3. Po tuljavi s 30 ovoji in presekom 15 cm^2 teče tok. Pri majhnih odmikih niha tuljava v zemeljskem magnetnem polju s frekvenco 1,5 s^{-1}. Kolikšen tok teče skozi tuljavo, če je njen vztrajnostni moment 10 gcm^2? Vodoravna komponenta zemeljskega magnetnega polja ima gostoto $2,1 \cdot 10^{-4}$ T. Vrtilna os tuljave je pravokotna na njeno geometrijsko os in na smer mag. polja. 1994

$N = 30$

$S = 15\ cm^2$

$\nu = 1,5\ s^{-1}$

$B = 2,1 \cdot 10^{-4}\ T$

$$\boxed{p_m = NIS}$$

$$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{NISB}{J}}}$$

$$p_m B \varphi = -J\alpha$$

$$\alpha = -\frac{p_m B}{J}\alpha$$

$$\omega_0^2 = (2\pi\nu)^2 = \frac{p_m B}{J} = \frac{NISB}{J}$$

$$\boxed{I = \frac{\omega_0^2 J}{NSB} = \frac{(2\pi\nu)^2 J}{NSB}}$$

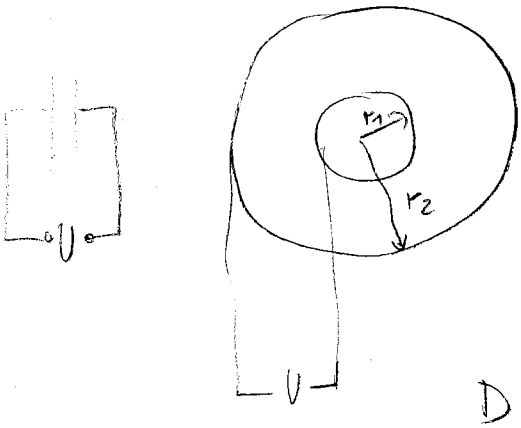
$J = 5 \times 10^{-5}\ kg\ m^2$; $I = \frac{(2\pi \cdot 1,5)^2 \cdot 5 \cdot 10^{-5}}{30 \cdot 15 \cdot 10^{-4} \cdot 2,1 \cdot 10^{-4}} = \frac{9\pi^2 \cdot 5 \cdot 10^{-1}}{4,5 \cdot 10^2 \cdot 2,1} =$

$= \frac{45 \cdot \pi^2}{9,45} \cdot 10 = 470\ A = I$ ✓

$J = 10\ g\ cm^2 = 10 \cdot 10^{-3} \cdot 10^{-4}\ kg\ m^2 = 10^{-6}\ kg\ m^2$

$I = I \frac{10^{-6}\ 10^{-1}}{5 \cdot 10^{-5}}$ $\boxed{I = 9,4\ A}$ ✓

Med dvema votlima koncentričnima kovinskima kroglama polmerov r_1=3 cm in r_2=6 cm je izolator z dielektričnostjo 6 in s prebojno jakostjo električnega polja 40 kV/cm. Kolikšna je največja napetost, ki jo smemo priključiti med obe krogli ?



$r_1 = 3$ cm

$r_2 = 6$ cm

$E_p = 40$ kV/cm

$U_{max} = ?$

$$D\, 4\pi r^2 = e \Rightarrow D = \frac{e}{4\pi r^2} = \varepsilon\varepsilon_0 E$$

$$\Rightarrow \boxed{E = \frac{e}{4\pi\varepsilon_0\varepsilon r^2}} \qquad E = -\frac{d\varphi}{dr}$$

$$\Delta\varphi = -\int_{r_1}^{r_2} E\, dr = -\frac{e}{4\pi\varepsilon\varepsilon_0}\frac{1}{r}\Big|_{r_1}^{r_2} = \frac{e}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\boxed{E_{max} = \frac{e}{4\pi\varepsilon\varepsilon_0 r_1^2}} \Rightarrow \boxed{e = E_{max}\cdot 4\pi\varepsilon\varepsilon_0 r_1^2}$$

