

7. Razredčeni plin vodika (H_2) mase 10 g, temperature $27^\circ C$ in volumena V_1 najprej adiabatno raztegnemo na štirikratni začetni volumen ($4 \cdot V_1$), nato pa ga izotermno stisnemo na polovični začetni volumen ($V_1/2$). Za koliko se spremeni notranja energija plina vodika pri opisanem procesu?

14. 4

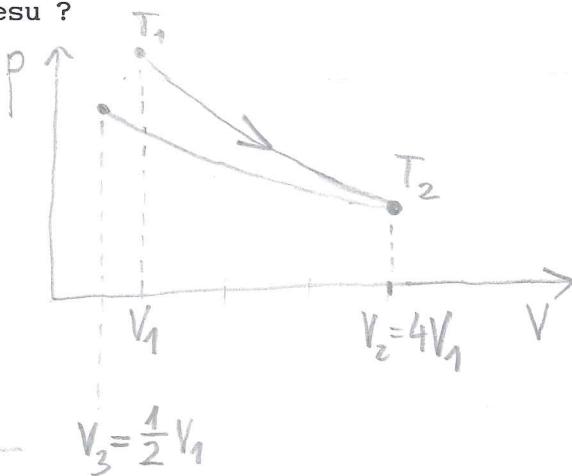
$$m = 10 \text{ g}$$

$$T_1 = 300 \text{ K}$$

$$c_V = \frac{5}{2} \frac{R}{M}$$

$$c_p = \frac{5}{2} \frac{R}{M} + \frac{R}{M} = \frac{7}{2} \frac{R}{M}$$

$$\kappa = \frac{c_p}{c_V} = \frac{7}{5} = 1.4$$



$$V_3 = \frac{1}{2} V_1$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\kappa-1} = 172,3 \text{ K} , \quad \Delta T = T_2 - T_1$$

$$\Delta W_n = c_V m \Delta T = \frac{5}{2} \frac{R}{M} m \Delta T = \underline{-13,27 \text{ kJ}}$$

3. Enoatomni idealni plin pri temperaturi $T_1 = 27^\circ\text{C}$ in tlaku $p_1 = 1,25 \cdot 10^6 \text{ Pa}$ ima volumen $V_1 = 4 \cdot 10^{-3} \text{ m}^3$. Plinu izobarno dovedemo toploto $Q = 2 \cdot 10^4 \text{ J}$. Kolikšna bosta končna temperatura in volumen plina?

4. Kolikšna je sprememba notranje energije (ΔW_n) in kolikšno delo je opravil plin (A) za izobarno spremebno stanja enoatomnega idealnega plina, ki je opisana pri prejšnji nalogi?

$$T_1 = 300 \text{ K}$$

$$p_1 = 1,25 \cdot 10^6 \text{ Pa}$$

$$V_1 = 4 \cdot 10^{-3} \text{ m}^3$$

$$Q = 2 \cdot 10^4 \text{ J}$$

$$T_2, V_2, \Delta W_n, A$$

$$c_p = c_v + \frac{R}{M} = \frac{3}{2} \frac{R}{M} + \frac{R}{M} = \frac{5}{2} \frac{R}{M} \quad (1. \text{ et. plin})$$

$$\boxed{p = \text{konst.}} \quad p_1 = p_2$$

$$p_1 V_1 = \frac{m}{M} R T_1 \Rightarrow m = \frac{p_1 V_1 M}{R T_1}$$

$$\Delta T = \frac{Q}{m c_p} = \frac{Q R T_1 2 M}{p_1 V_1 M 5 R} = \frac{2 Q T_1}{5 p_1 V_1} = 481 \text{ K}$$

$$\underline{\underline{T_2 = T_1 + \Delta T = 781 \text{ K} = 508^\circ\text{C}}}$$

$$\underline{\underline{\frac{T_1}{V_1} = \frac{T_2}{V_2}}} \Rightarrow V_2 = \frac{T_2}{T_1} V_1 = \underline{\underline{1,04 \cdot 10^{-2} \text{ m}^3}}$$

$$\underline{\underline{\Delta W_n = c_v m \Delta T = \frac{3}{2} \frac{R}{M} \frac{p_1 V_1 M}{R T_1} \Delta T = \frac{3}{2} \frac{p_1 V_1}{T_1} \Delta T = 1,2 \cdot 10^4 \text{ J}}}$$

$$\boxed{\Delta W_n = A + Q} \Rightarrow A = \underline{\underline{\Delta W_n - Q = -8000 \text{ J}}}$$

$$A = - \int_{V_1}^{V_2} p dV = p(V_1 - V_2)$$

3.

