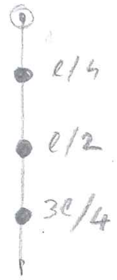
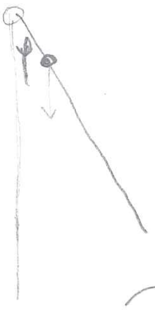
1994

1. Zelo lahla palica je na enem koncu pritrjena na vodoravno os. Na 1/4, 1/2, 3/4 dolžine palice, merjeno od osi, so pritrjene enake svinčene kroglice. Kolikšna je dolžina palice, če je nihajni čas tega nihala 1,5 s ? Kolikšen je tangetni pospešek kroglice na polovici dolžine palice, ko gre le ta skozi svojo mirovno lego ?



$$\boxed{\vec{F}\cdot\vec{r} = J\cdot\alpha}$$

$$\boxed{\ddot{\varphi} = -(2\pi\nu)^2\varphi}$$



$$\sum_i \vec{F}\cdot\vec{r} = mgr\sin\varphi$$

$$\underbrace{\cancel{m}g\varphi\left(\frac{l}{4}+\frac{l}{2}+\frac{3l}{4}\right)} = -\cancel{m}\overbrace{\left[\frac{l^2}{16}+\frac{l^2}{4}+\frac{9l^2}{16}\right]}^{J}\cdot\ddot{\varphi}$$

$$-\varphi g\left(\frac{(1+2+3)l}{4}\right) = \ddot{\varphi}\left[\frac{1+4+9}{16}\right]l^2$$

$$\ddot{\varphi} = -\frac{g}{l}\,\frac{24}{14}\,\varphi \Rightarrow (2\pi\nu)^2 = \frac{g}{l}\cdot\frac{24}{14}$$

$$l = \frac{24}{14}\cdot g\cdot\frac{1}{(2\pi\nu)^2} = \underbrace{\frac{24}{14}\cdot g\cdot\frac{t_0^2}{4\pi^2}} = \frac{24}{14}\cdot 10\cdot\frac{(1.5)^2}{4\pi^2}\Rightarrow$$

$$\Rightarrow l = 0.98\ \text{m}$$

$d = \frac{l}{2}$

$J =$

$$\boxed{a_t = 0}$$

obrni

3. Homogena valjasta plošča s polmerom 10 cm in debelino 0,5 cm, se lahko praktično brez trenja vrti okoli svoje vodoravno ležeče geometrijske osi, ki je pravokotna na ravnino plošče. Na obod plošč pritrdimo majhno svinčeno utež mase 20 g. S kolikšnim nihajnim časom niha plošča pri majhnih odmikih ? Gostota plošče je 8 g/cm^3.

1992

A – L

$g = 10\,ms^{-2}$

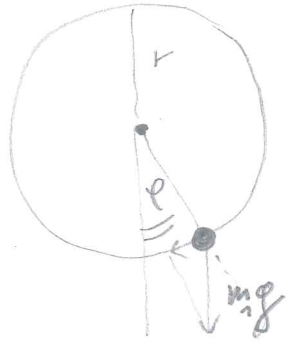
$r = 0.1\,m$

$h = 0.005\,m$

$m_1 = 2 \cdot 10^{-2}\,kg$

$\rho = 8\,g/cm^3 = 8000\,kg/m^3$

$$\boxed{J = \frac{1}{2} m r^2}$$



$\varphi \ll 1$

$$F \cdot r = J \cdot \alpha$$

$$m_1 g r \varphi = -\left(\frac{1}{2} m r^2 + m_1 r^2\right)\alpha$$

$$\alpha \cong -\left(\frac{m_1 g r}{\frac{1}{2} m r^2}\right)\varphi \Rightarrow$$

$\varphi = \varphi_0 \cos(2\pi\nu t)$

$\alpha = \ddot{\varphi} = -\varphi_0 (2\pi\nu)^2 \cos(2\pi\nu t)$

$$\boxed{\alpha = -(2\pi\nu)^2 \cdot \varphi}$$

$$2\pi\nu = \frac{2\pi}{t_0} = \left(\frac{2 m_1 g}{m r}\right)^{1/2} \Rightarrow \boxed{t_0 = 2\pi\sqrt{\frac{\pi r^2 \cdot h \cdot \rho \cdot r}{2 m_1 g}} = 3.52\,s}$$

$$\boxed{\ddot{x} = -\frac{k}{m} x}$$

$x = x_0 \sin \omega t$

$v = \dot{x} = x_0 \omega \cos \omega t = v_0 \cos \omega t$

$a = \ddot{x} = -x_0 \omega^2 \sin \omega t = -\underline{\omega^2 x}$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

$\sin' = \cos$
$\cos' = -\sin$

PODATKI:

$m = 10^{-2}$ kg
$x_0 = 0.02$ m
$\nu = 2\ s^{-1}$

1) S kolikšno hitrostjo utež skozi mirovno lego?

$v_0 = x_0 \omega = 0.25$ m/s

2) Kolikšen pospešek uteži v mirovni legi?

$a = -x_0 \omega^2 \underbrace{\sin \omega t}_{0} = 0$

3) Kolikšna sila vleče utež nazaj v mirovno lego, ko ima utež največji odmik?

$F = ma = m\ddot{x} = -m x_0 \omega^2 \underbrace{\sin \omega t}_{1} = \underline{\underline{-0.032\ N}}$

$F = -kx_r - kx_0$

4) Kolikšen je koeficient vzmeti?

$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m = 1.6 \frac{N}{m}$

$(\omega = 2\pi\nu)$ $\nu = 2\ s^{-1}$ 10^{-2} kg

E2*

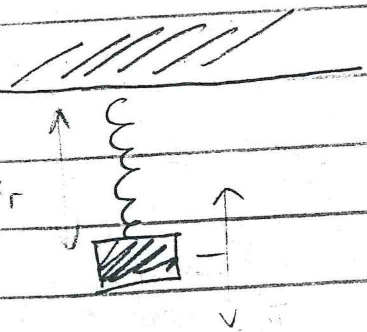
M - Naloga

$m = 10^{-2}$ kg

$x_0 = 0.02$ m

$\nu = 2\,s^{-1}$

Na vijačni vzmeti niha 10 gramska utež ~~s frekvenco~~ NIHAJNIM ČASOM 0.5 s in z amplitudo 2 cm. S kolikšno hitrostjo gre utež skozi mirovno lego? Kolikšen je pospešek uteži v mirovni legi? Kolikšna sila vleče utež nazaj v mirovno lego, ko ima utež največji odmik? Kolikšen je koeficient vijačne vzmeti?



$\tilde{x}$ = celoten odmik

x_r = ravnovesni odmik

x = odstopanje od ravnovesja

$$\boxed{\tilde{x} = x_r + x}$$

ravnovesje:

$$mg = kx_r$$

$$\boxed{x_r = \frac{m}{k} \cdot g}$$

$$\boxed{g = \frac{k}{m} x_r}$$

$$\boxed{x_r \equiv \text{konstanta}} \Rightarrow \ddot{x}_r = 0$$

$$m\ddot{\tilde{x}} = mg - k\tilde{x}$$

$$\ddot{\tilde{x}} = g - \frac{k}{m}\tilde{x}$$

$$\ddot{x} = g - \frac{k}{m}(x_r + x) = g - \underbrace{\frac{k}{m}x_r}_{g} - \frac{k}{m}x = -\frac{k}{m}x$$

$$\boxed{\ddot{x} = -\frac{k}{m} \cdot x}$$

$\dot{\sin} = \cos$

$\dot{\cos} = -\sin$

$\longrightarrow$

3. Na zelo gladki plošči se brez trenja s frekvenco $\nu_0 = 2\ s^{-1}$ vrti utež mase 0,2 kg, ki je z vzmetjo (k = 50 N/m) pripeta v os vrtenja. S kolikšno frekvenco zaniha kocka, če jo med vrtenjem izmaknemo iz ravnovesne lege v radialni smeri ?