$$\Delta\varphi = \frac{e}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{E_{max}\,4\pi\varepsilon\varepsilon_0 r_1^2}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\boxed{\Delta\varphi = E_{max}\cdot r_1^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} =$$

$$= \frac{40\cdot 10^3}{cm}\, 9\, cm^2\left(\frac{1}{3}-\frac{1}{6}\right) = 360\cdot 10^3\left(\frac{1}{6}\right) = 60\ kV$$

$$\boxed{\Delta\varphi = 60\ kV}$$

1. Galvanometer na vrtljivo tuljavico, ki ima 180 ovojev, se nahaja med zaobljenima poloma podkvastega magneta tako, da ima polje na mestu tuljavice (B = 1 T) radialno smer glede na os vrtenja tuljavice. Tuljavica je vrtljiva okoli osi, ki je pravokotna na njen magnetni dipolni moment ter pravokotna na smer magnetnega polja. Površina posameznega ovoja tuljavice je 2 cm^2. S tuljavico je togo povezan kazalec, ki kaže v smeri magnetnega dipolnega momemnta tuljavice pravokotno na njeno os vrtenja. Kolikšna je amplituda kazalčevega odklona, če spustimo skozi tuljavico galvanometra kratkotrajen tokovni sunek 5 μAs ? Vztrajnostni moment tuljavice s kazalcem vred je $1{,}5\cdot10^{-6}$ $kg\cdot m^2$, koeficient obeh polžastih vzmeti, ki sta povezani s tuljavico (z namenom, da jo vračata v ravnovesno stanje), pa je skupaj enak $2{,}5\cdot10^{-6}$ N·m.

$\boxed{M - Z}$

$N = 180$

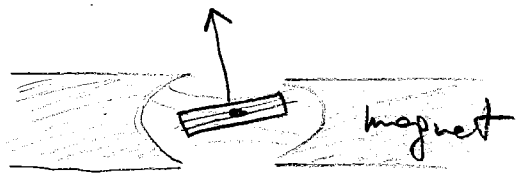
$B = 1\ T$

$\int I\,dt = 5\,\mu AS$

$J = 1{,}5\cdot10^{-6}\ kg\cdot m^2$

$D = 2{,}5\cdot10^{-6}\ N\cdot m$

$S = 2\ cm^2$



$M = p_m B \underbrace{\sin\varphi}_{1} = p_m B = NISB$

φ v rad

$(M_D = D\cdot\varphi)$

N·m

$$\boxed{\int M\,dt = J\omega_0}$$

$$\int NISB\,dt = J\cdot\omega_0$$

$$NSB\int I\,dt = J\cdot\varphi_0\sqrt{\frac{D}{J}}$$

$$J\cdot\ddot{\varphi} = -D\varphi$$

$$\ddot{\varphi} = -\frac{D}{J}\cdot\varphi$$

$$\boxed{(2\pi\nu)^2 = \frac{D}{J}}$$

$$\varphi = \varphi_0 \sin(2\pi\nu t)$$

$$\omega = \dot{\varphi} = \underbrace{\varphi_0(2\pi\nu)}_{\omega_0}\cos(2\pi\nu t)$$

$$\boxed{\varphi_0 = \frac{NSB\int I\,dt}{\sqrt{D\cdot J}} = 0{,}093\ (5{,}33^\circ)}$$

$$\frac{180\cdot2\cdot10^{-4}\ 5\cdot\cancel{10^{-6}}}{(2{,}5\cdot1{,}5)^{1/2}\cdot\cancel{10^{-6}}} = 0{,}093$$

1895

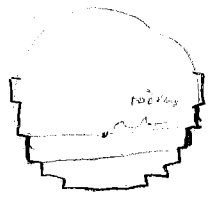
3. Okrogla ploščata tuljava ima 100 navojev in [redacted] radij 5 cm. Skozi tuljavo teče tok 0.1 A, tuljava pa je v magnetnem polju 1.5 T. Kolikšno delo moramo opraviti, če zavrtimo tuljavo za 180° iz začetnega položaja, kjer je magnetni dipolni moment tuljave vzporeden magnetnemu polju ?

