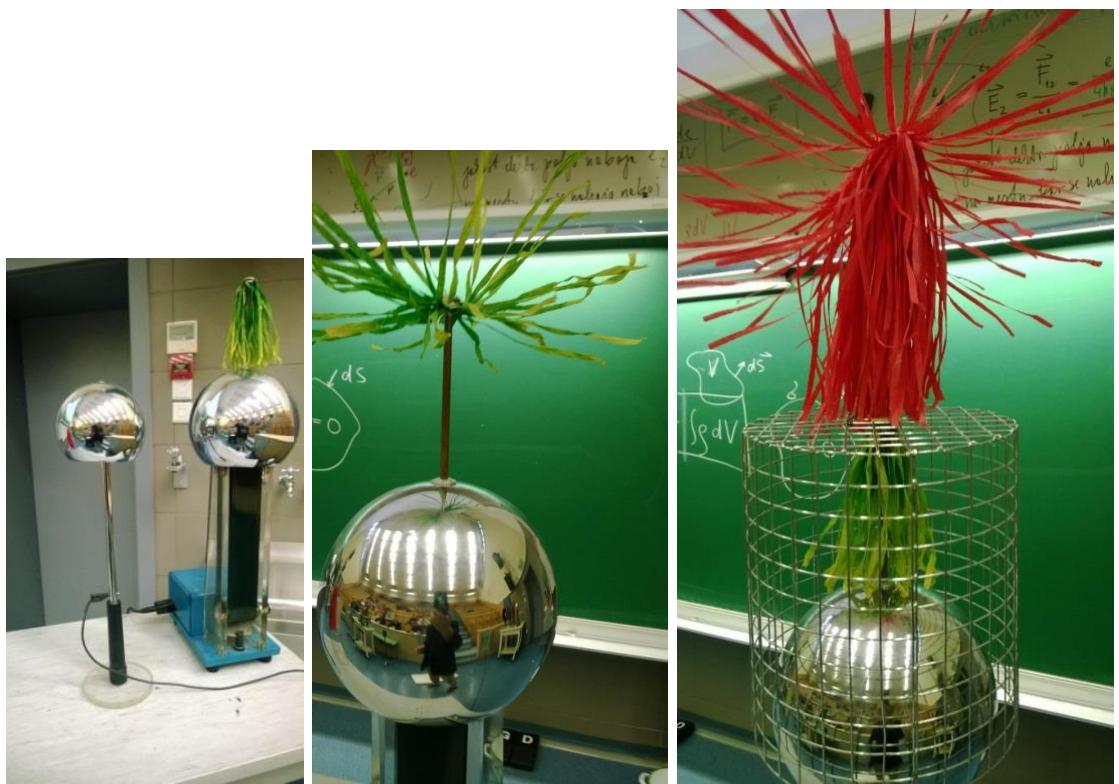
$$F \propto K \frac{e_1 e_2}{r^2}$$


$$K = 9 \cdot 10^9 \text{ N m}^2 / (\text{As})^2$$

$$K = \frac{1}{4\pi\epsilon_0} \quad \epsilon_0 = 8.85 \cdot 10^{-12} \text{ As/Vm}$$

elektroskop





van de Graafov generator napetosti

film



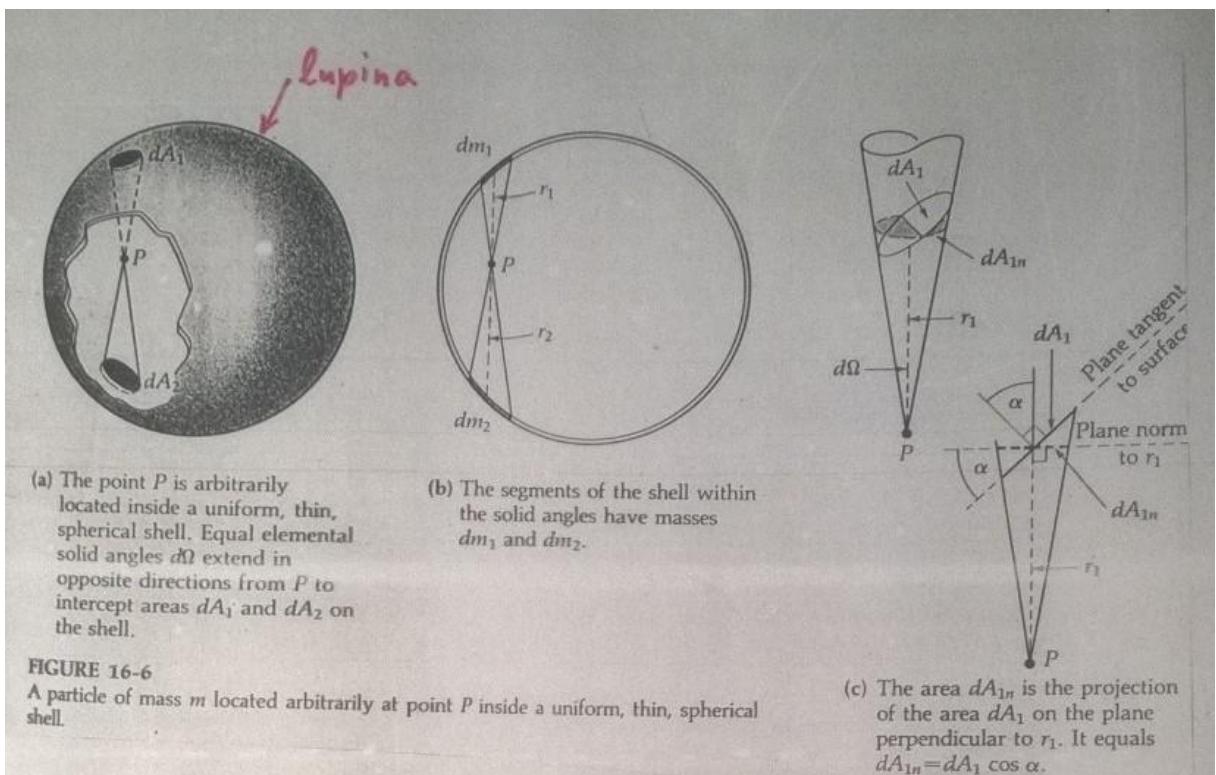


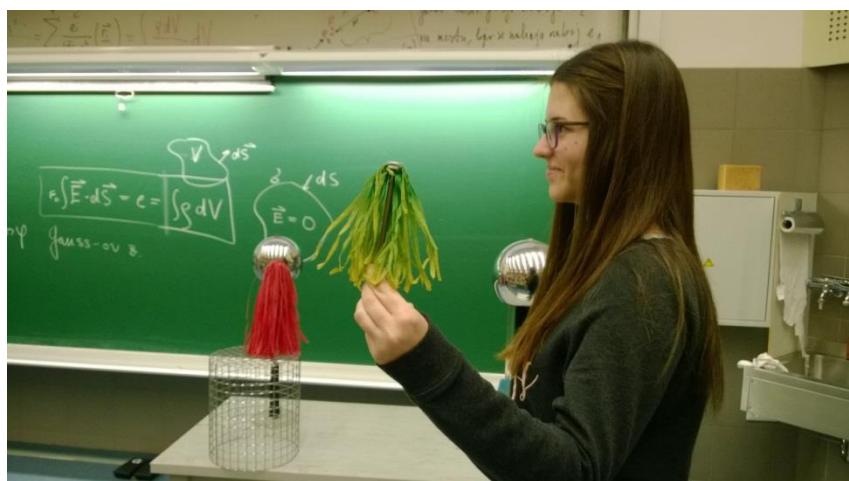
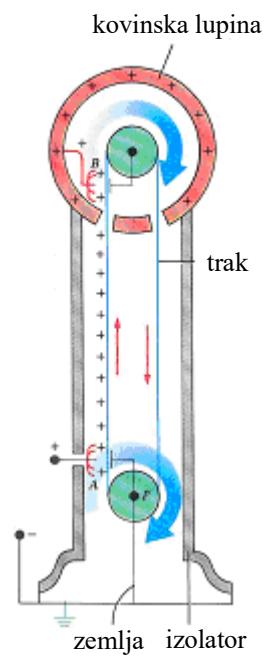
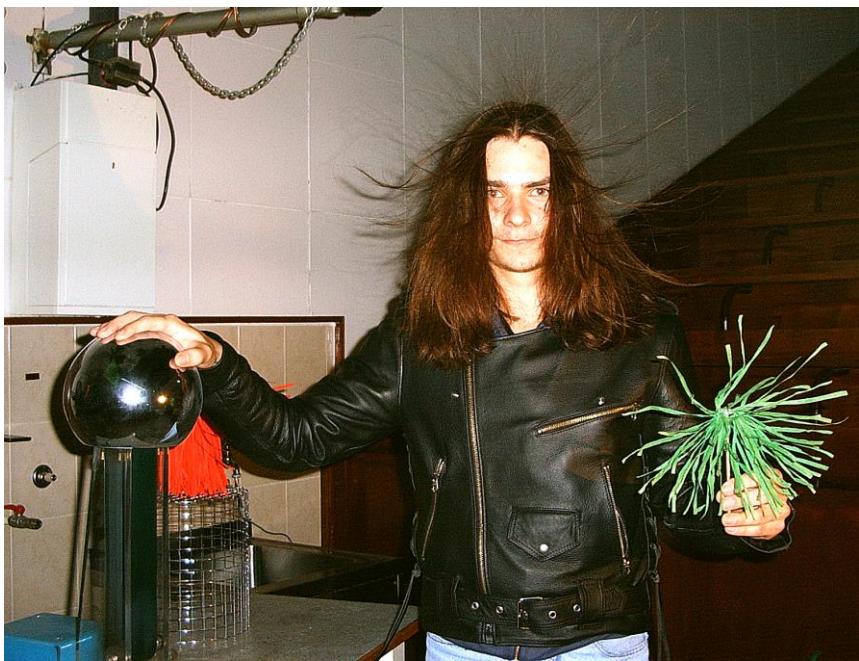
FIGURE 16-6
A particle of mass m located arbitrarily at point P inside a uniform, thin, spherical shell.