4)

1. 5 g dvoatomnega idealnega plina reverzibilno in adiabatno stisnemo iz začetnega stanja pri temperaturi 20°C in volumen 6 dm^3 na petkrat manjši končni volumen. Kolikšno delo moramo opraviti pri tem? ($M = 28 \text{ kg/kmol}$) A-L

OT,

2. Kolikšen je končni tlak pri adiabatni spremembi, ki je opisana pri prejšnji nalogi? Kolikšna pa je sprememba entropije in notranje energije plina? A-L

$$m = 5 \text{ g}, M = 28 \text{ kg/kmol}$$

$$T_1 = 293 \text{ K}$$

$$V_1 = 6 \cdot 10^{-3} \text{ m}^3$$

$$V_2 = V_1 / 5$$

$$c_v = \frac{5}{2} \frac{R}{M}, c_p = \frac{7}{2} \frac{R}{M}$$

$$\kappa = \frac{c_p}{c_v} = \frac{7}{5} = 1.4$$

① $Q=0 \Rightarrow A = \Delta W_n = c_v m \Delta T = \frac{5}{2} \frac{R}{M} m (T_2 - T_1) = \underline{\underline{981.2 \text{ J}}}$

$$T_1 V_1^{\kappa-1} = T_2 V_2^{\kappa-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\kappa-1} = T_1 \cdot 5^{0.4} = \underline{\underline{557.8 \text{ K}}}$$

$$\Theta = - \int_{V_1}^{V_2} p dV = \left[+ p_1 V_1^x \frac{1}{x-1} \right]_{V_1}^{V_2}$$

② $\boxed{p_1 V_1^x = p_2 V_2^x} \Rightarrow p_2 = p_1 \left(\frac{V_1}{V_2} \right)^x = \frac{1}{V_1} \frac{m}{M} R T_1 \left(5 \right)^x = \underline{\underline{6,89 \cdot 10^5 \text{ Pa}}}$

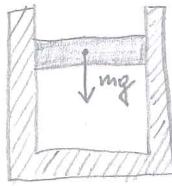
$$p_1 = \frac{1}{V_1} \frac{m}{M} R T_1$$

$$\Delta S = \int \frac{dQ}{T} = 0$$

$$\Delta W_n = A = \underline{\underline{981.2 \text{ J}}}$$

6. V cilindru preseka 1 cm^2 je z batom mase $0,3 \text{ kg}$ zaprt enoatomen plin z volumnom 2 dm^3 . Bat in stene posode so topotno izolirani. Bat se lahko giblje po cilindru skoraj brez trenja. Zunanji tlak je 10^5 Pa . Kolikšen je nihajni čas bata pri majhnih odmikih? Cilinder je postavljen vertikalno.

$$\begin{aligned} S &= 10^{-4} \text{ m}^2 \\ m &= 0,3 \text{ kg} \\ V_0 &= 2 \cdot 10^{-3} \text{ m}^3 \\ p_0 &= 10^5 \text{ Pa} \\ t_0 &=? \end{aligned}$$



$$p_{\text{no}} S = p_0 S + mg$$

$$p_{\text{no}} = p_0 + \frac{mg}{S}$$

$$\text{enoatome plin: } \left. \begin{array}{l} c_v = \frac{3}{2} \frac{R}{M} \\ c_p = \frac{5}{2} \frac{R}{M} \end{array} \right\} \kappa = \frac{c_p}{c_v} = \frac{5}{3} = 1,66$$

$$pV^\kappa = \text{konst.}$$

$$\Delta V = S \times x \quad \times \ll 1$$

$$p \kappa V^{\kappa-1} dV + V^\kappa dp = 0 \Rightarrow \Delta p = - \frac{p \kappa V^{\kappa-1}}{V^\kappa} = - \frac{p \kappa}{V} dV$$

$$= - \kappa \frac{p_{\text{no}}}{V_0} S x$$

$$m \ddot{x} = S \Delta p = - \kappa \frac{p_{\text{no}}}{V_0} S^2 x$$

$$\ddot{x} = - \kappa \underbrace{\frac{p_{\text{no}} S^2}{m V_0} \cdot x}_{\left(\frac{2\pi}{t_0}\right)^2}$$

II,

$$t_0 = 2\pi \sqrt{\frac{m V_0}{\kappa \cdot p_{\text{no}} \cdot S^2}} = 2\pi \sqrt{\frac{m V_0}{\kappa \cdot S^2 \left(p_0 + \frac{mg}{S}\right)}} = 3.31 \text{ s}$$