$N = 100$

$I = 0.1\ A$

$B = 1.5\ T$

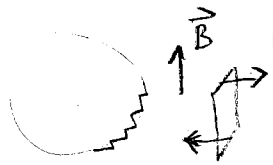
$\varphi_1 = 0°, \quad \varphi_2 = 180°$



$$\boxed{p_m = N \cdot I \cdot \pi a^2}$$

$$\boxed{W_m = -p_m B \cos\varphi}$$

$$\underline{\underline{A}} = \Delta W_m = -p_m B \left[\cos(180°) - \cos(0°)\right] = \underline{\underline{2 p_m B}} = \underline{\underline{0.24\ J}}$$



$\boxed{E53}$

$$\vec{F} = I \int d\vec{l} \times \vec{B}$$

$$\vec{M} = \vec{p}_m \times \vec{B}$$

$$\boxed{\vec{p}_m = N I \vec{S}}$$

$$\Delta W = \Delta A = \int_{\varphi_1}^{\varphi_2} M\, d\varphi = -\vec{p}_m \cdot B \cdot \cos\varphi \Big|_{\varphi_1}^{\varphi_2}$$

$$\boxed{W = -\vec{p}_m \cdot \vec{B}}$$

$$M = p_m B \cdot \sin\varphi$$

1. Kvadraten okvir iz žice s presekom 2 mm^2 in specifičnim uporom 0,02 $\Omega mm^2/m$ ima površino 100 cm^2. Homogeno magnetno polje z gostoto 0,8 T je nagnjeno za kot 50° glede na ravnino okvirja. Kolikšen tok teče po okvirju, če magnetno polje v 20 sekundah enakomerno zmanjšamo na 0,1 T ?

A-6

1994

$S_0 = 2\,mm^2$

$\rho = 0{,}02\ \Omega mm^2/m$

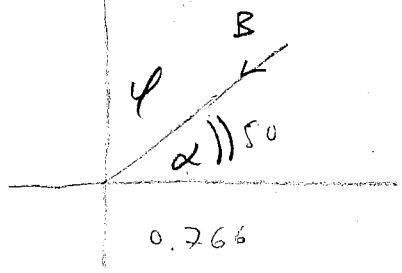
$S = 100\ cm^2$

$B_1 = 0{,}8\ T$

$\alpha = 50°$

$t_0 = 20\ s$

$B_2 = 0{,}1\ T$



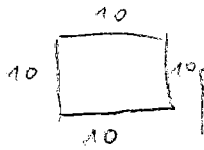
$\cos(90-\alpha) = \cos 90 \cos\alpha + \sin 90 \sin\alpha$

$\phi = \vec{B}\cdot\vec{S}$

$\phi = BS\cos\varphi$

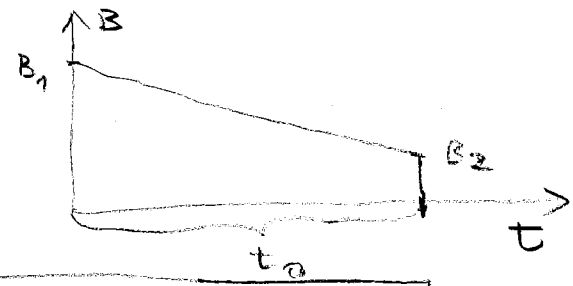
$\phi = BS\cos(90-\alpha)$

$\boxed{\phi = B\cdot S\sin\alpha}$



$\boxed{l = 0.4\ m}$

$\boxed{R = \frac{\rho l}{S_0}}$



$\boxed{B(t) = B_1 + \frac{B_2 - B_1}{t_0}\,t}$

$\boxed{\phi = B(t)\cdot S\cdot\sin\alpha} \Rightarrow \boxed{U_i = \frac{d\phi_m}{dt} = S\cdot\sin\alpha\cdot\frac{B_2-B_1}{t_0}}$

$I = \frac{U}{R} = \frac{S\cdot\sin\alpha\cdot(B_2-B_1)\cdot S_0}{t_0\cdot\rho\cdot l} =$