We may also obtain this last result by an interesting chain of reasoning (first pointed out by Newton) that does not involve complicated integrals. Consider the particle of mass m to be located at an arbitrary point P inside the shell. Imagine that a narrow cone is constructed with its apex at the point, extending out in an arbitrary fashion. The cone will intercept, on the shell, an element of area dA_1 (Figure 16-6a). If we project a similar cone with an equal solid angle⁶ in the opposite direction, it will intercept an area dA_2 . An elemental solid angle $d\Omega$ (measured in steradians) is defined as the ratio dA_n/r^2 where dA_n is an area *normal* to the distance r from the apex of the solid angle. For the sphere, how are dA_1 and dA_2 related to their projections dA_{1n} and dA_{2n} on planes normal to r_1 and r_2 ? Any straight line such as r_1 and r_2 together will intersect the sphere at two points, making the same angles α with the normal to the surface of the sphere. That is, $d\Omega = dA_{1n}/r_1^2 = (dA_1 \cos \alpha)/r_1^2 = (dA_2 \cos \alpha)/r_2^2$. With $\sigma =$ the mass per unit area, the two mass elements are

$$dm_1 = \sigma dA_1 = \frac{\sigma d\Omega r_1^2}{\cos \alpha} \quad \text{and} \quad dm_2 = \sigma dA_2 = \frac{d\Omega r_2^2}{\cos \alpha}$$

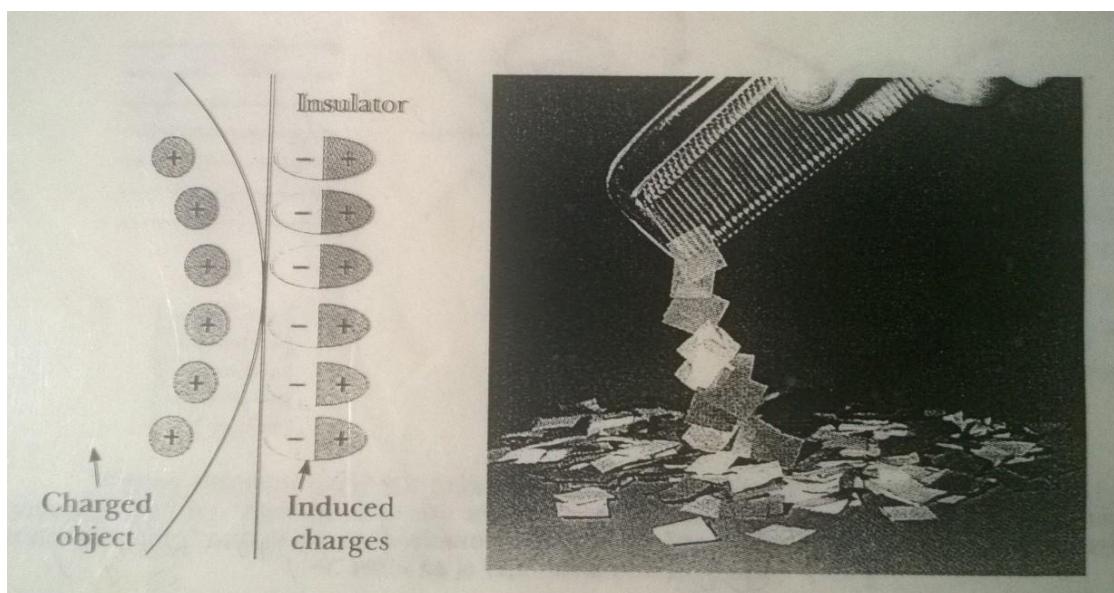
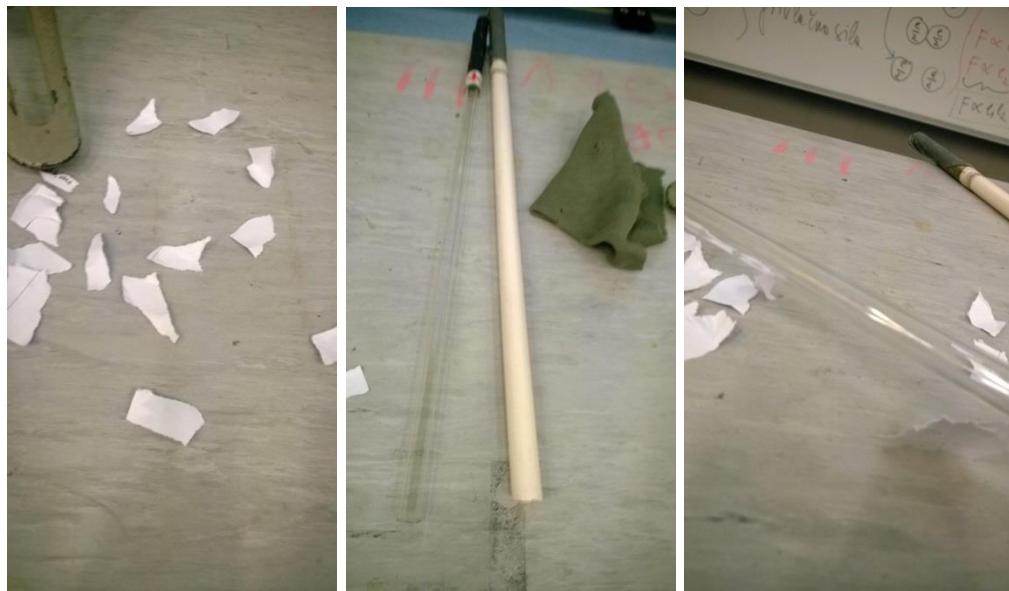
These two elements of mass exert gravitational forces on the particle m . The two forces are in opposite directions, and the ratio of their magnitudes is

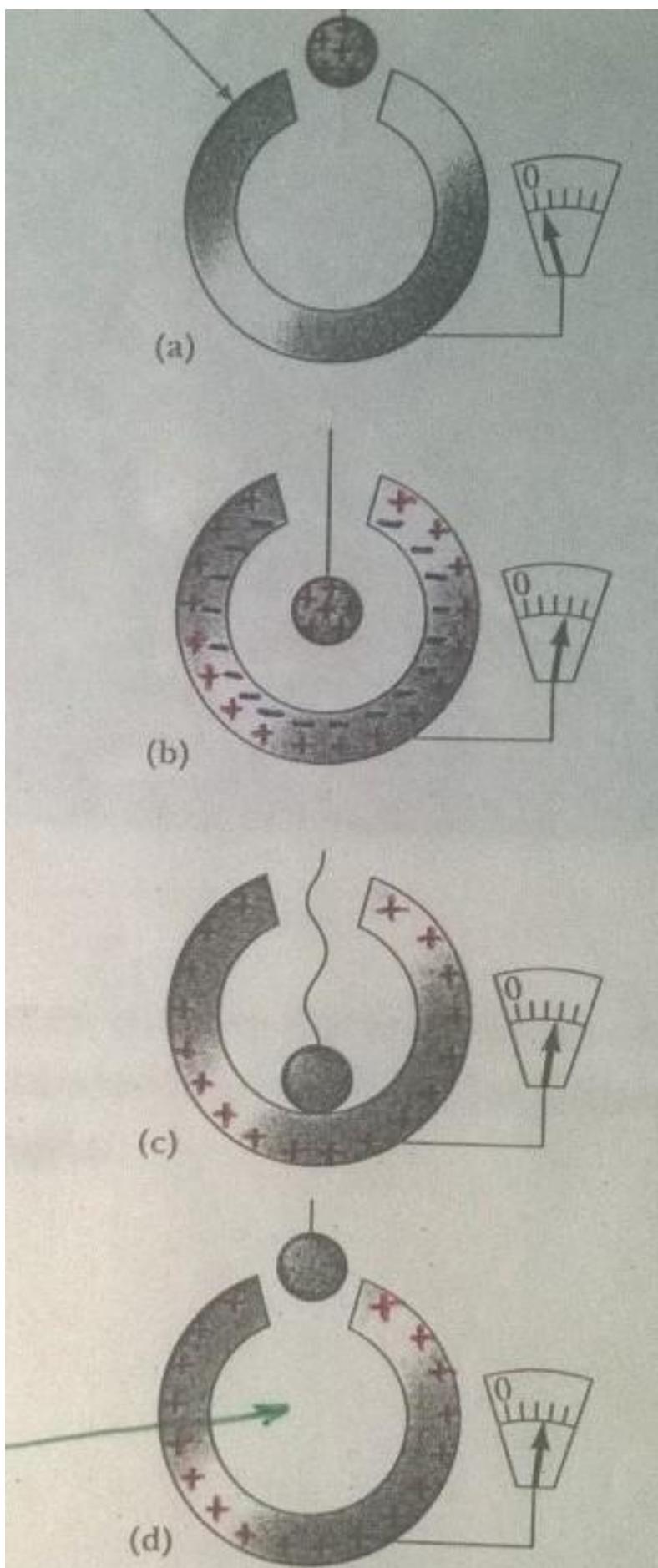
$$\frac{dF_1}{dF_2} = \frac{\left(\frac{Gm dm_1}{r_1^2} \right)}{\left(\frac{Gm dm_2}{r_2^2} \right)} = \frac{\left(\frac{\sigma d\Omega r_1^2}{\cos \alpha r_1^2} \right)}{\left(\frac{\sigma d\Omega r_2^2}{\cos \alpha r_2^2} \right)} = 1$$