14.9.

$$l_0 = 10\ cm$$
$$m = 0{,}2\ kg$$
$$k = 50\ N/m$$
$$\nu = 2\ s^{-1}$$

$$\boxed{k x_r = m\omega^2(l_0 + x_r)}$$ ko ne niha

$$\boxed{\tilde{x} = l_0 + x_r + x}$$

$$\boxed{m\ddot{\tilde{x}} = -k(x_r + x) + m\omega^2\tilde{x}}$$

$$m\ddot{x} = -kx_r - kx + m\omega^2(l_0 + x_r) + m\omega^2 x$$

0

$$\boxed{m\ddot{x} = -kx + m\omega^2 x}$$

$$x = x_0 \cos\omega_0 t$$

$$\boxed{\ddot{x} = -\omega_0^2 x}$$

$$\ddot{x} = -\left(\frac{k}{m} - \omega^2\right)x$$

$$\boxed{\omega_0^2 = \frac{k}{m} - \omega^2} \Rightarrow \boxed{\nu_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \omega^2}}$$

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{50}{0.2} - (2\pi\cdot 2)^2} = 1{,}52\ s^{-1}$$

$$\boxed{\nu_0 = 1{,}52\ s^{-1}}$$

Coriolisov pospešek smo zanemarili!

str. 28 nal. 18 april 1991

2. Tanek obroč mase 2500 kg in polmera 5 m lebdi v breztežnem prostoru. V smeri geometrijske osi obroča, ki je pravokotna na ravnino obroča in gre skozi center kroga obroča, niha majhna kroglica, katere masa je veliko manjša od mase obroča. Amplituda nihanja kroglice je veliko manjša od polmera obroča. Izračunaj nihajni čas kroglice ?

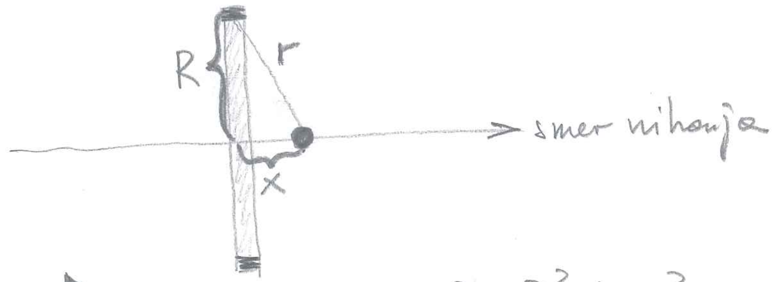
FI

$R = 5\,\mathrm{m}$

$M = 2500\,\mathrm{kg}$

$t_0 = ?$

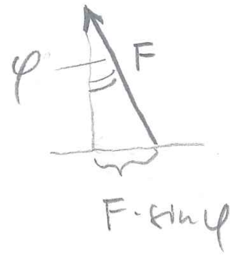
$\kappa = 6{,}67 \cdot 10^{-11}\ \mathrm{Nm^2/kg^2}$



$$dF = -\frac{\kappa\, m\, dM}{r^2}$$

$$\Downarrow$$

$$\boxed{F = -\frac{\kappa\, m\, M}{r^2}}$$



$$r^2 = R^2 + x^2$$

$$\boxed{\sin\varphi = \frac{x}{\sqrt{R^2+x^2}}}$$

$$\boxed{F \cdot \sin\varphi = m\ddot{x}}$$

$$-\frac{\kappa\, m\, M}{(x^2+R^2)} \cdot \frac{x}{\sqrt{R^2+x^2}} = m\ddot{x}$$

$$\Downarrow$$

$$\boxed{\ddot{x} = -\frac{\kappa M}{R^3}\, x} \Rightarrow \left(\frac{2\pi}{t_0}\right)^2 = \frac{\kappa M}{R^3}$$

$$\boxed{t_0 = 2\pi\sqrt{\frac{R^3}{\kappa \cdot M}}} = \underline{172029\ \mathrm{s}} \cong 47{,}8\ \mathrm{ur}$$

$$2\pi\sqrt{\frac{125}{6{,}67\cdot 10^{-11}\cdot 2{,}5\cdot 10^{3}}} = 2\pi \cdot 10^{4}\sqrt{\frac{125}{6{,}67\cdot 2{,}5}} =$$

1. Ravna palica z dolžino 1 m in maso 4 kg je vrtljiva okoli vodoravne osi skozi zgornje krajišče. Na spodnjem krajišču palice je pritrjena valjasta plošča z maso 1 kg in polmerom 0,2 m tako, da je geometrijska os plošče vzporedna z osjo vrtenja palice in od nje oddaljena 1,2 m. S kolikšnim nihajnim časom zaniha nihalo, če ga za malenkost izmaknemo iz ravnovesne lege ?

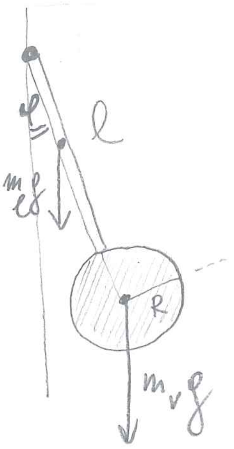
$l = 1\,m$

$m_l = 4\,kg$

$m_v = 1\,kg$

$R = 0.2\,m$

$t_0 = ?$



$J_l = m_l l^2/3$

$$-\left(m_l \cdot g \cdot \frac{l}{2}\cdot \varphi + (l+R)\, m_v\, g \cdot \varphi\right) = \left[\frac{m_l \cdot l^2}{3} + \frac{m_v R^2}{2} + m_v (l+R)^2\right]\ddot{\varphi}$$

$$\ddot{\varphi} = -\left[\frac{\left(\frac{m_l \cdot l}{2} + (l+R)\, m_v\right) g}{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}\right]\varphi$$

$$t_0 = 2\pi \cdot \sqrt{\frac{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}{\left(\frac{m_l \cdot l}{2} + (l+R)\, m_v\right)\cdot g}} = \underline{\underline{1.86\ s}}$$

$J =$

$$\int_0^l x^2 \rho S\, dx = \frac{l^3}{3}\rho S = \underline{\underline{\frac{m l^2}{3}}}$$

(še ena naloga)

OBRNI!

4. Maksimalna sila, ki vrača harmonično nihajoče telo v ravnovesno lego je $3\cdot10^{-3}$ N, celotna energija nihanja pa je $7\cdot10^{-5}$ J. Kolikšna je amplituda nihanja tega telesa ?

1992

$$F_{max} = 3\cdot10^{-3}\ N$$
$$E = 7\cdot10^{-5}\ J$$
$$x_0 = ?$$

$$x = x_0 \sin(\omega_0 t)$$
$$v = \dot{x} = x_0 \omega \cos(\omega_0 t)$$
$$a = \ddot{x} = \underbrace{-x_0 \omega^2}_{a_0} \sin(\omega_0 t) = -\omega^2 x$$

$$\frac{E}{F_{max}} = \frac{\frac{1}{2} m v_0^2}{m a_0} = \frac{\frac{1}{2} m x_0^2 \omega_0^2}{m x_0 \omega_0^2} = \frac{1}{2} x_0 \Longrightarrow$$

$$\boxed{x_0 = \frac{2E}{F_{max}} = \frac{14\cdot10^{-5}\ J}{3\cdot10^{-3}\ N} = 4{,}7\cdot10^{-2}\ m}$$

$$\varphi_1 = \varphi_0 \cos\omega_1 t + \varphi_0 \cos\omega_2 t = \qquad \boxed{\omega = \tfrac{1}{2}(\omega_1+\omega_2)}$$

$$= 2\varphi_0 \cos\left[\underbrace{\tfrac{1}{2}(\omega_2-\omega_1)}\, t\right]\cos\omega t$$

$$\boxed{\nu_u = \tfrac{1}{2}(\omega_2-\omega_1)} \quad \boxed{\omega_1 = \sqrt{\tfrac{g}{l_1}}} \quad \boxed{\omega_2 = \sqrt{\tfrac{g}{l_2}}}$$

Naloga 1988

Železni valj s polmerom 5 cm in višina 10 cm visi na žici debeli 1mm in dolgi 2m. Os valja in žice ležita na isti premici. Perioda nihanja valja je 1s. Izračunaj sučno konstanto in strižni modul žice. Gostota železa $\rho_{Fe} = 7,25\, g/cm^3$.

