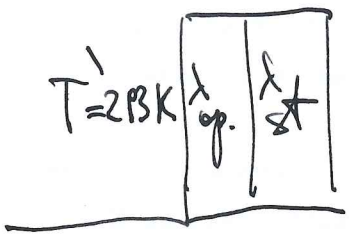
4. Soba ima obliko kocke z notranjim robom 2 m. Strop in tla ne prevajajo toplote, vse štiri stene pa so v stiku z okolico temperature 10°C. Najmanj kolikšno moč ima električni grelec v sobi, če je stacionarna temperatura v sobi 20°C ? Stranske stene sobe so narejene iz opečnatega zidu debeline 10 cm (λ_{op} = 0.7 W/(Km), ki je prekrit s slojem stiropora debeline 2 cm in toplotne prevodnosti λ_{st} = 0.04 W/(Km).

$a = 2\,m$



$T = 283\,K$

$d_{op} = 10\,cm$

$d_{st} = 2\,cm$

$$P = -\lambda \frac{T - T'}{x} S = \frac{T - T'}{R}$$

$$R = \frac{x}{\lambda S}$$

$$R = R_{op} + R_{st} = \frac{d_{op}}{S\lambda_{op}} + \frac{d_{st}}{S\lambda_{st}}$$

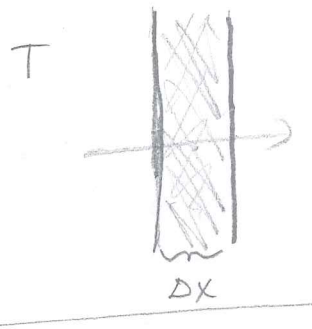
$$S = 4a^2 = 16\,m^2$$

$$P = \frac{T' - T}{\frac{d_{op}}{S\lambda_{op}} + \frac{d_{st}}{S\lambda_{st}}} = 249\,W$$

Rapid 1991

7. Telo z maso 4 kg in specifično toploto c_p = 1000 J/kg·K se ohlaja preko stene debeline 10 cm, površine 1 m^2 in s toplotno prevodnostjo 0,8 W/m·K. V času t=0 je temperatura telesa 25°C, tenperatura na drugi strani stene pa je ves čas 0°C. Kdaj bo temperatura telesa 0°C?

12,5 C



0 - 25

$m = 4\,kg$

$c_p = 1000\ J/kg \cdot K$

$\Delta x = 0{,}1\ m$

$S = 1\ m^2$

$\lambda = 0{,}8\ W/m \cdot K$

$T(t=0) = 25\,^\circ C$

$T_0 = 0\,^\circ C$

$$j = -\lambda\, \mathrm{grad}\, T = -\lambda \frac{T_0 - T}{\Delta x}$$

$$-j \cdot S \cdot dt = c_p\, m\, dT$$

$$+\frac{\lambda}{\Delta x} \cdot S (T_0 - T)\, dt = c_p \cdot m \cdot dT \quad \leftarrow \quad \alpha = \frac{\lambda S}{\Delta x \cdot c_p \cdot m}$$

$$\int_0^t +\alpha\, dt = \int_{T(0)}^{T} \frac{dT}{T_0 - T}$$

$T_0 - T = u$, $du = -dT$

$$+\alpha t = \int_{T_0 - T(0)}^{T_0 - T} \frac{-du}{u} = -\ln u \Big|_{T_0 - T(0)}^{T_0 - T} = -\ln \frac{T_0 - T}{T_0 - T(0)} = \ln \frac{T_0 - T(0)}{T_0 - T}$$

$$\frac{T_0 - T(0)}{T_0 - T} = e^{+\alpha t} \Rightarrow T_0 - T = [T_0 - T(0)]\, e^{-\alpha t}$$

$$(T - T_0) = [T(0) - T_0]\, e^{-\alpha t} \Rightarrow \frac{12.5}{25} = e^{-\alpha t_x}$$

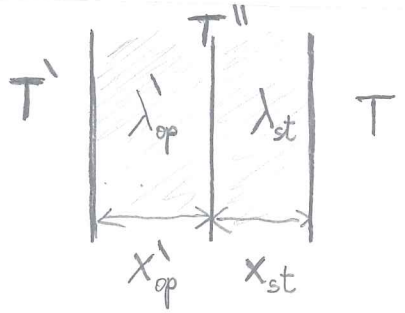
$$\ln \frac{1}{2} = -\alpha t_x \Rightarrow t_x = \frac{\ln 2}{\alpha} = \frac{\ln 2 \cdot \Delta x \cdot c_p \cdot m}{\lambda \cdot S}$$

$$= \frac{\ln 2 \cdot 0{,}1 \cdot 1000 \cdot 4}{0.8 \cdot 1} = 346.6\ s$$

$t_x = 346.6\ s$ ✓

M-7

4. Opečnati zid debeline 20 cm (λ_{op} = 0,7 W/(K·m)) je prekrit s slojem stiropora debeline 2 cm in toplotne prevodnosti λ_{st} = 0,04 W/(K·m). Kolikšna je temperatura na stikališču obeh plasti, če je temperatura v sobi 20°C, zunanja temperatura pa je 0°C ?