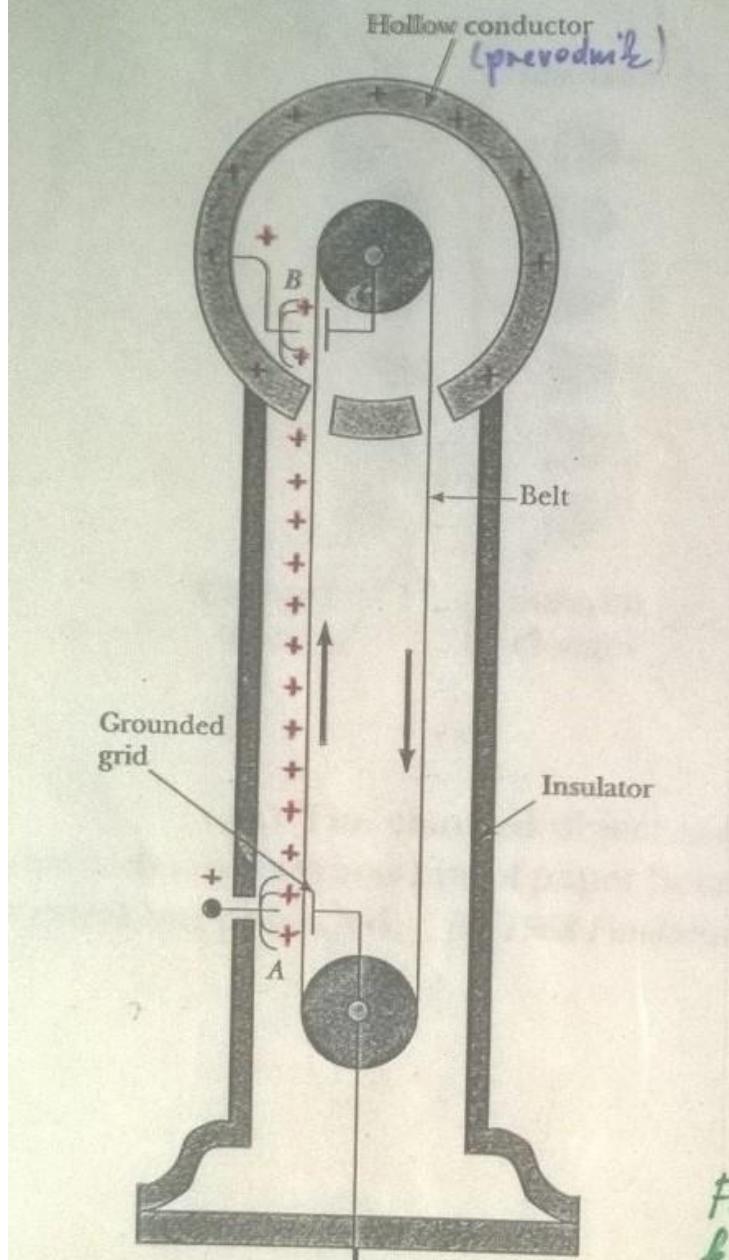
INFLUENCA

lističi papirja ter steklena in polivinilna palica, volnena krpa





Van de Graaff generator

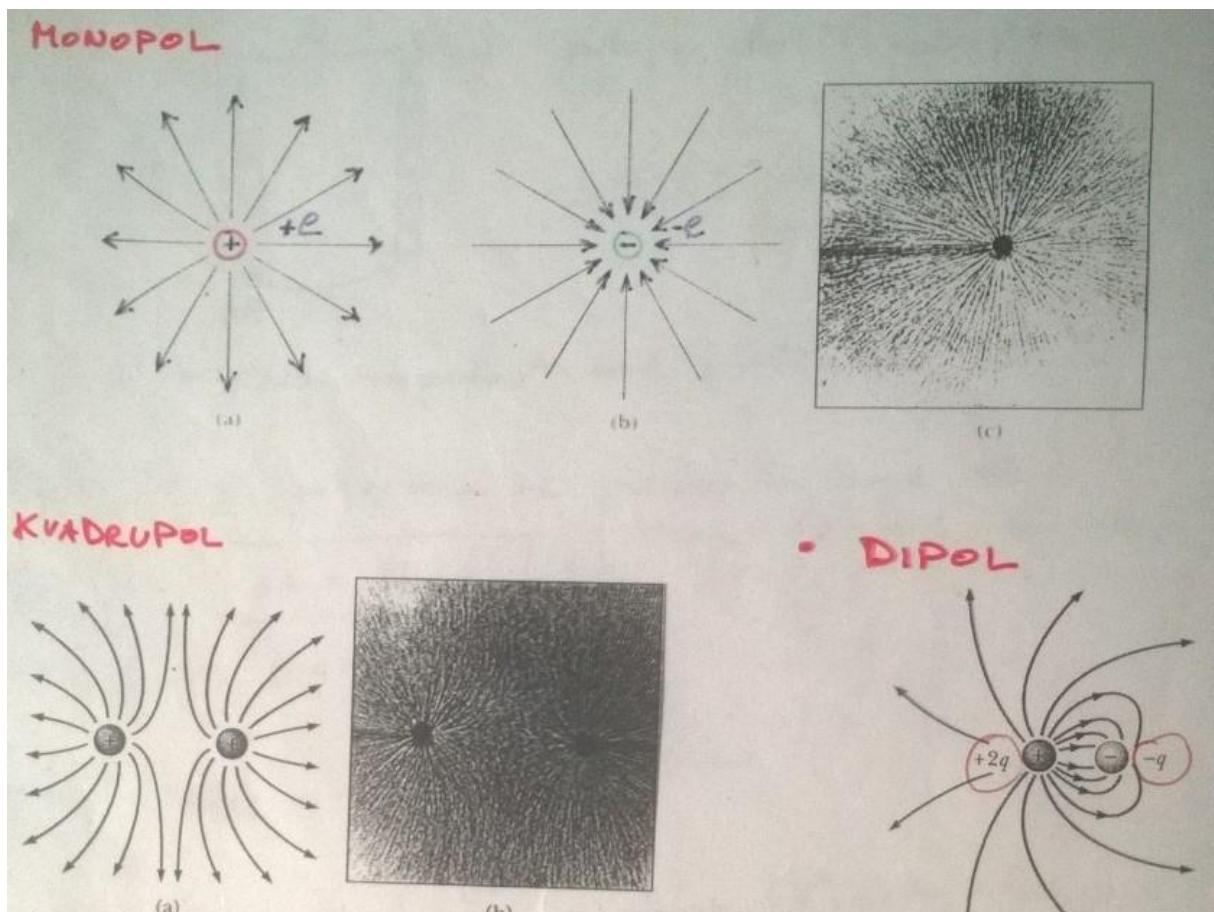


Električno polje točkastega delca (definicija)

vektorski zapis

$$\vec{F}_{12} = \frac{e_1 e_2}{4\pi \epsilon_0 r^2} \frac{\vec{r}}{r} \quad \vec{F}_{12} = \vec{E}_1 e_2$$

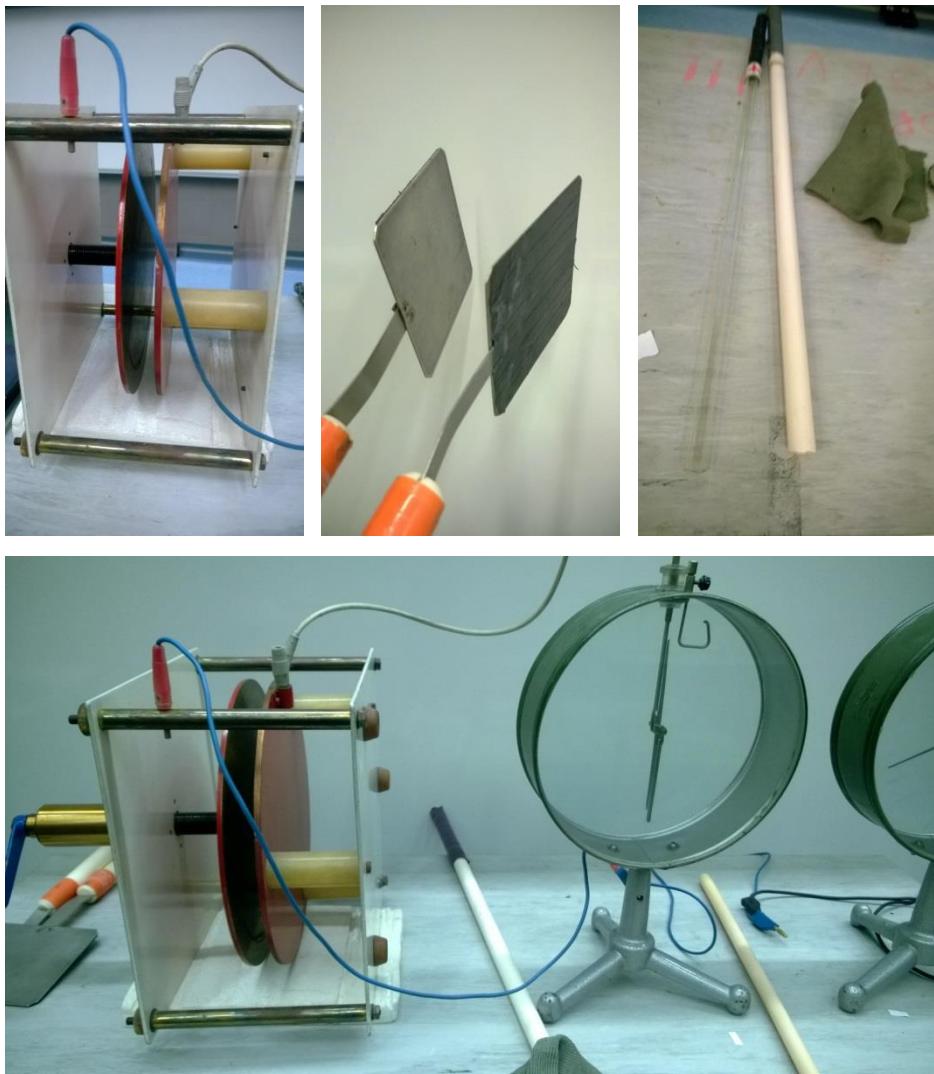
$$\left| \frac{\vec{r}}{r} \right| = 1 \quad \vec{E}_1 = \frac{e_1}{4\pi \epsilon_0 r^2} \frac{\vec{r}}{r}$$

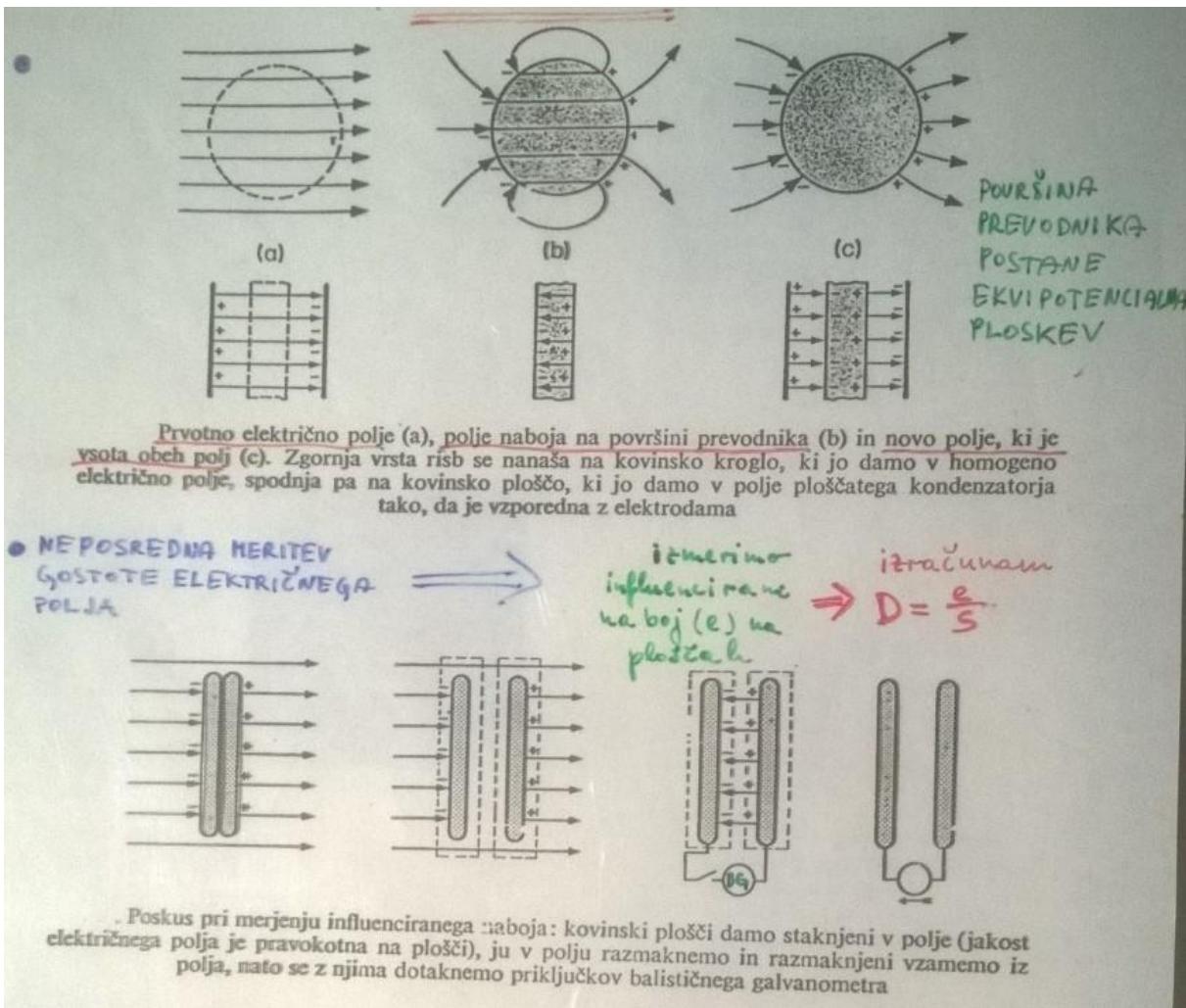


SLIKE ELEKTRIČNEGA POLJA makroskopskih sistemov



Influenca (primer) : dve staknjeni kovinski plošči razmagnemo v električnem polju in izmerimo njun naboj na elektroskopu





GAUSSOV ZAKON (zakon o električnem pretoku)