$2r = 5\,cm$

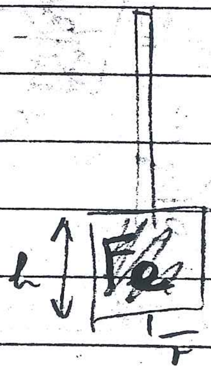
$h = 10\,cm$

$2R = 1\,mm$

$a = 2\,m$

$t_0 = 1\,s$

$\rho_{Fe} = 7.25\, g/cm^3$



$$m_{Fe} = \pi r^2 h \rho_{Fe}$$

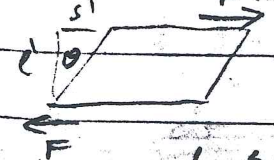
$$\boxed{J = \tfrac{1}{2} m_{Fe} r^2}$$

$$-D\cdot\varphi = J\cdot\ddot{\varphi}$$

$$\omega = \frac{2\pi}{t_0} \longrightarrow \boxed{\omega = \sqrt{\frac{D}{J}}} \Rightarrow D$$

Za palico valja (Kladnik str. 138): $D = \pi G R^4 / 2a \Rightarrow G$ (strižni modul)

Strnad str. 103



$$\boxed{\frac{F}{S} = G\cdot\frac{s'}{l'}} \Rightarrow \boxed{\tau = G\cdot\Theta}$$

STRIŽNA DEFORMACIJA

$$\tan\Theta = \frac{s'}{l'} = \Theta$$

$$\frac{F}{S} = \tau$$

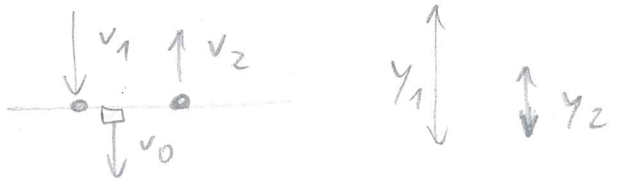
torzija palice

Kolokvij 90/91

M14

4. Na vzmetno tehtnico spustimo žogo mase m_1= 1 kg z višine y_1= 2 m. Žoga odskoči do višine y_2= 0,8 m. Koliko največ pokaže tehtnica (v kg)? Nihajni čas tehtnice je T = 1 s. (65 N)

$$\boxed{\frac{2\pi}{T} = \omega = \sqrt{\frac{k}{m}}}$$



$$\boxed{mgh_1 = \frac{mv_1^2}{2}} \Rightarrow v_1 = \sqrt{2gy_1} = 6.32\ \frac{m}{s}$$

$$v_2 = \sqrt{2gy_2} = 4\ m/s$$

$$\boxed{\begin{aligned} x &= x_0 \sin \omega t \\ v = \dot{x} &= x_0 \omega \cos \omega t \end{aligned}}$$

$$\Downarrow$$

$$\boxed{v_0 = \omega x_0}$$

$$\boxed{\frac{kx_0^2}{2} = \frac{mv_0^2}{2}}$$

$$\Downarrow$$

$$kx_0 = mv_0 \cdot \overbrace{\frac{v_0}{x_0}}^{\omega} = mv_0 \cdot \omega = mv_0 \cdot \frac{2\pi}{T} = 64.84\ N$$

$$\Downarrow$$

pokaže težo 6,5 kg

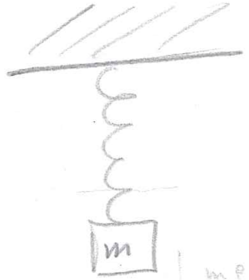
$$m_1 v_1 + m_1 v_2 = mv_0 = 10.32\ kgms^{-1}$$

kolokvij 90/91 — N5

7. Utež visi na vijačni vzmeti (konstanta vzmeti je 80N/m) in niha z nihajnim časom 2s. Kakšno vijačno vzmet moramo zaporedno zvezati z obstoječo vzmetjo, da se bo nihajni čas povečal za 2s ?

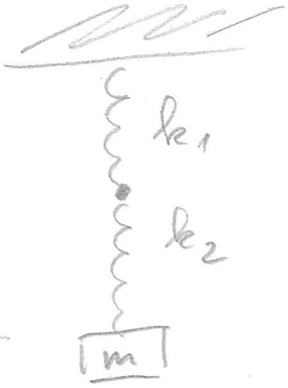
A-M

Hl,198/434



$$k_1 = 80\,N/m$$
$$t_{01} = 2\,s$$

$mg \qquad kx_0 = mg$



$$t_{02} = 4\,s$$

$$mg - k_1(x + x_0) = ma$$
$$mg - k_1 x - k x_0 = ma$$
$$-k_1 x = ma$$
$$-\frac{k_1}{m} x = a \Rightarrow \omega = \sqrt{\frac{k_1}{m}}$$
$$\frac{2\pi}{t_{01}} = \sqrt{\frac{k_1}{m}} \Rightarrow \boxed{t_{01} = 2\pi\sqrt{\frac{m}{k_1}}}$$

$$x = x_0 \cos\omega t$$
$$\dot{x} = -x\omega \sin\omega t$$
$$\ddot{x} = -\omega^2 x$$

$$k_1 x_1 = mg$$
$$k_2 x_2 = mg$$

$$x = x_1 + x_2 = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = mg \cdot \frac{1}{k}$$
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{k_2 + k_1}{k_1 k_2}$$
$$\boxed{k = \frac{k_1 k_2}{k_2 + k_1}}$$
$$\boxed{t_{02} = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}}$$

$$\frac{t_{02}^2}{t_{01}^2} = \frac{4\pi^2 m(k_1 + k_2)k_1}{(k_1 k_2)4\pi^2 m} = \frac{k_1}{k_2} + 1$$
$$\Downarrow$$
$$\boxed{\frac{k_1}{k_2} = \frac{t_{02}^2}{t_{01}^2} - 1}$$
$$\boxed{k_2 = k_1 \Big/ \left[\frac{t_{02}^2}{t_{01}^2} - 1\right]}$$

$$\frac{t_{02}^2}{t_{01}^2} = \frac{16\,s^2}{4\,s^2} = \underline{\underline{4}}$$

80 : 3 = 26,66
20
20

$$\underline{\underline{k_2 = k_1/3 = 26{,}66\ N/m}}$$

7. Valj polmera 5 cm se kotali sem ter tja znotraj valjaste ploskve, ki ima polmer 30 cm. Izračunaj nihajni čas nihanja valja za majhne odmike.