Toplotna - dolo pri izotermnem stiskanju

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Koliko dela opravimo, če izotermno stisnemo 5 kg idealnega plina z molsko maso 29 kg.kmol⁻¹ pri temperaturi 0°C od tlaka 10⁵N.m⁻² na 5.10⁵ N.m⁻². (Splošna plinska konstanta R = 8,3.10³ J.kmol⁻¹.K⁻¹).

$$A = \int_{V_2}^{V_1} p dV \quad \text{pri } \text{zobenim spremembri veljki:}$$

$$pV = nRT = p_1 V_1 \text{ ali } p = \frac{nRT}{V} \text{ je to dobro}$$

$$A = nRT \int_{V_2}^{V_1} \frac{1}{V} dV = nRT \ln \frac{V_1}{V_2}$$

$$\begin{aligned} A &= nRT \ln \frac{V_1}{V_2} = p_1 V_1 \ln \frac{V_1}{V_2} = p_2 V_1 \ln \frac{p_2}{p_1} \\ &= -p_1 V_1 \ln \frac{V_2}{V_1} \end{aligned}$$

$$A = nRT \ln \frac{V_1}{V_2}$$

$$A = -nRT \ln \frac{V_2}{V_1}$$

$$A = nRT \ln \frac{p_2}{p_1}$$

$$A = -nRT \ln \frac{p_1}{p_2}$$

$$A = nRT \ln \frac{p_2}{p_1} = \frac{5 \text{ kg.kmol} \cdot 8,3 \cdot 10^3 \text{ J} \cdot 273 \text{ K} \cdot 1,6094}{29 \text{ kg.kmol.K}} = \underline{\underline{6,287 \cdot 10^5}}$$

$$\ln 5 = 1,6094$$

$$\frac{6,287 \cdot 10^5}{6,287 \cdot 10^4}$$

$$6,287 \cdot 10^5 = 628700$$

$$6,287 \cdot 10^4 = 628700$$

Rješenje 1841 FIZIKA I

1. Liter idealnega dvoatomnega plina s temperaturo 0°C pri tlaku 10^5 Pa izredno hitro stisnemo na polovico začetne prostornine. Plin nato pri konstantnem tlaku počasi ohladimo nazaj na temperaturo 0°C . Kakšna je končna prostornina plina? Nekateri podatki so odveč! FI

$$V_3 = ? , V_1 = 10^{-3} \text{ m}^3$$

adiabata:

$$V_1, T_1 \rightarrow \frac{V_1}{2}, T_2 \Rightarrow \boxed{\frac{T_1}{T_2} = \left(\frac{V_1/2}{V_1}\right)^{k-1}}$$

dvoatomni plin:

$$k = \frac{c_p}{c_v} = 1.4$$

$$pV = \frac{m}{M} RT$$

$p = \text{konst.}$:

$$\frac{V_1}{2}, T_2 \rightarrow V_3, T_1 \Rightarrow \boxed{\frac{T_1}{V_3} = \frac{T_2}{(V_1/2)}}$$



$$\underline{V_3 = \frac{T_1}{T_2} \cdot \frac{V_1}{2}} = \left(\frac{1}{2}\right)^{k-1} \cdot \left(\frac{V_1}{2}\right) = \underline{\left(\frac{1}{2}\right)^k \cdot V_1} =$$

$$= 0,375 \cdot 10^{-3} \text{ m}^3$$

1. Valjasto posodo, napolnjeno z enoatomnim plinom, zapira zelo lahek in tanek bat. Temperatura plina je enaka temperaturi zraka zunaj posode, ki je enaka 20°C . Presek posode je 1 dm^2 , bat pa miruje 1 m nad dnem. Na bat položimo utež z maso 10 kg . Za koliko centimetrov se premakne bat takoj, ko položimo utež na bat? Kolikšno delo opravi utež na plinu v tem kratkem času? Stene posode niso izolirane.

2. Nadaljavanje 1. naloge :

Za koliko centimetrov pa se še premakne bat v primeru, če počakamo dalj časa? Kolikšna je skupna končna sprememba notranje energije plina v batu potem, ko se bat spusti na svojo končno ravnočesno višino? Čas štejemo od trenutka, ko položimo utež na bat. Plin v batu obravnavamo kot idealni plin.