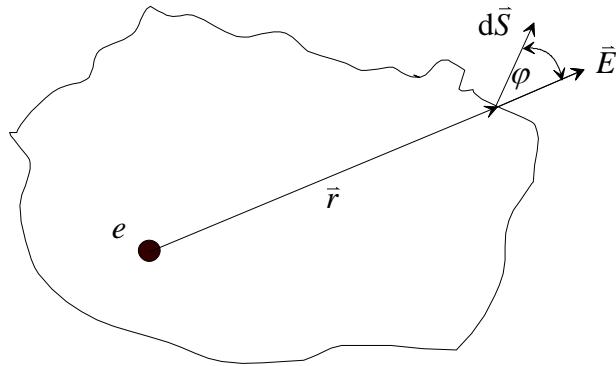
Gaussov zakon je skladen s Coulombovim zakonom

$$\vec{F}_{12} = \frac{e_1 e_2}{4\pi \epsilon_0 r^2} \frac{\vec{r}}{r}$$

$$\vec{F}_{12} = \vec{E}_1 e_2$$

$$\vec{E} = \frac{e}{4\pi \epsilon_0 r^2} \frac{\vec{r}}{r}$$

$$\vec{r} \cdot d\vec{S} = r dS \cos \varphi = r dS', \quad \text{kjer} \quad dS' = dS \cos \varphi$$



Integriramo po **zaključeni ploskvi** okoli točkastega naboja e:

$$\begin{aligned} \oint \vec{E} \cdot d\vec{S} &= \oint \frac{e}{4\pi \epsilon_0 r^2} \frac{\vec{r} \cdot d\vec{S}}{r} = \oint \frac{e}{4\pi \epsilon_0 r^2} \frac{r dS'}{r} = \oint \frac{e}{4\pi \epsilon_0} \frac{dS'}{r^2} = \\ &= \oint \frac{e}{4\pi \epsilon_0} d\Omega = \frac{e}{4\pi \epsilon_0} \oint d\Omega = \frac{e}{4\pi \epsilon_0} 4\pi \end{aligned}$$

$$d\Omega = \frac{dS'}{r^2}$$

$$\oint d\Omega = 4\pi$$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = e$$

$$\epsilon_0 \oint_S \vec{E} \cdot d\vec{S} = \sum_i e_i$$

$$\epsilon_0 \oint_S \vec{E} \cdot d\vec{S} = \int_V \rho dV$$

GAUSSOV ZAKON O ELEKTRIČNEM PRETOKU

ELEKTROSTATSKA POTENCIJALNA ENERGIJA

DELO SILE TOČKASTEGA NABOJA e_1 NA TOČKASTEM NABOJU e_2

električno polje naboja e_1

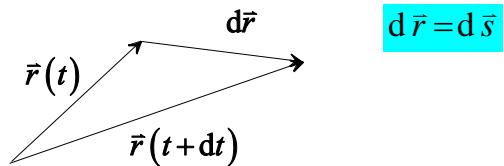
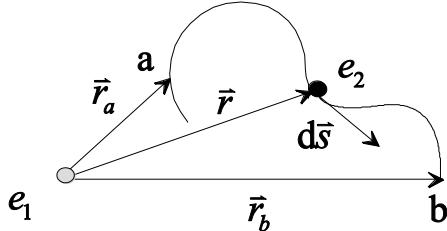
sila na naboj e_2 :

$$\vec{E}_1 = \frac{e_1}{4\pi\epsilon_0 r^2} \frac{\vec{r}}{r}$$

$$\vec{F}_2 = e_2 \vec{E}_1 = \frac{e_1 e_2}{4\pi\epsilon_0 r^2} \frac{\vec{r}}{r}$$

$$d(\vec{r} \cdot \vec{r}) = \vec{r} \cdot d\vec{r} + d\vec{r} \cdot \vec{r} = 2\vec{r} \cdot d\vec{r}$$

$$\vec{r} \cdot d\vec{r} = \frac{1}{2} d(\vec{r} \cdot \vec{r}) = \frac{1}{2} d(r^2) = r dr$$



$$d\vec{r} = d\vec{s}$$

$$A_e = \int_{r_a}^{r_b} \vec{F}_2 \cdot d\vec{s} = \frac{e_1 e_2}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{\vec{r} \cdot d\vec{s}}{r^3} = \frac{e_1 e_2}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{r dr}{r^3} = \frac{e_1 e_2}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{dr}{r^2} = -\frac{e_1 e_2}{4\pi\epsilon_0} \frac{1}{r} \Big|_{r_a}^{r_b}$$

ENERGIJSKI ZAKON (poslošitev)

$$A = \Delta W_k + \Delta W_{g,p}$$

$$A = A_{ost} + A_e$$

$$A_{ost} + A_e = \Delta W_k + \Delta W_{g,p}$$

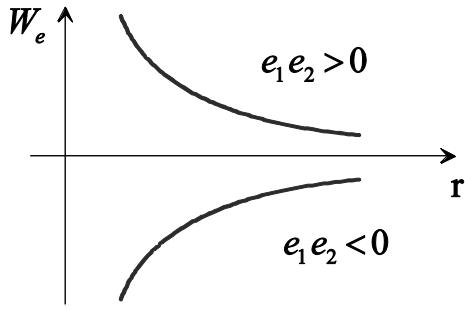
$$A_{ost} = \Delta W_k + \Delta W_{g,p} - A_e$$

$$A_{ost} = \Delta W_k + \Delta W_{g,p} + \frac{e_1 e_2}{4\pi\epsilon_0} \frac{1}{r} \Big|_{r_a}^{r_b}$$

$$W_e = \frac{e_1 e_2}{4\pi\epsilon_0 r} + \text{konst.}$$

$$A_{ost} = \Delta W_k + \Delta W_{g,p} + \Delta W_e$$

$$W_e = \frac{e_1 e_2}{4\pi \epsilon_0 r} + \text{konst.}$$



$$A_{ost} = 0$$

$$0 = \Delta W_k + \Delta W_{g,p} + \Delta W_e$$

$$\Delta(W_k + W_{g,p} + W_e) = 0$$

$$W_k + W_{g,p} + W_e = \text{konst.}$$

Elektrostatska potencialna energija **sistema nabojev**

$$W_{e,p} = \frac{1}{2} \sum_{ij} \frac{e_i e_j}{4\pi \epsilon_0 r_{ij}}$$

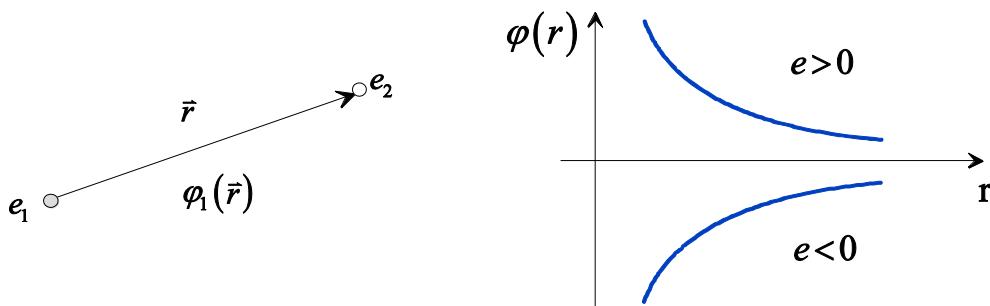
DEFINICIJA ELEKTRIČNEGA POTENCIALA

električna potencialna energija preračunana na testni nabojo:

$$\varphi \equiv \frac{W_e}{e} \quad \left[\frac{\text{J}}{\text{As}} \equiv \text{V} \right]$$

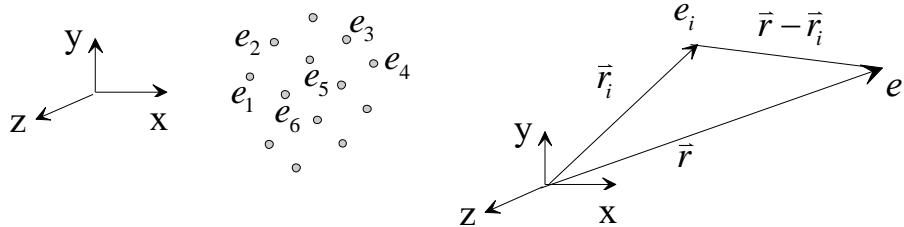
Primer dveh točkastih nabojev (električna potencialna energija naboja e_2 v polju naboja e_1)

$$W_e = e_2 \frac{e_1}{4\pi \epsilon_0 r} = e_2 \varphi_1 \quad \varphi_1 = \frac{e_1}{4\pi \epsilon_0 r} = \frac{W_e}{e_2}$$



Potencialna energija naboja e v polju sistema točkastih nabojev e_i :

$$W_e = e \sum_i \frac{e_i}{4\pi \epsilon_0 |\vec{r} - \vec{r}_i|} = e \varphi \quad \varphi(x, y, z) = \sum_i \frac{e_i}{4\pi \epsilon_0 |\vec{r} - \vec{r}_i|}.$$



Vsaki točki v prostoru pripada neka vrednost električnega potenciala $\varphi(x, y, z)$,
ki je posledica porazdelitve nabojev e_i v prostoru

ZVEZA MED \vec{E} IN φ

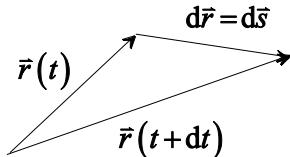
$$\varphi(\vec{r}) \equiv \text{skalarno polje}$$

$$\vec{E}(\vec{r}) \equiv \text{vektorsko polje}$$

$$d\varphi = \frac{\partial \varphi}{\partial x} dx + \frac{\partial \varphi}{\partial y} dy + \frac{\partial \varphi}{\partial z} dz = \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z} \right) \cdot (dx, dy, dz)$$

Hamiltonov operator (nable): $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$ $d\varphi = \vec{\nabla} \varphi \cdot d\vec{s}$

$$d\vec{s} = d\vec{r} = (dx, dy, dz)$$



Drugi način

$$A_{ost} = \Delta W_k + \Delta W_{g,p} - A_e = \Delta W_k + \Delta W_{g,p} + \Delta W \quad - A_e = \Delta W$$

$$W_e = e \sum_i \frac{e_i}{4\pi \epsilon_0 |\vec{r} - \vec{r}_i|} = e \varphi$$

$$d(W_e) = d(e \varphi) = -A_e = -\vec{F}_e \cdot d\vec{s} = -e \vec{E} \cdot d\vec{s} \quad d(e \varphi) = -e \vec{E} \cdot d\vec{s}$$

$$d\varphi = -\vec{E} \cdot d\vec{s}$$

$$d\varphi = \vec{\nabla} \varphi \cdot d\vec{s} \quad d\varphi = -\vec{E} \cdot d\vec{s}$$

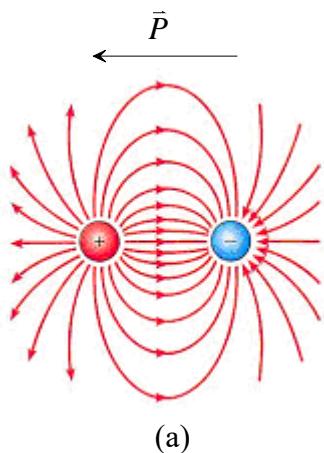
$$\vec{E} = -\vec{\nabla} \varphi = -\left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z}\right)$$

ekvipotencialne ploskve

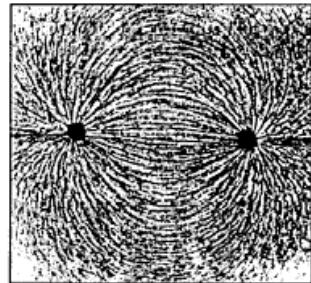
$$d\varphi = -\vec{E} \cdot d\vec{s}$$

$\check{c}e \quad \vec{E} \perp d\vec{s} \Rightarrow \underbrace{d\varphi = 0}_{\text{EKVIPOTENCIJALNE PLOSKVE}} \Rightarrow$ vektor \vec{E} pravokoten na ekvipotencialne ploskve