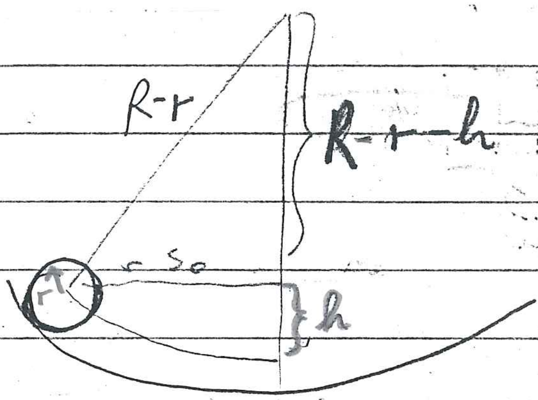
1983

Klodnik str. 185

$g = 9{,}8\ ms^{-2}$

$r = 0.05\ m$

$R = 0.3\ m$



$h \ll l$

$$\boxed{J = \frac{1}{2} m r^2}$$

$R\omega_0$

$$\boxed{mgh = \frac{1}{2} m v_0^2 + \frac{1}{2} J \omega_0^2}, \quad \boxed{\omega_0 = \frac{v_0}{r}}$$

$$mgh = \frac{1}{2} m v_0^2 + \frac{1}{2} \cdot \frac{1}{2}\frac{m r^2 v_0^2}{r^2}$$

$$mgh = \frac{1}{2} m v_0^2 + \frac{1}{4} m v_0^2 = \frac{3}{4} m v_0^2$$

$$\boxed{v_0^2 = \frac{4}{3} g h}$$

$$\boxed{\begin{aligned} \varphi &= \varphi_0 \sin(2\pi\nu t) \\ \omega = \dot{\varphi} &= \varphi_0 (2\pi\nu) \cos(2\pi\nu t) \\ \alpha = \ddot{\varphi} &= \varphi_0 (2\pi\nu)^2 \sin(2\pi\nu t) \end{aligned}}$$

$$\omega = \varphi_0 (2\pi\nu) \cos 2\pi\nu t$$

$$v = (R-r)\omega = \underbrace{\varphi_0 (R-r) 2\pi\nu}_{v_0} \cos 2\pi\nu t$$

$$v_0 = \varphi_0 (R-r) 2\pi\nu \Rightarrow 2\pi\nu = \frac{v_0}{\varphi_0 (R-r)} \Rightarrow \boxed{(2\pi\nu)^2 = \frac{v_0^2}{\varphi_0^2 (R-r)^2} = \cong \frac{v_0^2}{s_0^2}}$$

$$\varphi_0 (R-r) \cong s_0 \longrightarrow$$

$$s_0^2 = (R-r)^2 - ((R-r)-h)^2 = 2(R-r)h - h^2 \cong 2(R-r)h$$

$$\boxed{(2\pi\nu)^2 = \frac{v_0^2}{s_0^2}} = \frac{4gh}{3 \cdot 2(R-h) \cdot h} = \frac{2g}{3(R-h)} \Rightarrow \boxed{(2\pi\nu)^2 = \frac{2g}{3(R-r)}}$$

$$t_0 = 2\pi \sqrt{\frac{3}{2} \frac{(R-r)}{g}} = 1{,}23\ s$$

$$\varphi_1 = \varphi_0 \cos\omega_1 t + \varphi_0 \cos\omega_2 t = \qquad \boxed{\omega = \frac{1}{2}(\omega_1+\omega_2)}$$

$$= 2\varphi_0 \cos\Big[\underbrace{\tfrac{1}{2}(\omega_2-\omega_1)}_{}\, t\Big] \cos\omega t$$

$$\boxed{\nu_u = \frac{1}{2}(\omega_2-\omega_1)} \quad \boxed{\omega_1 = \sqrt{\frac{g}{l_1}}} \quad \boxed{\omega_2 = \sqrt{\frac{g}{l_2}}}$$

Naloga 1988

Železni valj s polmerom 5 cm in višino 10 cm visi na žici debeli 1mm in dolgi 2m. Os valja in žice ležita na isti premici. Perioda nihanja valja je 1s. Izračunaj sučno konstanto in strižni modul žice. Gostota železa $\rho_{Fe} = 7,25\, g/cm^3$.

$2r = 5\,cm$

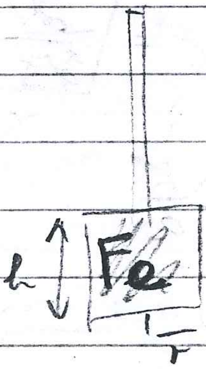
$h = 10\,cm$

$2R = 1\,mm$

$a = 2\,m$

$t_0 = 1\,s$

$\rho_{Fe} = 7.25\, g/cm^3$



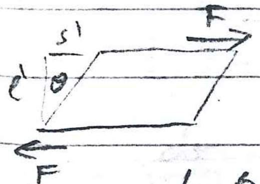
$$m_{Fe} = \pi r^2 h \cdot \rho_{Fe}$$

$$\boxed{J = \frac{1}{2} m^2_{Fe} r^2}$$

$$-D\cdot\varphi = J\cdot\ddot{\varphi}$$

$$\omega = \frac{2\pi}{t_0} \longrightarrow \boxed{\omega = \sqrt{\frac{D}{J}}} \Rightarrow D$$

za palico valja (Kladnik str. 138; Strnad str. 103): $D = \pi G R^4 / 2a \Rightarrow G$ (strižni modul)



$$\boxed{\frac{F}{S} = G\cdot\frac{s'}{l'}} \Rightarrow \boxed{\tau = G\cdot\Theta}$$

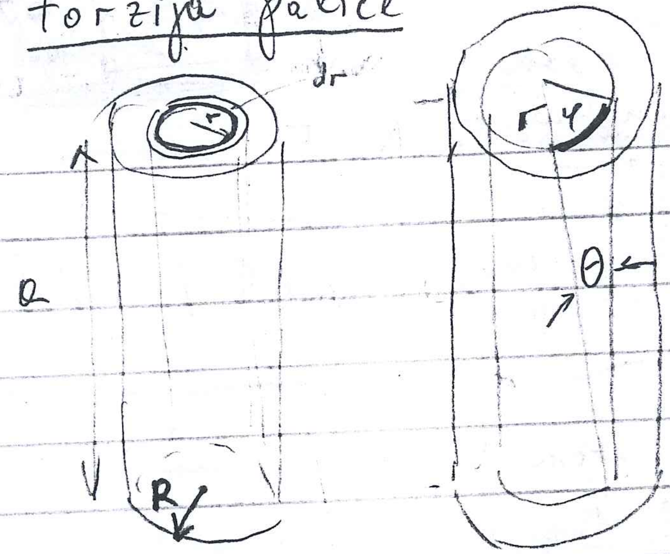
STRIŽNA DEFORMACIJA

$$\operatorname{tg}\Theta = \frac{s'}{l'} = \Theta$$

$$\frac{F}{S} = \tau$$

torzija palice:

68 torzija palice

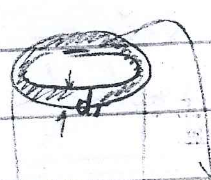


$$r\varphi = a\theta$$

$$\Downarrow$$

$$\boxed{\theta = \varphi \cdot \frac{r}{a}}$$

na kolobarjast pas od pade strižna sila dF



$$dS = 2\pi r\, dr$$

$$\tau = G \cdot \theta \quad , \quad \tau = \frac{dF}{dS} \text{ (za en kolobar)}$$

$$\frac{dF}{dS} = \frac{dF}{2\pi r\, dr} = G \cdot \theta = G \cdot \varphi \frac{r}{a}$$