$R = \frac{\rho l}{S_0} = \frac{0{,}02\cdot 0.4}{2} =$

$= 0.004\ \Omega$

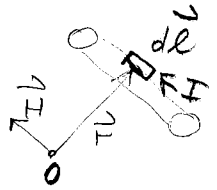
$= \frac{100\cdot 10^{-4}\cdot\sin\alpha\,(-0.7)}{20\cdot 0.004} = \frac{0.7}{0.08}\cdot 10^{-2}\cdot\sin\alpha \Rightarrow$

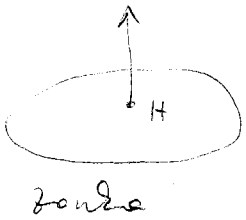
$\boxed{I = 6{,}7\cdot 10^{-2}\ A}$

E35

3. Dve ploščati tuljavi navijemo z žico enake dolžine tako, da imata isto število ovojev N. Prva tuljava je navita na kvadraten okvir, druga pa na okrogel obroč. Kolikšno je razmerje magnetnih poljskih jakosti H_1/H_2 v osi tuljav, če teče po obeh isti tok? (rezultat: $H_1/H_2 = \frac{\pi^2}{8\sqrt{2}}$)

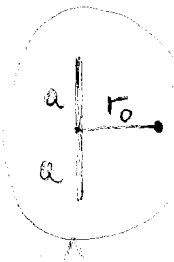
Biot-Savartov zakon: $\boxed{\vec{H} = \frac{I}{4\pi}\int \vec{r}\times d\vec{l}/r^3}$





$$\boxed{H_1 = \frac{I}{4\pi}\frac{2\pi r}{r^2} = \frac{I}{2r}}$$

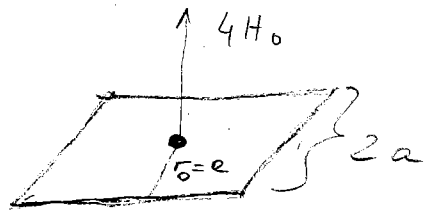
glej Klad. str. 117:



$$\boxed{H_0 = \left(\frac{I}{2\pi r_0}\right) a/\sqrt{a^2+r_0^2}}$$

$r_0 = a$

$$H_0 = \frac{I}{2\pi a}\cdot a/(\sqrt{2}\cdot a) = \frac{I\sqrt{2}}{4\pi a}$$

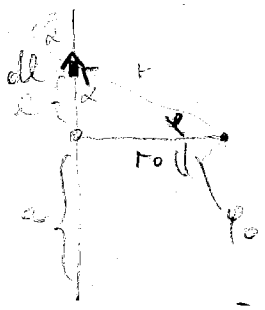


$$\boxed{H_2 = 4H_0 = I\sqrt{2}/\pi a}$$

$$\boxed{B = \mu_0 H}$$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\cos\beta$

izpeljava:



$$dH = \frac{I}{4\pi}\int\frac{r\,dl\sin\alpha}{r^3} = \frac{I}{4\pi}\int\frac{dl\cos\varphi}{r^2} =$$

$$= \frac{I}{4\pi}\int\frac{\cos^2\varphi\cdot r_0\,d\varphi\cdot\cos\varphi}{r_0^2\cos^2\varphi} = \frac{I}{4\pi}\int_{-\varphi_0}^{+\varphi_0}\frac{\cos\varphi\,d\varphi}{r_0}$$

$$= \frac{I}{4\pi r_0}\sin\varphi\Big|_{-\varphi_0}^{\varphi_0} = \frac{I}{4\pi r_0}\{\sin\varphi_0 - \sin(-\varphi_0)\}$$

$$= \frac{I}{4\pi r_0}2\sin\varphi_0 = \frac{I}{2\pi r_0}\sin\varphi_0 = \left(\frac{I}{2\pi r_0}\right)\frac{a}{\sqrt{a^2+r_0^2}}$$

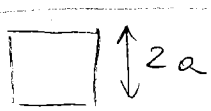
$\sin\alpha = \sin(90-\varphi) = \cos\varphi$