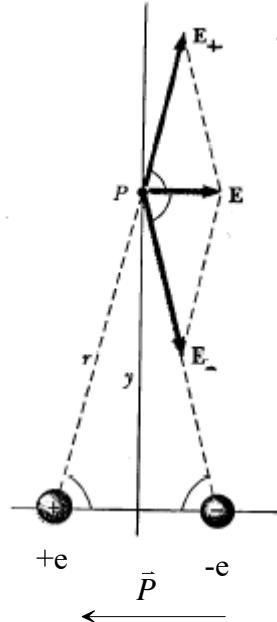
PRIMER : električni dipol



(a)



(b)



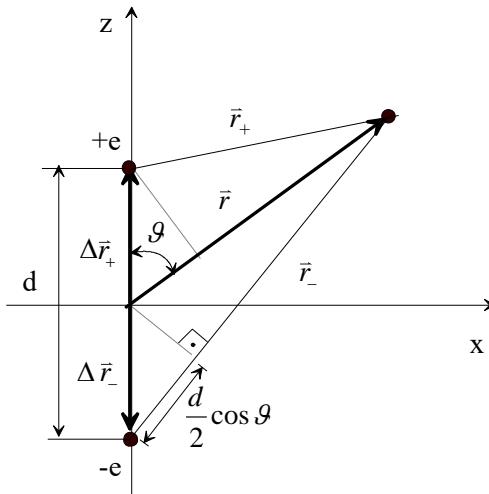
○ Prvi način:

vektorski zapis

\vec{r}_+ in \vec{r}_- :

$$\vec{r}_+ = \vec{r} - \Delta\vec{r}_+$$

$$\vec{r}_- = \vec{r} - \Delta\vec{r}_-$$



Električno polje dipola = vektorska vsota prispevkov obeh točkastih nabojev

$$\bar{E} = \frac{e}{4\pi\epsilon_0} \left[\frac{\vec{r}_+}{r_+^3} - \frac{\vec{r}_-}{r_-^3} \right] = \frac{e}{4\pi\epsilon_0} \left[\frac{\vec{r} - \Delta\vec{r}_+}{r_+^3} - \frac{\vec{r} - \Delta\vec{r}_-}{r_-^3} \right],$$

$$r_+ \cong r - \frac{d}{2} \cos \theta = r \left(1 - \frac{d}{2r} \cos \theta \right),$$

$$r_- \cong r + \frac{d}{2} \cos \theta = r \left(1 + \frac{d}{2r} \cos \theta \right).$$

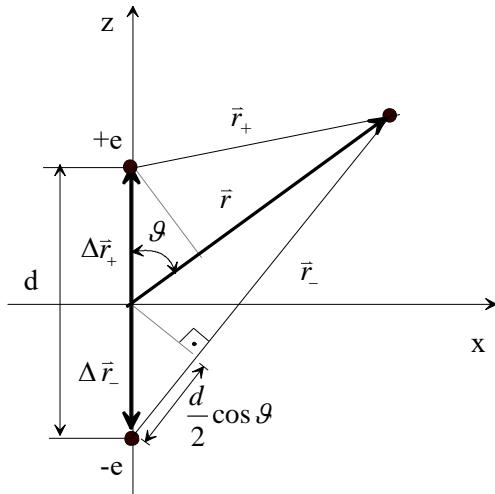
še dva približka :

$$\frac{1}{r_+^3} = \frac{1}{r^3 \left(1 - \frac{d}{2r} \cos \theta \right)^3} \cong \frac{1}{r^3} \left(1 + 3 \frac{d}{2r} \cos \theta \right),$$

$$\frac{1}{r_-^3} = \frac{1}{r^3 \left(1 + \frac{d}{2r} \cos \theta \right)^3} \cong \frac{1}{r^3} \left(1 - 3 \frac{d}{2r} \cos \theta \right),$$

$$\text{upoštevali : } \frac{1}{(1 \pm x)^3} \cong 1 \mp 3x$$

$$\begin{aligned} \bar{E} &= \frac{e}{4\pi\epsilon_0} \left[(\vec{r} - \Delta\vec{r}_+) \frac{1}{r^3} \left(1 + \frac{3}{2} \frac{d}{r} \cos \theta \right) - (\vec{r} - \Delta\vec{r}_-) \frac{1}{r^3} \left(1 - \frac{3}{2} \frac{d}{r} \cos \theta \right) \right] = \\ &= \frac{e}{4\pi\epsilon_0} \frac{1}{r^3} \left[\frac{\vec{r}}{r} 3d \cos \theta + (\Delta\vec{r}_- - \Delta\vec{r}_+) - \frac{3d}{2r} (\Delta\vec{r}_- + \Delta\vec{r}_+) \cos \theta \right] = \\ &= \frac{e}{4\pi\epsilon_0} \frac{1}{r^3} [3d \cos \theta \sin \theta, 0, 3d \cos^2 \theta - d], \end{aligned}$$



kjer smo upoštevali:

$$\vec{r} = (r \sin \theta, 0, r \cos \theta),$$

$$\Delta \vec{r}_+ = \left(0, 0, \frac{d}{2} \right),$$

$$\Delta \vec{r}_- = \left(0, 0, -\frac{d}{2} \right).$$

Torej:

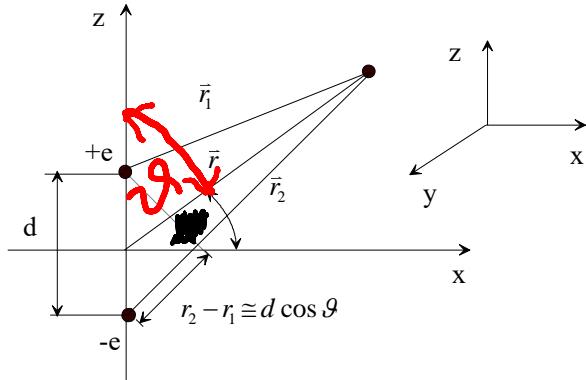
$$E_x = \frac{3 p_e \cos \theta \sin \theta}{4 \pi \epsilon_0 r^3}$$

$$E_y = 0$$

$$E_z = \frac{p_e (3 \cos^2 \theta - 1)}{4 \pi \epsilon_0 r^3}$$

električni dipolni moment: $p_e = d e$

Drugi način:



Električni potencial dipola:

$$\varphi = \varphi_1 + \varphi_2 = \frac{1}{4\pi\epsilon_0} \left(\frac{e}{r_1} - \frac{e}{r_2} \right) = \frac{e}{4\pi\epsilon_0} \left(\frac{r_2 - r_1}{r_1 r_2} \right) \cong \frac{e}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2}$$

kjer smo upoštevali: $r_2 - r_1 \cong d \cos \theta$ $r_1 r_2 \cong r^2$

$$\varphi = \frac{1}{4\pi\epsilon_0} \frac{p_e \cos \theta}{r^2} \quad p_e = e d$$

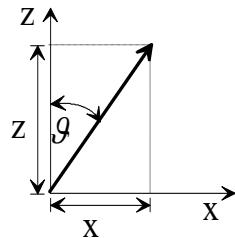
$$\theta = 0: \quad \varphi = \frac{1}{4\pi\epsilon_0} \frac{p_e}{r^2}$$

$$\theta = 90^\circ: \quad \varphi = 0$$

$$\varphi = \frac{1}{4\pi\epsilon_0} \frac{p_e z}{(x^2 + z^2)^{3/2}}$$

$$\cos \theta = \frac{z}{\sqrt{x^2 + z^2}}$$

$$r^2 = x^2 + z^2$$



Velja:

$$\vec{E} = -\vec{\nabla} \varphi$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \quad \vec{E} = - \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z} \right)$$

Torej :

$$\vec{E} = (E_x, E_y, E_z) = \left(-\frac{\partial \varphi}{\partial x}, -\frac{\partial \varphi}{\partial y}, -\frac{\partial \varphi}{\partial z} \right)$$

$$\varphi = \frac{1}{4\pi \epsilon_0} \frac{p_e z}{(x^2 + z^2)^{3/2}}$$

$$E_x = \frac{3 p_e \cos \vartheta \sin \vartheta}{4\pi \epsilon_0 r^3}$$

$$E_y = 0$$

$$E_z = \frac{p_e (3 \cos^2 \vartheta - 1)}{4\pi \epsilon_0 r^3}$$

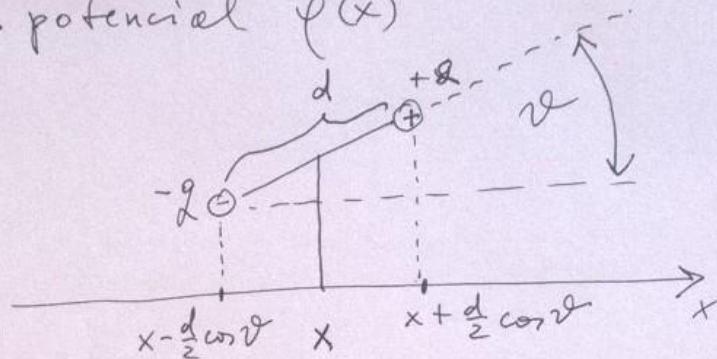
kjer veljajo izrazi v ravni $y = 0$. Zaradi osne simetrije lahko rezultat brez težav posplošimo.

$$\begin{aligned}
 \varphi &= \frac{1}{4\pi\epsilon_0} \frac{p_e \cos\vartheta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p_e z}{(x^2+z^2)^{3/2}} \\
 \vec{E} &= -\nabla\varphi = -\left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right) \\
 E_x &= -\frac{\partial\varphi}{\partial x} = +\frac{1}{4\pi\epsilon_0} \frac{3}{2} \frac{p_e z (k_z)}{(x^2+z^2)^{5/2}} = \\
 &= \underbrace{\frac{3 p_e}{4\pi\epsilon_0}}_{\frac{1}{r^3}} \underbrace{\frac{1}{(x^2+z^2)^{3/2}}}_{\cos\vartheta} \underbrace{\frac{z}{\sqrt{x^2+z^2}}}_{\sin\vartheta} \underbrace{\frac{x}{\sqrt{x^2+z^2}}}_{\sin\vartheta} = \frac{3 p_e \cos\vartheta \sin\vartheta}{4\pi\epsilon_0 r^3} \\
 E_y &= -\frac{\partial\varphi}{\partial y} = 0 \\
 E_z &= -\frac{\partial\varphi}{\partial z} = \frac{p_e (3 \cos^2\vartheta - 1)}{4\pi\epsilon_0 r^3}
 \end{aligned}$$

$$\begin{aligned}
 -\frac{\partial\varphi}{\partial z} &= -\frac{p_e}{4\pi\epsilon_0} \left\{ \frac{1}{(x^2+z^2)^{3/2}} + \left(\frac{3}{2}\right) \frac{z (k_z)}{(x^2+z^2)^{5/2}} \right\} = \\
 &= \underbrace{\frac{p_e}{4\pi\epsilon_0 (x^2+z^2)^{3/2}}}_{\frac{1}{r^3}} \left[\underbrace{\frac{3 z^2}{(x^2+z^2)}}_{3 \cos^2\vartheta} - 1 \right] = \\
 &= \frac{p_e (3 \cos^2\vartheta - 1)}{4\pi\epsilon_0 r^3}
 \end{aligned}$$