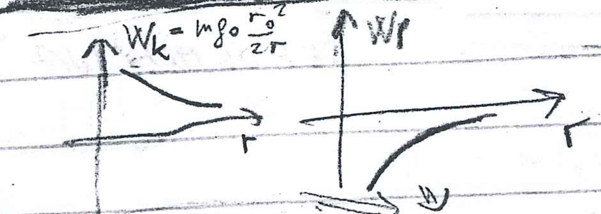
$$dF = (2\pi G\varphi / a)\, r^2\, dr$$

[kot φ je parameter $\equiv$ trenutni zasuk]

$$\boxed{dM = r\, dF}$$

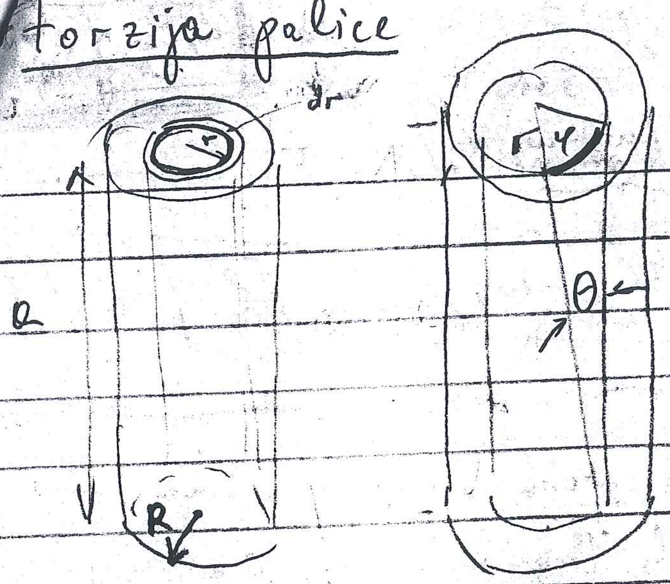
$$M = \int dM = \int_0^R r\, dF = \pi G R^4 \varphi / 2a = D \cdot \varphi$$

$$\boxed{D = \pi G R^4 / 2a}$$



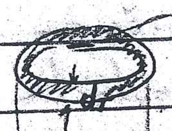
$$\frac{1}{1+x} \cong 1 - x + \frac{1}{2}x^2$$

torzija palice



$$\boxed{r\varphi = a\Theta \;\Downarrow\; \boxed{\Theta = \varphi \cdot \frac{r}{a}}}$$

na kolobarjast pas odpade strižna sila dF



$dS = 2\pi r\, dr$

$$\tau = G \cdot \Theta \quad , \quad \tau = \frac{dF}{dS} \text{ (za en kolobar)}$$

$$\frac{dF}{dS} = \frac{dF}{2\pi r\, dr} = G \cdot \Theta = G \cdot \varphi \frac{r}{a}$$

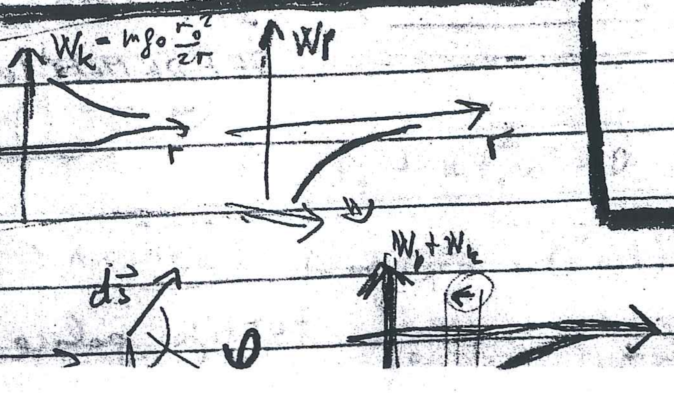
$$dF = (2\pi G \varphi / a)\, r^2\, dr$$

[kot φ je parameter $\equiv$ trenutni zasuk]

$$\boxed{dM = r\, dF}$$

$$M = \int dM = \int_0^R r\, dF = \pi G R^4 \varphi / 2a = D \cdot \varphi$$

$$\boxed{D = \pi G R^4 / 2a}$$



$$\frac{1}{1+x} \cong 1 - x + \frac{1}{2}x^2$$

4. Ravna palica dolžine 1 m in mase 1 kg je vrtljiva okoli vodoravne osi skozi zgornje krajišče. Na spodnjem krajišču palice je pritrjena valjasta plošča z maso 1 kg in polmerom 0.4 m tako, da je geometrijska os plošče vzporedna z osjo vrtenja palice. Izračunajte nihajni čas nihala pri majhnih odmikih ! Razdalja od osi vrtenja do geometrijske osi plošče je 1,4 m.

1995, M-7

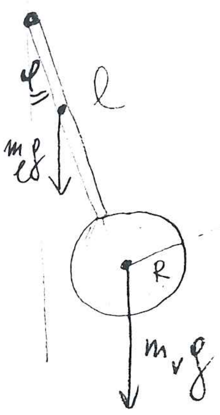
$l = 1\,m$

$m_l = 1\,kg$

$m_v = 1\,kg$

$R = 0.4\,m$

$t_0 = ?$



$J_l = m_l l^2/3$

$$-\left(m_l \cdot g \cdot \frac{l}{2} \cdot \varphi + (l+R) m_v g \cdot \varphi\right) = \left[\frac{m_l \cdot l^2}{3} + \frac{m_v R^2}{2} + m_v (l+R)^2\right]\ddot{\varphi}$$

$$\ddot{\varphi} = -\left[\frac{\left(\frac{m_l \cdot l}{2} + (l+R) m_v\right) g}{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}\right]\varphi$$

$$t_0 = 2\pi \cdot \sqrt{\frac{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}{\left(\frac{m_l \cdot l}{2} + (l+R) m_v\right) \cdot g}} = 2{,}22\,s$$

2.22

$$\int_0^l x^2 \rho S\, dx = \frac{l^3}{3}\rho S = \frac{m l^2}{3}$$

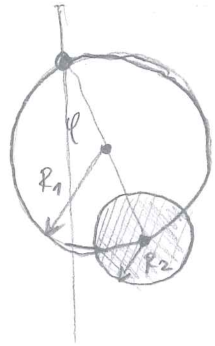
M-7

6. Tanek obroč mase 1 kg (R_1 = 0.3 m) je vrtljiv okoli osi, ki je vzporedna z geometrijsko osjo obroča in je od nje oddaljena 0,3 m v vertikalni smeri. Na spodnji del obroča je pritrjena krogla mase 1 kg in polmera R_2 = 0,1 m tako, da je središče krogle oddaljeno v vertikalni smeri 0,6 m od osi vrtenja. Kolikšen je nihajni čas pri majhnih odmikih ?

$m = 1\,kg$

$R_1 = 0.3\,m$

$R_2 = 0.1\,m$



$M = J\alpha$

$\varphi = \varphi_0 \cos\left(\frac{2\pi}{t_0} t\right)$

$\boxed{\alpha = \ddot{\varphi} = -\left(\frac{2\pi}{t_0}\right)^2 \varphi}$

$\sin\varphi \approx \varphi$

$$-\left(\cancel{m}gR_1\varphi + \cancel{m}g2R_1\varphi\right) = \left(\cancel{m}R_1^2 + \frac{2}{5}\cancel{m}R_2^2 + \cancel{m}(2R_1)^2\right)\ddot{\varphi}$$

$$\ddot{\varphi} = -\left[\frac{3gR_1}{R_1^2 + 4.4R_2^2}\right]\varphi$$

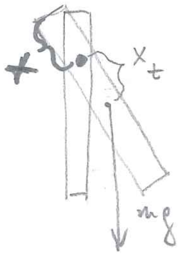
$$t_0 = 2\pi\cdot\sqrt{\frac{R_1^2 + 4.4R_2^2}{3gR_1}} = \underline{\underline{0.77\,s}} \quad (?)$$

$l = 1\,m$

$a = g/2$

$g = 10\,m s^{-2}$

4. Palica z dolžino 1 m niha v mirujočem dvigalu okoli vodoravno ležeče osi, ki gre skozi krajišče palice. Kam je treba prestaviti vodoravno os nihanja palice, da bo nihajni čas palice v dvigalu, ki se dviga s pospeškom g/2, enak nihajnemu času palice v mirujočen nihalu ?