$T' = 293\,K$

$T = 273\,K$

$x'_{op} = 0.2\,m$, $\lambda_{op} = 0.7\ W/(K\cdot m)$

$x_{st} = 0.02\,m$, $\lambda_{st} = 0{,}04\ W/(K\cdot m)$

V stacionarnem stanju toplotni tok enak skozi obe plasti:

$$S_j = S\lambda'_{op}\frac{T' - T''}{x'_{op}} = S\lambda_{st}\frac{T'' - T}{x_{st}}$$

$$\Downarrow$$

$$T'' = \left(\frac{\lambda'_{op}T'}{x'_{op}} + \frac{\lambda_{st}T}{x_{st}}\right) \Big/ \left(\frac{\lambda'_{op}}{x'_{op}} + \frac{\lambda_{st}}{x_{st}}\right) = 285.7\,K\ (?)$$

1571.5

$$P = -(T' - T) \Big/ \left(\frac{x'_{op}}{S\lambda'_{op}} + \frac{x_{st}}{S\lambda_{st}}\right) = (T - T') \Big/ (R_{op} + R_{st})$$

$$P = -\lambda\frac{T - T'}{x}\cdot S = \frac{T - T'}{R}, \quad R = \frac{x}{\lambda S}$$

Upori se seštevajo

1 BP 2

6. Ocenite, kako debelo odejo s toplotno prevodnostjo $1{,}8 \cdot 10^{-2}$ J/(m·s·K) smemo zaviti onesvešcenca (ponesrečenca), ki ima temperaturo 37°C, da se ne bo pregrel. Temperatura okolice je 10°C. Površina deke je 1,8 m^2, vpliv potenja zanemarimo. Telo oddaja vsako sekundo 40 J toplote.

$$j = -\lambda \frac{dT}{dx}$$

$$P_{od} = \frac{dQ}{dt}$$

$\lambda = 1{,}8 \cdot 10^{-2}$ J/(m s K)

$T_č = 37\,°C$, $T_{okr} = 10\,°C$, $\Delta T = 27\,°C$

$S = 1{,}8\ m^2$

$P_{od} = 40$ J/s

$$P_{od} = P_{pre} = \lambda \frac{\Delta T}{\Delta x} \cdot S$$

$$\Delta x = \frac{\lambda \cdot \Delta T \cdot S}{P_{od}} = 2{,}19 \cdot 10^{-2}\ m = 2{,}2\ cm$$

2) Posoda ima obliko krogle z notranjim radijem 10 cm in zunanjim radijem 20 cm. Toplotna prevodnost stene je 0.2 W/mK. V posodi je voda s temperaturo 95° C, zunaj pa je temperatura 10° C. Kolikšno moč mora imeti grelec, da temperaturo vode v posodi vzdržuje pri 95° C?

1 pts

2) $r_1 = 10\,cm$

$r_2 = 20\,cm$

$T_1 = 95°C$

$T_2 = 10°C$

$\lambda = 0{,}2\,W/mK$

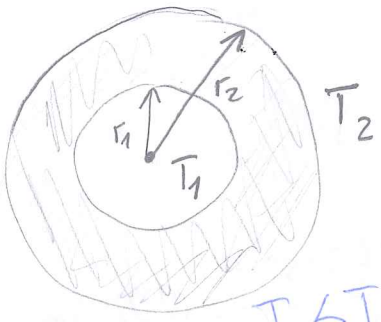
spremenljivka: T_1 = konst.

$j = -\lambda \frac{dT}{dr}$

$+P_{grelec} = P = \lambda 4\pi r^2 \frac{dT}{dr}$

$+\int_{T_1}^{T_2} dT = \frac{P_{grel}}{4\pi\lambda} \int_{r_1}^{r_2} \frac{dr}{r^2}$

$+\frac{1}{F}\Big|_{r_1}^{r_2} = \frac{1}{r_2} - \frac{1}{r_1}$



$T_2 < T_1$

$(T_1 - T_2) = \frac{P \cdot (r_2 - r_1)}{4\pi\lambda r_1 r_2}$

$P_{grel} = \frac{4\pi\lambda r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)} =$

$= \frac{4 \cdot \pi \cdot 0{,}2 \cdot 0{,}1 \cdot 0{,}2 \cdot 85}{0{,}1} = 42{,}73\,W$

✓

2

2. Zid se sestoji iz dveh slojev debeline d_1 = 8 cm in d_2 = 15 cm, katerih koeficienta toplotne prevodnosti sta λ_1 = 335 J/msK in λ_2 = 125 J/msK. Temperaturi zunanjih površin obeh slojev sta 15 °C in 32 °C. Izračunajte temperaturo na stiku obeh slojev v stacionarnem stanju !

Na osnovu zakona o očuvanju energije, količina toplote koja prođe kroz prvi sloj Q_1 jednaka je količini toplote koja prođe kroz drugi sloj Q_2

$$\lambda_1 S \frac{T_0 - T_1}{d_1} \tau = \lambda_2 S \frac{T_2 - T_0}{d_2} \tau, \qquad (1)$$

gdje su d_1 i d_2 debljine slojeva, T_1 i T_2 temperature vanjskih površina, λ_1 i λ_2 koeficijenti toplotne provodljivosti, S površina slojeva, τ vrijeme.
Iz (1) dobijamo

$$T_0 = \frac{\lambda_2 d_1 T_2 + \lambda_1 d_2 T_1}{\lambda_1 d_2 + \lambda_2 d_1}$$

$$T_0 = 290{,}822 \text{ K}.$$

$T_0 = 17{,}8$ K