DIPOL - energija

el. potencial $\varphi(x)$



$$\varphi'(x) \equiv \frac{d\varphi}{dx}$$

$$\varphi''(x) \equiv \frac{d^2\varphi}{dx^2}$$

$$E = -\frac{d\varphi}{dx} = -\varphi'(x)$$

$$W_e = -q\varphi(x - \frac{d}{2}\cos\vartheta) + q\varphi(x + \frac{d}{2}\cos\vartheta)$$

(+)

$$-q[\varphi(x) - \frac{d}{2}\cos\vartheta\varphi'(x) + \frac{d^2}{8}\cos^2\vartheta\varphi''(x)]$$

$$+q[\varphi(x) + \frac{d}{2}\cos\vartheta\varphi'(x) + \frac{d^2}{8}\cos^2\vartheta\varphi''(x)]$$

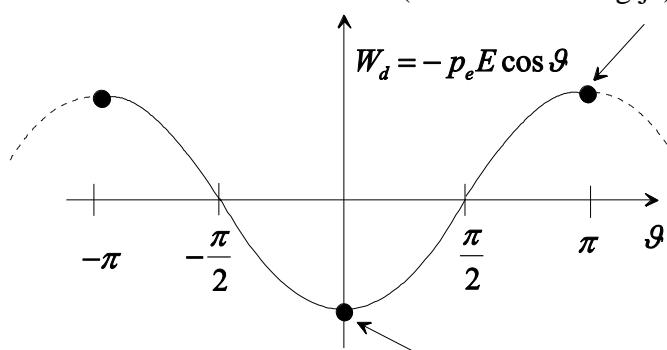
$$W_e = \underbrace{\frac{q}{2}d\cos\vartheta\varphi'(x)}_{p_e} = -p_e E \cos\vartheta$$

$$W_e = -p_e E \cos\vartheta$$

uporabili Taylorjevo vrsto:

$$\varphi(x + \Delta x) = \varphi(x) + \frac{\Delta x}{1!}\varphi'(x) + \frac{(\Delta x)^2}{2!}\varphi''(x) + \dots$$

labilni zasuk (labilna orientacija)
(maksimum energije)



stabilni zasuk (stabilna orientacija)
(minimum energije)

minimum energije: $\uparrow \overset{\vec{E}}{\uparrow} \vec{p}_e \vartheta = 0 : W_d = -p_e E$

maksimum energije: $\uparrow \overset{\vec{E}}{\downarrow} \vec{p}_e \vartheta = \pi : W_d = +p_e E$

KVADRUPOL - energija

elektr potenciel $\varphi(x)$

$\varphi'(x) \equiv \frac{d\varphi}{dx}$

$\varphi''(x) \equiv \frac{d^2\varphi}{dx^2}$

$$W_e = +g \varphi\left(x - \frac{d}{2} \cos \vartheta\right) + g \varphi\left(x + \frac{d}{2} \cos \vartheta\right)$$

$$= g \left[\varphi(x) - \frac{d}{2} \cos \vartheta \varphi'(x) + \frac{d^2}{8} \cos^2 \vartheta \varphi''(x) \right]$$

$$\textcircled{+} \quad g \left[\varphi(x) + \frac{d}{2} \cos \vartheta \varphi'(x) + \frac{d^2}{8} \cos^2 \vartheta \varphi''(x) \right]$$

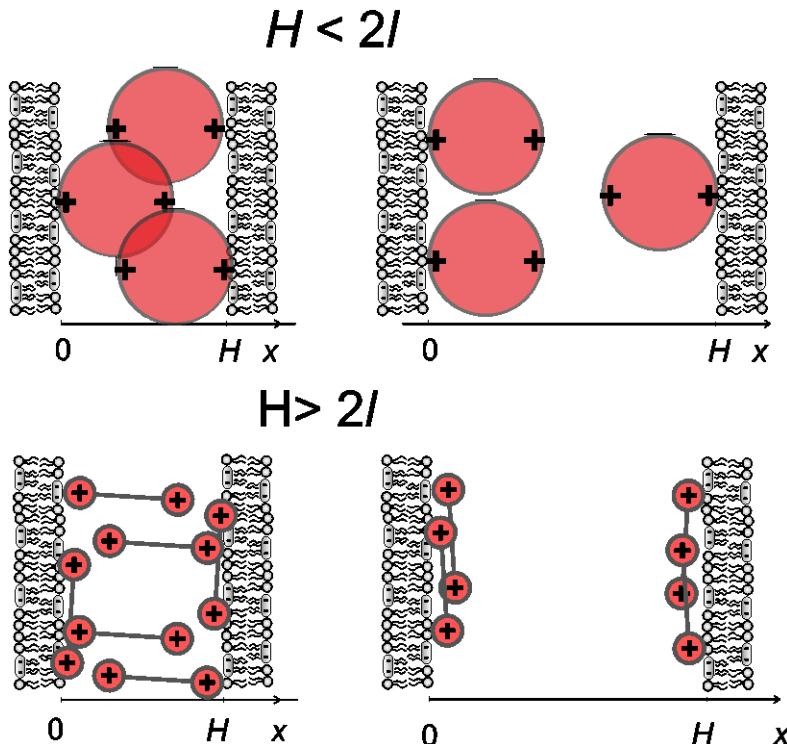
$$W_e = 2g \varphi(x) + g \frac{d^2}{4} \cos^2 \vartheta \varphi''(x) =$$

$$= 2g \varphi(x) - g \frac{d^2}{4} \cos^2 \vartheta \frac{dE}{dx}$$

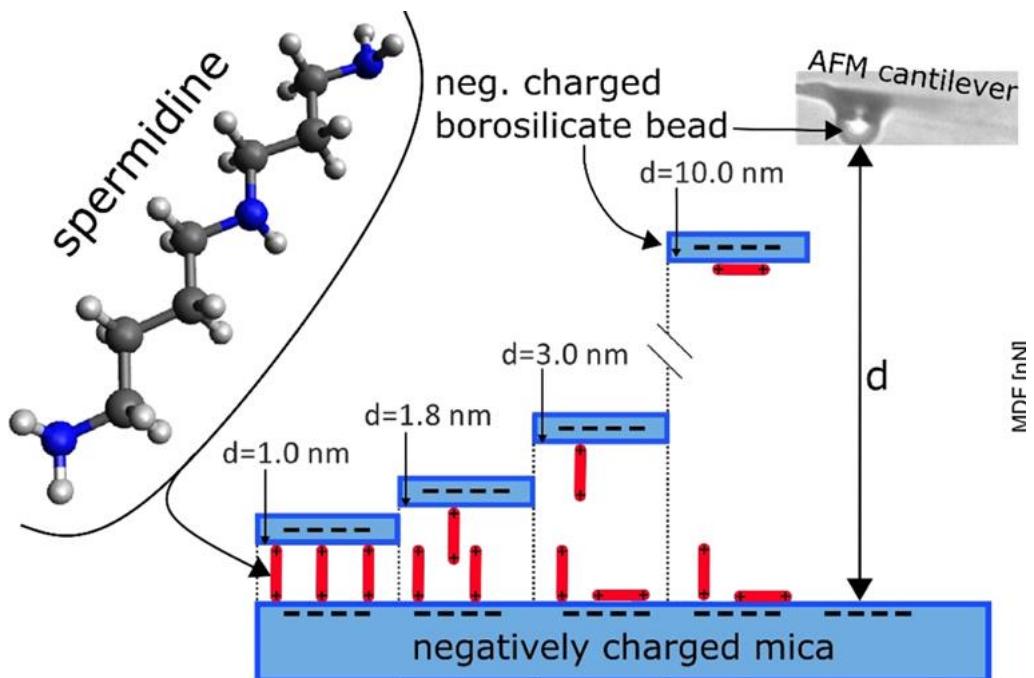
$E = -\frac{d\varphi}{dx} = -\varphi'(x)$

 $-\frac{dE}{dx} = \varphi''(x)$

$\therefore \text{je } E = \text{konst.} \Rightarrow \frac{dE}{dx} = 0$

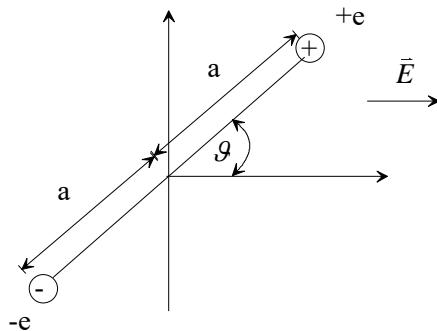


E. Gongadze, A. Velikonja, Š. Perutkova, P. Kramar, A. Maček-Lebar, V. Kralj-Iglič, A. Iglič, Ions and water molecules in an electrolyte solution in contact with charged and dipolar surfaces, *Electrochimica Acta*, 126: 42-60, 2014.
<http://physics.fe.uni-lj.si/publications/pdf/Gongadze et al Electr Acta 2014 PRINTED.pdf>



J. Gimša, P. Wysotzki, Š. Perutkova, T. Weihe, P. Elter, P.E. Marszałek, V. Kralj-Iglič, A. Iglič: The spermidine-induced attraction of like-charged surfaces is correlated with the pH-dependent spermidine charge: force spectroscopy characterization, *Langmuir*, 34: 2725-2733, 2018.
<http://physics.fe.uni-lj.si/publications/pdf/Clanek%20Gimša.pdf>

Drugi način : ENERGIJA ELEKTRIČNEGA DIPOLA V ZUNANJEM ELEKTRIČNEM POLJU



$$d = 2a$$

$$p_e = e d = e 2a$$

Navor na električni dipol:

$$+e: F_+ = +e E, M_+ = a e E \sin \theta$$

$$-e: F_- = -e E, M_- = (-a)(-e)E \sin \theta = a e E \sin \theta$$

Celoten navor:

$$M = M_- + M_+ = 2a e E \sin \theta = p_e E \sin \theta$$

Posplošitev:

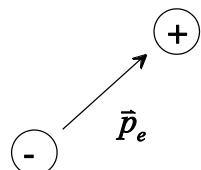
$$\vec{M} = \vec{p}_e \times \vec{E}$$

Energija dipola (delu, ki ga mora opraviti zunanji navor proti navoru zunanjega električnega polja)

$$W_d = \int M \, d\theta = \int_{\theta_1}^{\theta_2} p_e E \sin \theta \, d\theta = -p_e E \cos \theta \Big|_{\theta_1}^{\theta_2}$$