M-3 ✓

$$P_0$$

$$S = 10^{-2} \text{ m}^2$$

$$g = 10 \text{ m s}^{-2}$$

$$T_0 = 293 \text{ K}$$

$$\gamma = 1,66$$

$$m = 10 \text{ kg}$$

$$l_0 = 1 \text{ m}$$

$$P_0 = 10^5 \text{ Pa}$$

$$c_v = \frac{3}{2} \frac{R}{M}$$

$$c_p = c_v + \frac{R}{M} = \frac{5}{2} \frac{R}{M}$$

$$\kappa = \frac{c_p}{c_v} = \frac{5}{3} = 1,66$$

①

$$P_0 V_0^K = P_1 V_1^K$$

$$V_0 = S \cdot l_0$$

$$V_1 = S \cdot l_1$$

$$l_1 = l_0 \left(\frac{P_0}{P_1} \right)^{1/K} = l_0 \left[\frac{P_0}{P_0 + \frac{mg}{S}} \right]^{1/K} = 0,944 \text{ m}$$

$$\Delta h = 5,6 \text{ cm}$$

$$Q=0 \Rightarrow A = \Delta W_n = c_v m \Delta T = \frac{3}{2} \frac{R}{M} \left(\frac{P_0 V_0}{T_0} \frac{M}{R} \right) \Delta T = \frac{3}{2} \frac{P_0 V_0}{T_0} (T_1 - T_0) \Rightarrow$$

$$P_0 V_0 = \frac{m}{M} R T_0 \Rightarrow m = \frac{P_0 V_0 M}{T_0 R}$$

$$A = 58.15 \text{ J}$$

$$T_0 V_0^{x-1} = T_1 V_1^{x-1} \Rightarrow T_1 = T_0 \left(\frac{V_0}{V_1} \right)^{x-1} = T_0 \left(\frac{l_0}{l_1} \right)^{x-1} = 304.36 \text{ K}$$

②

$$P_0 V_0 = P_1 \tilde{V}_1 \Rightarrow \tilde{l}_1 = l_0 \cdot \frac{P_0}{P_1} = l_0 \cdot \frac{P_0}{P_0 + \frac{mg}{S}} = 0,91 \text{ m}$$

$$\Delta \tilde{h} = 8 \text{ cm}$$

$$\Delta W_n = c_v m \Delta T = 0$$

$$A = - \int_{V_1}^{V_2} P dV = \left[P_1 V_1^{\frac{x}{x-1}} \right]_{V_1}^{V_2}$$

6. 3 g enoatomnega idealnega plina adiabatno razpnemo iz začetnega stanja pri tlaku 10^5 Pa in volumnu 6 dm^3 na trikrat večji končni volumen. Kolikšno delo opravi plin? ($M = 12 \text{ kg/kmol}$)

$$m = 3 \text{ g}$$

$$p_1 = 10^5 \text{ Pa}$$

$$V_1 = 6 \cdot 10^{-3} \text{ m}^3$$

$$V_2 = 3 V_1$$

$$M = 12 \frac{\text{kg}}{\text{kmol}}$$

$$A = ?$$

$$T_1 V_1^{k-1} = T_2 V_2^{k-1} \Rightarrow T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{k-1} = 138 \text{ K}$$

$$A = \Delta W_h = c_v m \Delta T = \frac{3}{2} \frac{R}{M} \cdot m \cdot (T_2 - T_1) = -467 \text{ J}$$

-3

$$= \frac{3}{2} \frac{R}{M} m$$

6. Ocenji delo pare in spremembo notranje energije pri izparitvi 1 kg vode pri 100°C in konstantnem tlaku 10^5 Pa . Izparilna toplota vode je $2,26 \cdot 10^6 \text{ J} \cdot \text{kg}^{-1}$. Paro obravnavamo kot idealni plin.

FOR

$$m = 1 \text{ kg}$$

$$M_{\text{H}_2\text{O}} = 18 \text{ g/mol}$$

$$T = 373 \text{ K}$$

$$p = 10^5 \text{ Pa}$$

$$\underline{L_i = 2,26 \cdot 10^6 \text{ J/kg}}$$

$$pV = \frac{m}{M} RT \Rightarrow V = \frac{m}{M \cdot p} \cdot R \cdot T = \underline{\underline{1,72 \text{ m}^3}}$$

$$A = -p\Delta V = -pV = \underline{\underline{-1,72 \cdot 10^5 \text{ J}}}$$

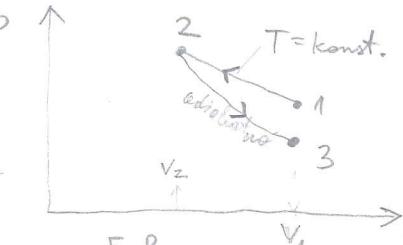
$$\underline{\Delta W_h = Q_{i,p} - p\Delta V = 2,26 \cdot 10^6 \text{ J} - 0,172 \cdot 10^6 \text{ J} = \underline{\underline{2,08 \cdot 10^6 \text{ J}}}}$$

2. 9 l kisika O_2 pri $T = 300 \text{ K}$ in $p = 10^5 \text{ Pa}$ izotermno stisnemo na prostornino 3 l, potem pa adiabatno razpnemo nazaj na prostornino 9 l. Prikazite ta proces na pV diagramu. Izračunajte delo, ki ga opravimo, spremembo notranje energije in spremembo entropije plina. Molska masa kisika O_2 : $M_{O_2} = 32 \text{ kg/kmol}$.