$$-x_t mg\varphi = J\ddot{\varphi}$$

$$J_t = \int_{-l/2}^{l/2} r^2 \rho S\,dr = \rho S 2 \frac{l^3}{8\cdot 3} = \frac{ml^2}{12}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{x_t mg}{J} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{J}{x_t mg}}}$$

$$J_{os} = \frac{ml^2}{12} + m\frac{l^2}{4} = \frac{1}{3}ml^2$$

$$\boxed{\frac{\frac{1}{3}ml^2}{\frac{l}{2}mg} = \frac{\frac{ml^2}{12} + m\left(\frac{l}{2}-x\right)^2}{m\left(\frac{l}{2}-x\right)(g+a)}}$$

$$\Downarrow$$

$$\left(\frac{l}{2}-x\right)^2 - l\left(\frac{l}{2}-x\right) + \frac{l^2}{12} = 0$$

$$\left(\frac{l}{2}-x\right) = \frac{l}{2} - \frac{1}{2}\sqrt{l^2 - 4\frac{l^2}{12}}$$

$$x = \frac{1}{2}\sqrt{\frac{8}{12}l^2} = \sqrt{\frac{1}{6}}\,l$$

$$\boxed{x = \sqrt{\frac{1}{6}}\cdot l} \cong \underline{0.41\,m}$$

4. Tanek obroč polmera 40 cm in mase 1 kg je podprt na obodu v eni točki, okrog katere lahko niha. Za koliko procentov se spremeni njegov nihajni čas pri majhnih amplitudah, če na obroč diametralno nasproti podporne točke (t.j. osi nihanja) pritrdimo majhno svinčeno kroglico z maso 0.1 kg ?

1995

$m = 0.1\,kg$

$R = 0.4\,m$

$\underline{M = 1\,kg}$

$J = MR^2$ (os v središču)

Steiner: $\underline{\underline{J}} = MR^2 + MR^2 = \underline{\underline{2MR^2}}$

samo obroč:

$$J\alpha = MgR\varphi \qquad \varphi = \varphi_0 \cos(2\pi\nu t)$$

$$\alpha = \frac{MgR}{J}\varphi \qquad \dot\varphi = -\varphi_0 (2\pi\nu)\sin(2\pi\nu t)$$

$$\alpha = \frac{MgR}{2MR^2}\varphi \qquad \ddot\varphi = -\varphi_0 (2\pi\nu)\cos(2\pi\nu t)$$

$$\ddot\varphi = -(2\pi\nu)^2\varphi$$

$$\alpha = \frac{g}{2R}\varphi, \qquad (2\pi\nu) = \sqrt{\frac{g}{2R}} \Rightarrow \boxed{t_0 = 2\pi\sqrt{\frac{2R}{g}}}$$

$$J^* = J + m(2R)^2$$

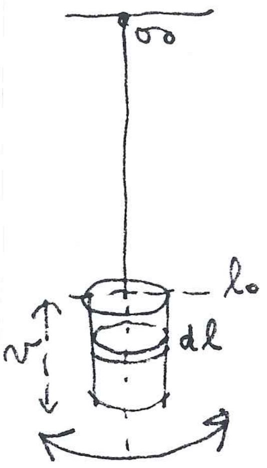
$$J^*\alpha = -(MgR\varphi + mg2R\varphi) = -(MgR + mg2R)\varphi$$

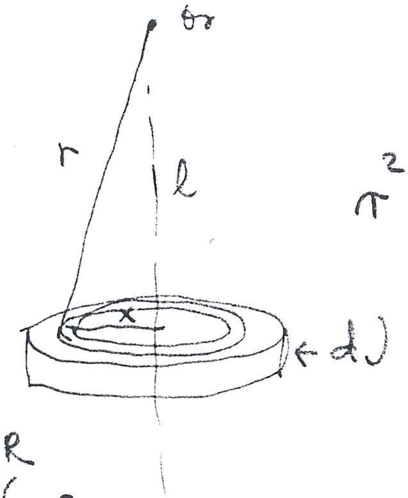
$$\alpha = -\frac{MgR + 2mgR}{2MR^2 + 4mR^2}\varphi, \qquad (2\pi\nu)^2 = \frac{Mg + 2mg}{2MR + 4mR} = \left(\frac{2\pi}{t_0'}\right)^2$$

$$\boxed{t_0' = 2\pi\sqrt{\frac{2MR + 4mR}{Mg + 2mg}} = 2\pi\sqrt{\frac{2R(M+2m)}{g(M+2m)}} = 2\pi\sqrt{\frac{2R}{g}}}$$

Odg.: Nihajni čas se <u>ne</u> spremeni $(t_0 = t_0')$

6. Kakšen je nihajni čas valja z radijem osnovne ploskve 10 cm in višino 20 cm, ki je pritrjen na 1 m dolgi žici. Težo žice lahko zanemarimo v primerjavi s težo valja. Nihalo niha v ravnini.





$$r^2 = l^2 + x^2$$

dJ : en obroč

$$dJ = \rho\, dl \int_0^R r^2 \cdot 2\pi x\, dx = \rho\, dl \int_0^R (l^2 + x^2)\, 2\pi x\, dx =$$

$$= \rho \cdot 2\pi \left(l^2 \frac{R^2}{2} + \frac{R^4}{4} \right) dl$$

$$J = \int_{l_0}^{l_0+v} \rho \cdot 2\pi \left(\frac{R^2}{2} l^2 + \frac{R^4}{4} \right) dl = \rho \cdot 2\pi \left[\frac{R^2}{2} \frac{l^3}{3} + \frac{R^4}{4} \cdot l \right]_{l_0}^{l_0+v} =$$

$$= \frac{\rho \cdot 2\pi R^2}{2} \left[\frac{l^3}{3} + \frac{R^2}{2} l \right]_{l_0}^{l_0+v} = \rho \pi R^2 \left[\frac{1}{3} \left(\cancel{l_0^3} + 3 l_0^2 v + 3 l_0 v^2 + v^3 - \cancel{l_0^3} \right) + \frac{R^2}{2} \left(\cancel{l_0} + v - \cancel{l_0} \right) \right] =$$

$$= \rho \pi R^2 \left[l_0^2 v + l_0 v^2 + \frac{v^3}{3} + \frac{R^2}{2} v \right] = \rho \pi R^2 l_0^3 \left[\frac{v}{l_0} + \frac{v^2}{l_0^2} + \frac{v^3}{3 l_0^3} + \frac{R^2}{2 \ldots} \right.$$

$$\omega = \sqrt{\frac{m g r^*}{J}} = \sqrt{\frac{\cancel{\rho} \cancel{\pi} \cancel{R^2} \cancel{v} \left(l_0 + \frac{v}{2} \right) g}{\cancel{\rho} \cancel{\pi} \cancel{R^2} l_0^2 \cancel{v} \left[1 + \frac{v}{l_0} + \frac{v^2}{3 l_0^2} + \frac{R^2}{2 l_0^2} \right]}}$$

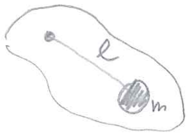
$$\omega = \sqrt{\frac{g}{l_0}} \sqrt{\frac{\left(1 + \frac{v}{2 l_0}\right)}{\left(1 + \frac{v}{l_0} + \frac{v^2}{3 l_0^2} + \frac{R^2}{2 l_0^2}\right)}} \qquad t_0 = \frac{\omega}{2\pi} = 0.48\ 1$$

5. Neko telo niha okoli izbrane osi z nihajnim časom 0,4 s. Če se na to telo 18 cm od osi nihanja pričvrsti 50 g svinčeno kroglico je nihajni čas telesa 0,6 s. Kolikšen je vztrajnostni moment telesa brez dodane kroglice ? 1994