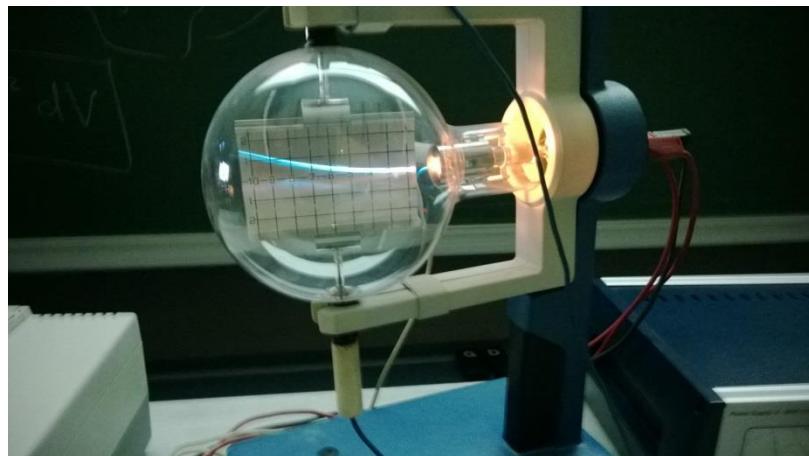
Posplošitev:

$$W_d = -\vec{p}_e \cdot \vec{E}$$



Gibanje električnih nabojev v električnem polju

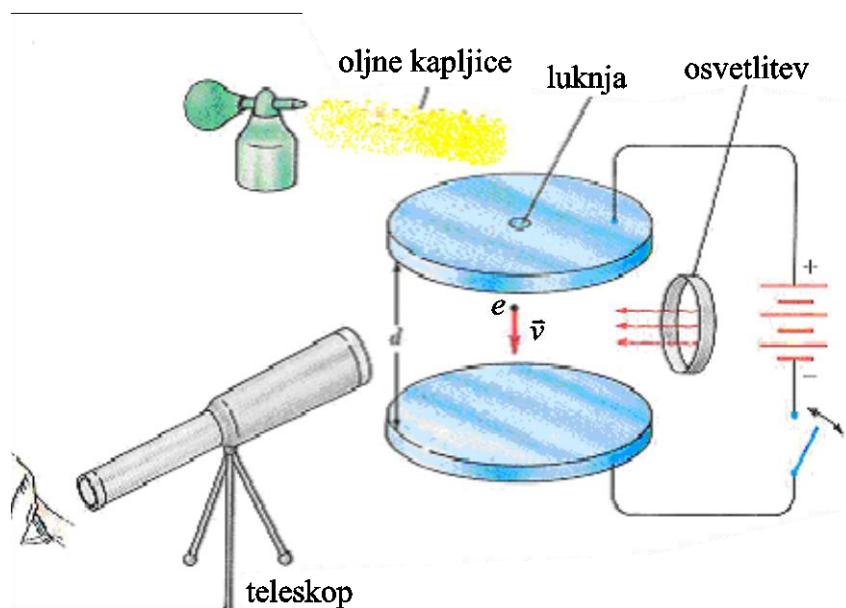
ODKLON CURKA ELEKTRONOV V ELEKTRIČNEM POLJU



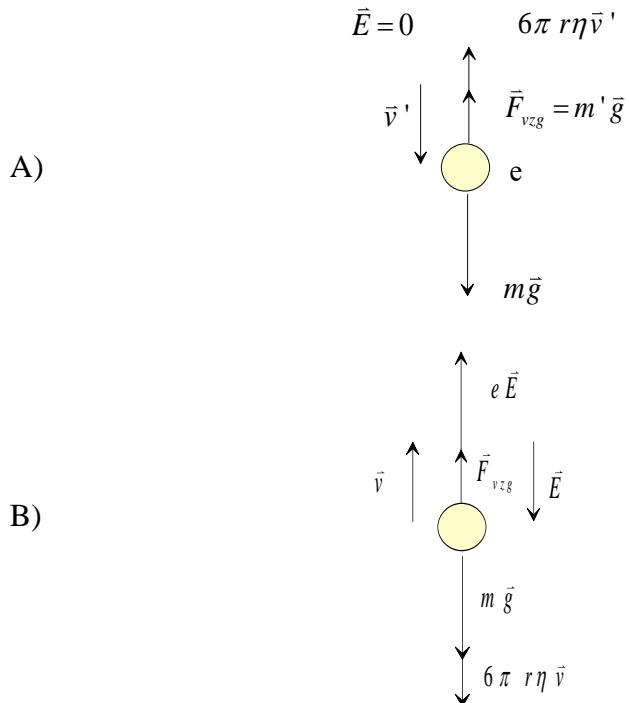
GIBANJE NABOJEV V VAKUUMU (konstantno električno polje)

$$m \frac{d\vec{v}}{dt} = e \vec{E}$$
$$\vec{v} = \vec{v}_0 + \frac{e}{m} \vec{E} t .$$

MILLIKANOV POSKUS (določitev osnovnega naboja)



Sile, ki delujejo na negativno nabito ($e < 0$) oljno kapljico:



$$\text{A primer: } E = 0: \quad m'g + 6\pi r \eta v' = mg$$

$$\text{B primer: } E \neq 0: \quad eE + m'g = mg + 6\pi r \eta v$$

m = masa oljne kapljice

Neznanki : e in r

m' = masa izpodrinjenega zraka

E = jakost električnega polja

v' , v = hitrosti kapljice

r = polmer oljne kapljice

$\eta \approx 1.8 \cdot 10^{-5}$ kg/ms viskoznost zraka

e = električni naboj oljne kapljice

$$r = \left[\frac{9\eta v'}{2(\rho - \rho')g} \right]^{1/2} \quad e = \frac{6\pi r \eta}{E} (v + v')$$

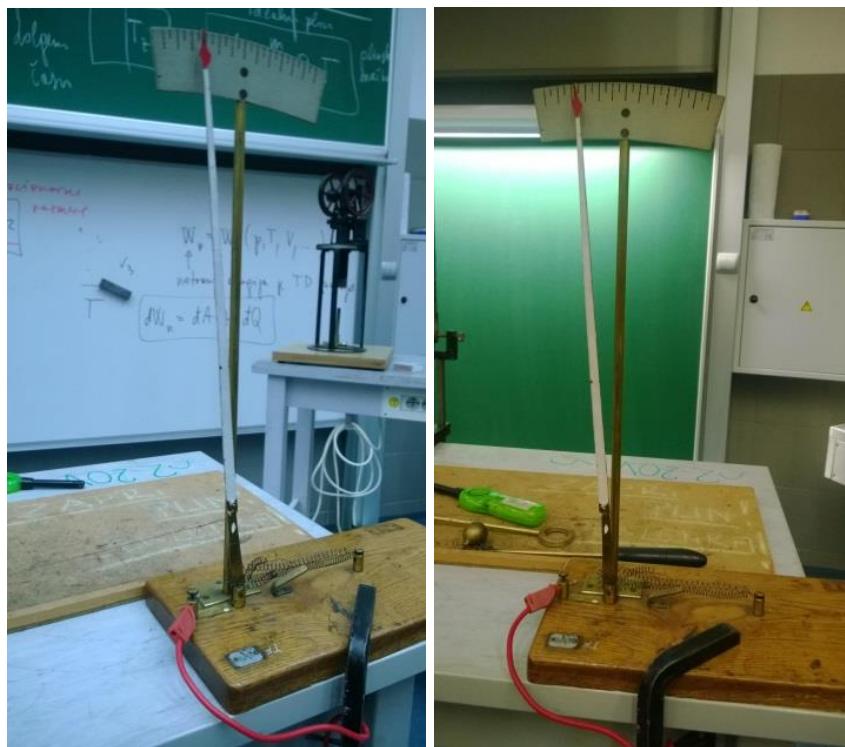
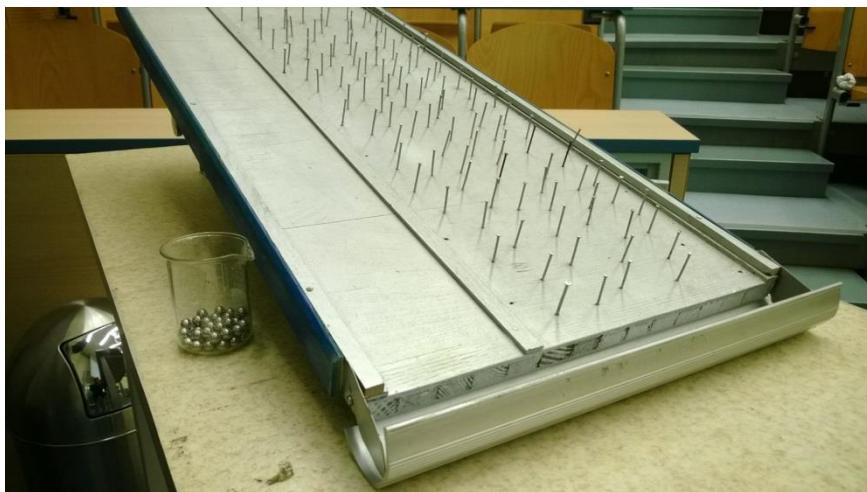
$$e = \frac{18\pi \eta (v + v')}{E} \left[\frac{\eta v'}{2(\rho - \rho')g} \right]^{1/2}$$

Rezultat poskusa:

$$e = n e_0, \quad n = 0, \pm 1, \pm 2, \pm 3 \quad e_0 = 1.6 \cdot 10^{-19} \text{ As}$$

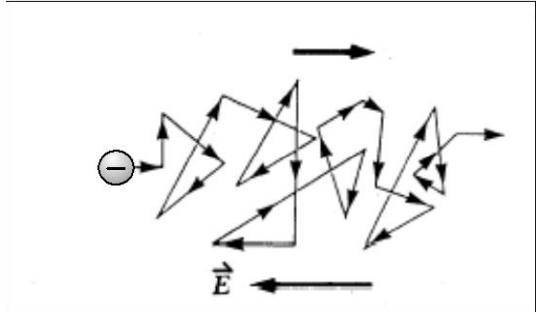
DELEC	NABOJ [As]	MASA [kg]
elektron (e)	$-1.6021917 \times 10^{-19}$	9.1095×10^{-31}
proton (p)	$+1.6021917 \times 10^{-19}$	1.67261×10^{-27}
nevtron (n)	0	1.67492×10^{-27}

GIBANJE PREVODNIŠKIH ELEKTRONOV V KOVINI



Naboji doživljajo trke, ki zavirajo njihovo gibanje \Rightarrow **zaviralna sila**

predpostavka: $\vec{F}_{zav} = -k \vec{v}$



$$m \frac{d\vec{v}}{dt} = e \vec{E} - k \vec{v} \quad \frac{d\vec{v}}{dt} + \frac{k}{m} \vec{v} = \frac{e}{m} \vec{E}$$

relaksacijski čas : $\tau = \frac{m}{k}$,

izključimo električno polje ($E = 0$) : $\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau} = 0$ $\vec{v} = \vec{v}_0 e^{-t/\tau}$
čas τ je približno enak času med dvema zaporednima trkoma

SNOV	τ [s]
kovine	$\sim 10^{-14}$
razelektritve v plinih	$\sim 10^{-9}$
sončna korona	$\sim 10^2$
medzvezdni plin	$\sim 10^5$

časovno povprečje v stacionarnem stanju:

$$\left\langle \frac{d\vec{v}}{dt} \right\rangle + \frac{\langle \vec{v} \rangle}{\tau} = \frac{e}{m} \langle \vec{E} \rangle$$

$$\langle \vec{a} \rangle \sim \left\langle \frac{d\vec{v}}{dt} \right\rangle = 0 \quad \text{in in } \langle \vec{E} \rangle \equiv \vec{E} = \text{konst.} \quad \langle \vec{v} \rangle = \frac{e\tau}{m} \vec{E}$$

$$\langle \bar{v} \rangle = \frac{e\tau}{m} \bar{E}$$

gostoto električnega toka (j) : $j = \frac{I}{S} = \frac{1}{S} \frac{de}{dt} = \frac{1}{S} \frac{n e dV}{dt} = \frac{1}{S} \frac{n e (S dx)}{dt} = n e \frac{dx}{dt} = n e v$

pospološitev: $\vec{j} = n e \langle \bar{v} \rangle$

$$n = \frac{N}{V}$$

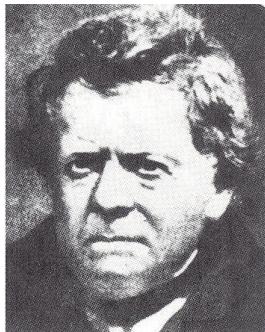
Ohmov zakon :

$$\vec{j} = n e \frac{e\tau}{m} \bar{E} = \sigma \bar{E},$$

$$\sigma = \frac{n e^2 \tau}{m}$$

za prevodne elektrone v kovini je $e = -e_0 = -1.6 \cdot 10^{-19}$ As in $m = m_e$ masa elektrona :

$$\sigma = \frac{n e_0^2 \tau}{m_e}$$



Georg Simon Ohm
(1787 – 1854)