1
2
3
4

$$p_1 = 10^5 \text{ Pa}, V_1 = 3 \cdot 10^{-3} \text{ m}^3 \\ T_1 = 300 \text{ K} \\ V_2 = 3 \cdot 10^{-3} \text{ m}^3 \quad \left\{ \begin{array}{l} T = \text{konst} \\ M = 32 \text{ kg/kmol} \end{array} \right.$$

dvoatomni plin: $\kappa = \frac{c_p}{c_v} = \frac{7}{5} = 1.4$, $c_v = \frac{5}{2} \frac{R}{M}$



$$\{A, \Delta W_n, \Delta S\} = ?$$

$$pV = \frac{m}{M} RT$$

① $T = \text{konst}$.

$$A_1 = - \int_{V_1}^{V_2} pdV = - \int_{V_1}^{V_2} \frac{m}{M} RT_1 \frac{dV}{V} = \frac{m}{M} RT_1 \ln \frac{V_1}{V_2} = p_1 V_1 \ln \frac{V_1}{V_2} = 989 \text{ J}$$

$\boxed{\Delta W_n = Q + A = 0}$, $\Delta S = \frac{Q}{T_1} = \frac{-A}{T_1} = -3.3 \text{ J/K}$

② $S = \text{konst}$.

$$\Delta W_n = Q = 0 + A \Rightarrow \Delta W_n = A_2 = c_v m \Delta T = \frac{5}{2} \frac{R \cdot m}{M} (T_2 - T_1) =$$

$$T_1 V_2^{x-1} = T_2 V_1^{x-1} \\ T_2 = T_1 \left(\frac{V_2}{V_1} \right)^{x-1} \\ = \frac{5}{2} \left(\frac{R \cdot m}{M} T_1 \right) \left[\left(\frac{V_2}{V_1} \right)^{x-1} - 1 \right] =$$

$$= \frac{5}{2} p_1 V_1 \left[\left(\frac{V_2}{V_1} \right)^{x-1} - 1 \right] = -800 \text{ J}$$

skupaj:

$$A = A_1 + A_2 = \underline{\underline{189 \text{ J}}}$$

$$\Delta W_n = \underline{\underline{-800 \text{ J}}}$$

$$\Delta S = \underline{\underline{-3.3 \text{ J/K}}}$$

4. Dva litra dvoatomnega idealnega plina s temperaturo 20°C in tlakom 10^5 Pa adiabatno stisnemo na polovično prostornino. Koliko dela pri tem opravimo? Kolikšna je sprememba notranje energije plina?

1893

$$V_1 = 2 \cdot 10^{-3} \text{ m}^3, p_1 = 10^5 \text{ Pa}, T_1 = 293 \text{ K}$$

$$V_2 = V_1/2 = 10^{-3} \text{ m}^3$$

$$\frac{\frac{m}{M} R}{M} : p_1 V_1 = \frac{m}{M} R T_1 \Rightarrow R \frac{m}{M} = \frac{p_1 V_1}{T_1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{k-1} \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1}$$

$$T_2 = T_1 (2)^{k-1} = 293 \cdot 2^{0.4} = 386.6 \text{ K}$$

$$\Delta W_h = Q + A, Q=0 \Rightarrow \Delta W_h = A = c_v m \Delta T$$

~~$$A = c_v m \Delta T = \left(\frac{5}{2} \frac{R}{M} \right) m \cdot \Delta T = \frac{5}{2} \cdot R \frac{m}{M} \Delta T =$$~~

$$= \frac{5}{2} \cdot \left(\frac{p_1 V_1}{T_1} \right) (T_2 - T_1) = \frac{5 \cdot 10^5 \cdot 2 \cdot 10^{-3} \cdot 386.6}{2 \cdot 293} = \frac{10^3}{2 \cdot 293} = 160 \text{ J}$$

$$\left. \begin{aligned} c_p &= c_v + \frac{R}{M} = \frac{7}{2} \frac{R}{M} \\ c_v &= \frac{5}{2} \frac{R}{M} \end{aligned} \right\} k = \frac{c_p}{c_v} = \frac{7}{5} = \underline{\underline{1.4}}$$