$T_1 = 0.4\ s$

$T_2 = 0.6\ s$

$l = 18\ cm,\ m = 50\ g$



M_0, d

$\varphi = \varphi_0 \cos(2\pi\nu t)$

$\ddot{\varphi} = -\varphi_0 (2\pi\nu)^2 \cos(2\pi\nu t)$

$\ddot{\varphi} = -(2\pi\nu)^2 \varphi$

$M = J \cdot \ddot{\varphi}$

$M_0 g d \varphi = J \cdot \ddot{\varphi}$

$\ddot{\varphi} = \frac{M_0 g d}{J} \varphi \Rightarrow \left(\frac{2\pi}{T_1}\right)^2 = \frac{M_0 g d}{J} \Rightarrow \boxed{T_1^2 = 4\pi^2 \frac{J}{M_0 g d}}$ (1)

$\boxed{T_2^2 = 4\pi^2 \frac{J + ml^2}{M_0 g d + mgl}}$ (2)

(1) $\Rightarrow M_0 g d = \frac{4\pi^2 J}{T_1^2}$ ← vstaviš v (2) ~~torej~~

(2): $T_2^2 = 4\pi^2 \frac{J + ml^2}{\frac{4\pi^2 J}{T_1^2} + mgl} \Rightarrow \boxed{4\pi^2 \frac{(J + ml^2) T_1^2}{(4\pi^2 J + mgl T_1^2)} = T_2^2}$

$4\pi^2 (J + ml^2) T_1^2 = T_2^2 (4\pi^2 J + mgl T_1^2)$

$4\pi^2 \cdot J \cdot T_1^2 + 4\pi^2 \cdot ml^2 \cdot T_1^2 = T_2^2 \cdot 4\pi^2 J + T_1^2 \cdot T_2^2 \cdot mgl$

$T_1^2 [4\pi^2 ml^2 - T_2^2 mgl] = 4\pi^2 J (T_2^2 - T_1^2)$

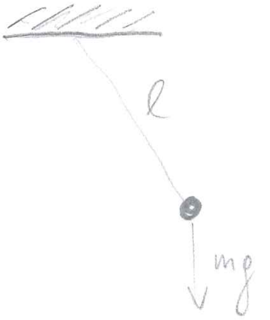
$J = \frac{T_1^2 ml [4\pi^2 l - T_2^2 g]}{(T_2^2 - T_1^2) 4\pi^2} \Rightarrow \boxed{J = \frac{T_1^2}{T_2^2 - T_1^2} \frac{ml}{4\pi^2} (4\pi^2 \cdot l - T_2^2 \cdot g)} \Rightarrow$

$J = 63{,}94 \cdot 10^{-5}\ kg\,m^2$

(od $19^h - 7^h$ kot podnevi od $7^h - 19^h$)

3. Stenska ura ima nihalo, sestavljeno iz kovinske palice na koncu katere je pritrjena polna krogla. Ocenite, kolikrat več zaniha nihalo ponoči, če je razlika med povprečnima temperaturama dneva in noči 15°C. Nihajni čas nihala podnevi je 2 s, koeficient dolžinskega temperaturnega raztezka kovine pa je $10 \cdot 10^{-6}\ K^{-1}$.

$t = 12^h$



$$\underline{\Delta T = 15^\circ C, \quad t_0 = 2s, \quad \alpha = 10^{-5}\ K^{-1}, \quad t = 12^h}$$

$$-mgl\varphi = ml^2\ddot{\varphi} \Rightarrow \boxed{t_0 = 2\pi\sqrt{\frac{l}{g}}}$$

$$\Delta l = \alpha l \Delta T$$

$$l_n = l - \Delta l = l(1 - \alpha\Delta T)$$

noč: $$t_0' = 2\pi\sqrt{\frac{l(1-\alpha\Delta T)}{g}} = t_0\sqrt{(1-\alpha\Delta T)}$$

$$\underline{N_n - N_d} = \frac{t}{t_0'} - \frac{t}{t_0} = t\left(\frac{1}{t_0'} - \frac{1}{t_0}\right) = t\left(\frac{1}{t_0\sqrt{1-\alpha\Delta T}} - \frac{1}{t_0}\right)$$

$$= \frac{t}{t_0}\left(\frac{1}{\sqrt{1-\alpha\Delta T}} - 1\right) \approx \frac{t}{t_0}\left(1 + \frac{\alpha}{2}\Delta T - 1\right) = \underline{\frac{t}{t_0}\frac{\alpha}{2}\Delta T}$$

$$\boxed{\Delta N = \frac{t}{t_0}\frac{\alpha}{2}\Delta T = \underline{\underline{1.62}}}$$

4. Celotna energija nihanja harmonično nihajočega telesa je $4 \cdot 10^{-5}$ J, Maksimalna sila, ki vrača telo v ravnovesno lego je $2 \cdot 10^{-3}$ N, nihajni čas 4 s, odmik ob čast t=0 pa je enak polovici maksimalnega odmika x_o. Napiši enačbo nihanja tega telesa $x = x(x_o, \upsilon, t, \varphi)$!

1442

FOR

$g = 9{,}82 \ m s^{-2}$

$E = 4 \cdot 10^{-5}$ J

$F_{max} = 2 \cdot 10^{-3}$ N

$t_o = 4$ s

$t = 0 : x = \frac{x_o}{2}$

$x = x_o \sin(\omega t + \varphi)$

$v = \dot{x} = x_o \omega \cos(\omega t + \varphi)$

$a = \ddot{x} = -x_o \omega^2 \sin(\omega t + \varphi)$

$\longrightarrow \frac{x_o}{2} = x_o \sin\varphi \Rightarrow \boxed{\varphi = \arcsin \frac{1}{2} = 30° = \frac{\pi}{6}}$

$$\boxed{\omega = \frac{2\pi}{t_o} = \frac{\pi}{2} s^{-1} = 1{,}57 \ s^{-1}}$$

$$\left.\begin{array}{l} E = \frac{1}{2} m v_o^2 = \frac{1}{2} m x_o^2 \omega^2 \\ F_{max} = m a_o = m x_o \omega^2 \end{array}\right\} \Rightarrow \underbrace{\frac{E}{F_{max}} = \frac{\frac{1}{2} m x_o^2 \omega^2}{m x_o \omega^2} = \frac{1}{2} x_o}$$

$$x_o = \frac{2E}{F_{max}} = 4 \cdot 10^{-2} \ m$$

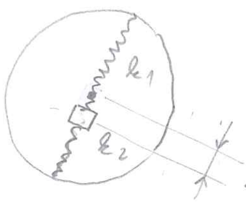
$$\boxed{x = 4 \cdot 10^{-2} \ m \cdot \sin\left[\frac{\pi}{2} s^{-1} \cdot t + \frac{\pi}{6}\right]}$$

to vzmet, lahko se F_u

1. Med dve vzmeti enakih dolžin ($l = 0,5$ m) s konstantama $k_1 = 90$ N/m in $k_2 = 30$ N/m je pričvrščena kvadratna kocka mase $m=0,3$ kg. Vzmeti sta na obeh prostih koncih privezani na rob plošče s premerom 1 m, ki se lahko vrti okoli svoje geometrijske osi. Koliko obratov v sekundi mora napraviti plošča, da bi kocka oscilirala z dvojno frekvenco v primeri s frekvenco vrtenja plošče ? Kocka se giblje po plošči brez trenja.