Poseben primer: dolg valjast vodnik dolžine l s konstantno površino preseka (S):

$$j = \sigma E = \sigma \frac{U}{l} \quad U = \Delta \varphi$$

$$\frac{I}{S} = \sigma \frac{U}{l}$$

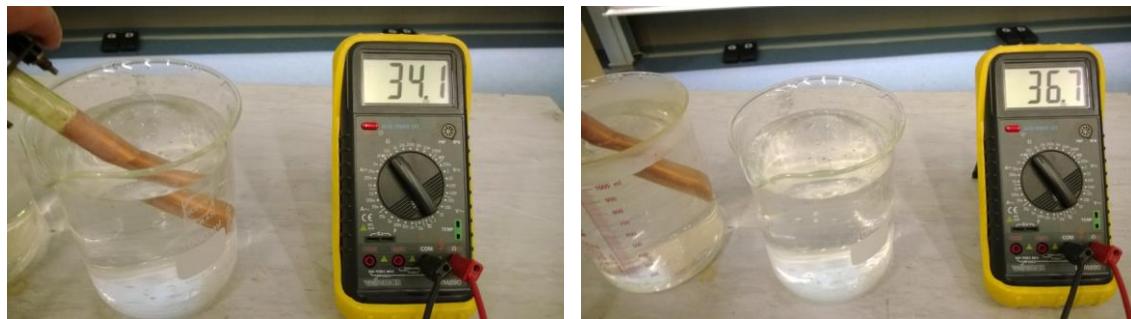
$$U = IR$$

$$R = \frac{1}{\sigma} \frac{l}{S}$$

$$\rho = \frac{1}{\sigma}$$

$$\rho = \rho_0 [1 + \alpha (T - T_0)], \quad \alpha = \text{temperaturni koeficient specifičnega upora}$$

MATERIAL	$\rho [\Omega \text{m}]$	$\alpha [\text{K}^{-1}]$
srebro	1.59×10^{-8}	3.8×10^{-3}
baker	1.7×10^{-8}	3.9×10^{-3}
zlato	2.44×10^{-8}	3.4×10^{-3}
aluminij	2.82×10^{-8}	3.9×10^{-3}
volfram	5.6×10^{-8}	4.5×10^{-3}
železo	10×10^{-8}	5.0×10^{-3}
platina	11×10^{-8}	3.92×10^{-3}
svinec	22×10^{-8}	3.9×10^{-3}



Padanje električnega toka v topotno izolirani žici zaradi segrevanja

$$R = \rho \frac{l}{S} \Rightarrow R = R_0 [1 + \alpha(T - T_0)],$$

$$\frac{R - R_0}{R_0} = \alpha(T - T_0), \quad \frac{dR}{R} = \alpha dT$$

$$I = \frac{U_0}{R}, \quad dI = U_0 \left(-\frac{1}{R^2} \right) dR$$

$$P dt = dQ = c_p m dT \Rightarrow dT = \frac{P dt}{c_p m}$$

$$dI = U_0 \left(-\frac{1}{R^2} \right) dR \quad \frac{dR}{R} = \alpha dT$$

$$dI = -\frac{U_0}{R^2} R \alpha dT = -\frac{U_0}{R^2} R \alpha \frac{P dt}{c_p m},$$

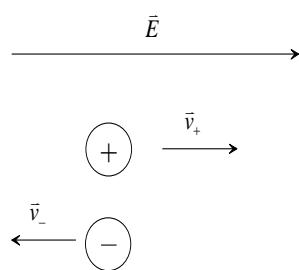
$$dT = \frac{P dt}{c_p m}$$

$$\frac{dI}{dt} = -\frac{U_0 \alpha P}{R c_p m} \quad m = \text{masa žice}$$

c_p = specifična toplota pri konstantnem pritisku

ELEKTRIČNI TOK V ELEKTROLITIH

Primer: $\text{K Cl} \rightarrow \text{K}^+ + \text{Cl}^-$
kation anion



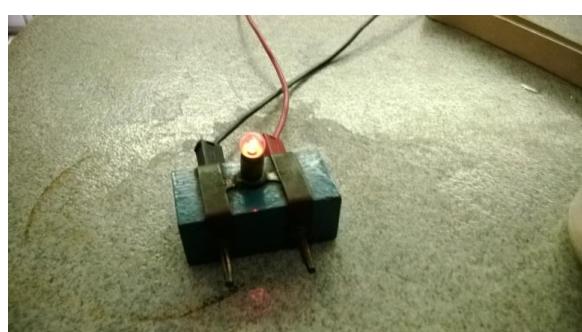
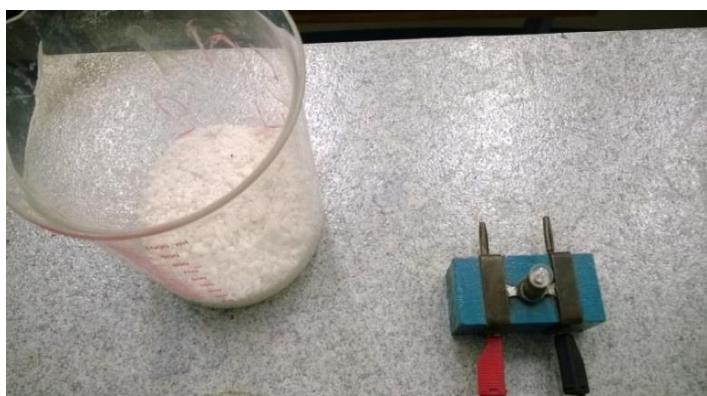
(+)

≡ kation

(-)

≡ anion

ELEKTROLIT ($\text{NaCl} + \text{H}_2\text{O}$) PREVAJA ELEKTRIČNI TOK



z_+ ≡ število osnovnih nabojev kationa
 z_- ≡ število osnovnih nabojev aniona

Stokesov zakon : $F_{up} = 6\pi r\eta v$

$$\langle v_+ \rangle = \beta_+ \vec{E} \quad \langle v_- \rangle = -\beta_- \vec{E}$$

β_+ = gibljivost kationov
 β_- = gibljivost anionov

$$j = z_+ e_0 n_+ \langle v_+ \rangle + z_- (-e_0) n_- \langle v_- \rangle,$$

$$n = n_+ = n_- \quad j = (z_+ e_0 n_+ \beta_+ + z_- e_0 n_- \beta_-) E$$

$$\sigma = (z_+ e_0 n_+ \beta_+ + z_- e_0 n_- \beta_-)$$

POLNjenje kondenzatorja

$$U_0 + U_R + U_C = 0$$

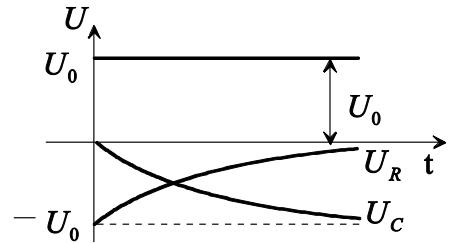
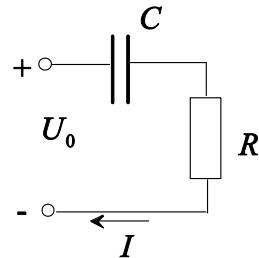
$$e = CU_C$$

$$U_0 - IR - \frac{e}{C} = 0 \quad / \frac{d}{dt}$$

$$-R \frac{dI}{dt} - \frac{I}{C} = 0$$

$$\frac{dI}{I} = -\frac{1}{RC} dt \quad / \int$$

$$\ln I \Big|_{I_0}^I = -\frac{1}{RC} t \Big|_0^t \Rightarrow I = I_0 e^{-\frac{t}{RC}}$$



$$U_C = -U_0 - U_R$$

$$U_C = -U_0 + IR = -U_0 + R I_0 e^{-\frac{t}{RC}}$$

velja: $RI_0 = U_o$, ker $U_C(t=0) = 0$

$$\text{torej: } U_C = -U_0 + U_0 e^{-\frac{t}{RC}}$$

$$U_C = -U_0 \left(1 - e^{-\frac{t}{RC}} \right)$$

$$U_R = -IR = -R I_0 e^{-\frac{t}{RC}} = -U_0 e^{-\frac{t}{RC}}$$

$$\text{Upoštevali smo } RI_0 = U_0$$

PRAZNJENJE KONDENZATORJA SKOZI UPOR

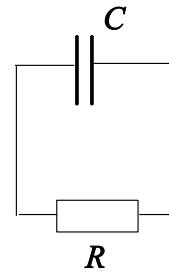
$$U_C + U_R = 0 \quad e = CU_C$$

$$-\frac{e}{C} - IR = 0$$

$$-\frac{1}{C} \frac{de}{dt} - R \frac{dI}{dt} = 0$$

$$-\frac{I}{C} - R \frac{dI}{dt} = 0$$

$$\frac{dI}{I} = -\frac{1}{RC} dt, \text{ torej}$$



$$I = I_0 e^{-\frac{t}{RC}}$$

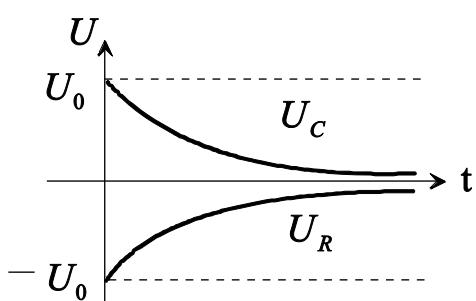
$$U_C = -U_R = IR = RI_0 e^{-\frac{t}{RC}}$$

$$U_C = U_0 e^{-\frac{t}{RC}}, \quad U_C > 0$$

upoštevali $RI_0 = U_0$

$$U_R = -IR = -RI_0 e^{-\frac{t}{RC}} = -U_0 e^{-\frac{t}{RC}}$$

$$U_R = -U_0 e^{-\frac{t}{RC}} \quad U_R < 0$$



SLIKA: polnjenje in praznjenje kondenzatorja (beleženje krivulje na osciloskopu)



PREMIKALNI TOK

- V krogih s kondenzatorjem dosežemo veljavnost kontinuitetne enačbe, če upoštevamo v kondenzatorju premikalni tok:

$$I = \frac{de}{dt} = C \frac{dU}{dt} = \epsilon_0 \frac{S}{d} \frac{dU}{dt} = \epsilon_0 S \frac{d\left(\frac{U}{d}\right)}{dt} = \epsilon_0 S \frac{dE}{dt} = S \frac{d(\epsilon_0 E)}{dt} = S \frac{dD}{dt}$$

$$I = S \frac{dD}{dt} \Rightarrow j_p = \frac{I}{S} = \frac{dD}{dt}$$

- vektorska oblika:

$$\vec{j}_p = \frac{d\vec{D}}{dt}.$$

- v nehomogenem polju velja:

$$I_p = \int \vec{j}_p \cdot d\vec{S} = \int \left(\frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

$$I_p = \int \left(\frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$