1PP2

$l = 0,5$ m
$k_1 = 90$ N/m
$k_2 = 30$ N/m
$m = 0,3$ kg
$R = 1$ m



ω_p = kotna hitrost plošče
$\omega_p = 2\pi \nu_p$

$$\boxed{m\ddot{x} = -(k_1+k_2)x + m\omega_p^2 \cdot x}$$

$$\ddot{x} = -\left(\frac{k_1+k_2-m\omega_p^2}{m}\right)\cdot x$$

$$\boxed{\omega_m^2 = \frac{k_1+k_2-m\omega_p^2}{m}}$$

če $\boxed{\omega_m = 2\omega_p}$ $(\nu_m = 2\cdot\nu_p)$

$\Downarrow$

$$m\cdot 4\omega_p^2 = k_1+k_2-m\omega_p^2$$

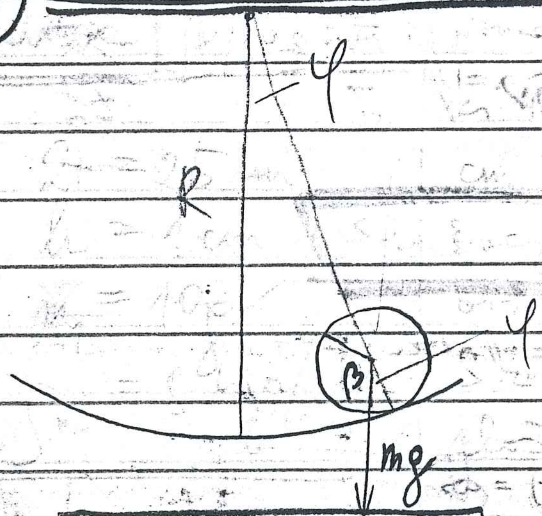
$$\omega_p^2 = \frac{k_1+k_2}{5m} \Rightarrow \nu_p^2 = \frac{k_1+k_2}{20\pi^2 m} \Rightarrow$$

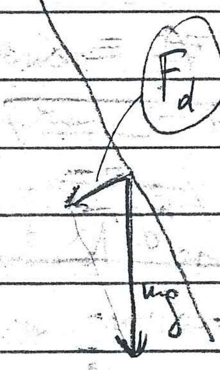
$$\boxed{\nu_p = \frac{1}{\pi}\sqrt{\frac{k_1+k_2}{20\cdot m}} = \underline{1,42 \text{ s}^{-1}}}$$

✓

$t_0 = 0.7$ s

(A) PRVI NAČIN IZPELJAVE (je že v Kladniku (veji) str. 195, zato ga tako NE DELAJ!)





$$\boxed{\sin\varphi \approx \varphi}$$

$$R\varphi = r\beta + r\varphi \Rightarrow \varphi(R-r) = \beta r$$

$$\boxed{R\varphi = r(\beta+\varphi)} \Longrightarrow \boxed{\beta = \varphi\frac{R-r}{r}}$$

os gre skozi dotikališče valja:

$$r\cdot F_d = r\,mg\sin\varphi = J\ddot{\beta}$$

– dis prov list V

$$\boxed{J = \frac{3}{2} m r^2}$$ skozi dotikališče

$$-rmg\varphi = J\ddot{\beta}$$

$$-\cancel{r}mg\varphi = J\ddot{\varphi}\frac{R-r}{r} = \frac{3}{2}\frac{mr^2(R-r)}{r(R-r)}\ddot{\varphi}$$

$$-\frac{2}{3}\frac{g}{(R-r)}\varphi = \ddot{\varphi} \Rightarrow \left(\frac{2\pi}{t_0}\right) = \sqrt{\frac{2}{3}\frac{g}{(R-r)}} \Rightarrow \boxed{t_0 = 2\pi\sqrt{\frac{3(R-r)}{2g}}}$$

3. Ravna palica dolžine 1 m in mase 1 kg je vrtljiva okoli vodoravne osi skozi zgornje krajišče. Na spodnjem krajišču palice je pritrjena valjasta plošča z maso 1 kg in polmerom 0.3 m tako, da je geometrijska os plošče vzporedna z osjo vrtenja palice. Izračunajte nihajni čas nihala pri majhnih odmikih ! Razdalja od osi vrtenja do geometrijske osi plošče je 1,3 m.

1995

A-L

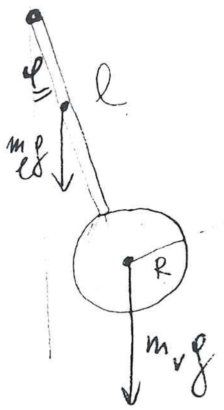
$l = 1\,m$

$m_l = 1\,kg$

$m_v = 1\,kg$

$R = 0.3\,m$

$t_0 = ?$



$J_l = m_l^2 l^2/3$

$$-\left(m_l \cdot g \cdot \frac{l}{2} \cdot \varphi + (l+R) m_v g \cdot \varphi\right) = \left[\frac{m_l \cdot l^2}{3} + \frac{m_v R^2}{2} + m_v (l+R)^2\right] \ddot{\varphi}$$

$$\ddot{\varphi} = -\left[\frac{\left(\frac{m_l \cdot l}{2} + (l+R) m_v\right) g}{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}\right] \varphi$$

$$t_0 = 2\pi \cdot \sqrt{\frac{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}{\left(\frac{m_l \cdot l}{2} + (l+R) m_v\right) \cdot g}} = \underline{\underline{2.13\,s}}$$

$g = 10$

2,11333

$$\int_0^l x^2 \rho S\, dx = \frac{l^3}{3} \rho S = \underline{\underline{\frac{m l^2}{3}}}$$

2.13

2,13

1992

3. V dvigalu, ki se dviga s pospeškom 2,1 m/s^2, zaniha nihalo 66-krat v minuti. Kolikšen je nihajni čas nihala v mirujočem dvigalu ?

M - Ž

$a = 2{,}1\ m/s^2$, $g = 10\ ms^{-2}$

$\frac{1}{t_0'} = 66/min$ (v dvigalu)

$t_0 = ?$

$$t_0 = 2\pi\left(\frac{J}{mgr}\right)^{1/2}$$

$$t_0' = 2\pi\left(J/[m(g+a)r]\right)^{1/2}$$

$$\frac{t_0}{t_0'} = \left(\frac{g+a}{g}\right)^{1/2} \Rightarrow t_0 = t_0'\left(1+\frac{a}{g}\right)^{1/2} = \underline{\underline{1\ s}}\ \checkmark$$

4

1. Ravna palica z dolžino 1 m in maso 4 kg je vrtljiva okoli vodoravne osi skozi zgornje krajišče. Na spodnjem krajišču palice je pritrjena valjasta plošča z maso 1 kg in polmerom 0,2 m tako, da je geometrijska os plošče vzporedna z osjo vrtenja palice in od nje oddaljena 1,2 m. S kolikšnim nihajnim časom zaniha nihalo, če ga za malenkost izmaknemo iz ravnovesne lege ?

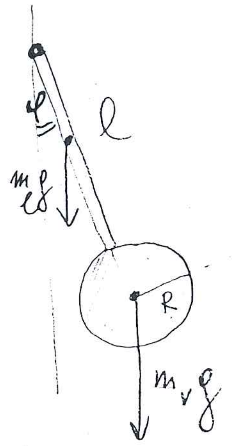
$l = 1\,m$

$m_l = 4\,kg$

$m_v = 1\,kg$

$R = 0.2\,m$

$t_0 = ?$



$J_l = m_l^2 l^2/3$

$$-\left(m_l \cdot g \cdot \frac{l}{2} \cdot \varphi + (l+R) m_v g \cdot \varphi\right) = \left[\frac{m_l \cdot l^2}{3} + \frac{m_v R^2}{2} + m_v (l+R)^2\right] \ddot{\varphi}$$

$$\ddot{\varphi} = -\left[\frac{\left(\frac{m_l \cdot l}{2} + (l+R) m_v\right) g}{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}\right] \varphi$$

$$t_0 = 2\pi \cdot \sqrt{\frac{\frac{m_l \cdot l^2}{3} + \frac{m_v \cdot R^2}{2} + m_v (l+R)^2}{\left(\frac{m_l \cdot l}{2} + (l+R) m_v\right) \cdot g}} = 1.86\ s$$

$J =$

$$\int_0^l x^2 \rho S\, dx = \frac{l^3}{3} \rho S = \frac{m l^2}{3}$$

(še ena naloga)

OBRNI!