$\operatorname{tg}\varphi = \frac{l}{r_0}$ $\quad l = \operatorname{tg}\varphi\cdot r_0$

$$\boxed{dl = \frac{1}{\cos^2\varphi}r_0\,d\varphi}$$

$\cos\varphi = \frac{r_0}{r} \Rightarrow \boxed{r = \frac{r_0}{\cos\varphi}}$

$$\sin\varphi_0 = \frac{a}{\sqrt{r_0^2+a^2}}$$



$8a = 2\pi r \Rightarrow \boxed{\frac{a}{r} = \frac{2\pi}{8} = \frac{\pi}{4}}$

r

$$\frac{NH_1}{NH_2} = \frac{N}{N}\left(\frac{I}{2r}\right)\left(\frac{\pi a}{I\sqrt{2}}\right) = \frac{\pi a}{2\sqrt{2}\cdot r} = \frac{\pi\cdot\pi}{2\cdot\sqrt{2}\cdot 4} = \frac{\pi^2}{8\sqrt{2}} \approx 0.87$$

3

3. Skozi upornik R = 10 Ω, ki je potopljen v 1 dm^3 vode, v 1,5 ure (enakomerno) steče naboj 10^4 As. Za koliko se v (prvih) 10 min po vklopu (dotoka) toka segreje voda, če izgube toplote v okolico lahko zanemarimo?

$R = 10\,\Omega$

$V = 1\,dm^3$

$A = cU$, $U = I \cdot R$

$P = I \cdot U = I^2 R$

$\underbrace{I \cdot t}_{} = \boxed{e_t = 10^4\,As} \leftrightarrow \boxed{t = 90\,min}$

$\boxed{t_1 = 10\,min}$

$\underline{\underline{I}} = \frac{e_t}{t} = \underline{\underline{1.85\,A}}$

$\boxed{m = 1\,kg}$

$\boxed{P \cdot t_1 = m \cdot c_p\,\Delta T}$

$\Downarrow$

$\underline{\underline{\Delta T}} = \frac{P \cdot t_1}{m \cdot c_p} = \frac{I^2 \cdot R \cdot t_1}{m \cdot c_p} = \underline{\underline{4.9\,K}}$

$I^2 \frac{e_t^2 R\, t_1^2}{t^2\, m\, c_p} = \frac{3.43 \cdot 10 \quad 600}{\quad \cdot 42 \cdot 10^2}$

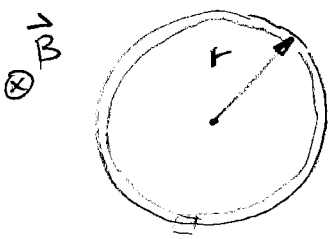
$= \frac{10^{16}}{10^8} \frac{36}{2346 \cdot 42}$

izpit IPQ1 A-L ← ODVEČ PODATEK

E-70

5. Tanek obroč z radijem 10 cm in maso 10^{-2} kg, ki je prosto vrtljiv okoli geometrijske osi, ima po vsem svojem volumnu enakomerno porazdeljen naboj 10^{-2} As. Obroč damo v homogeno magnetno polje z gostoto 1 T, tako da je smer magnetnega polja vzporedna z geometrijsko osjo obroča. S kaksno frekvenco se začne vrteti obroč? Nosilci naboja v obroču niso gibljivi.

$$\boxed{\int \vec{E}\,d\vec{s} = U_i = -\frac{d\Phi_m}{dt}}$$



$r = 0{,}1\ m$

$m = 10^{-2}\ kg$

$e_T = 10^{-2}\ As$

$B = 1\ T$

$\boxed{J = mr^2}$

$\boxed{\nu = ?}$

$$\boxed{de = \frac{e_T}{2\pi r}\,dl}$$

$$dM = r\,dF = r\,(de \cdot E) = r\left(\frac{e_T}{2\pi r}\,dl \cdot E\right) \Rightarrow \boxed{M = r\,e_T\,E}$$

$$\int M\,dt = J\omega$$

$$\Downarrow$$

$$\boxed{\int r\,e_T\,E\,dt = J\omega}$$

$$U_i = -\frac{d\Phi_m}{dt} \Rightarrow \boxed{\int U_i\,dt = \Delta\Phi_m = BS}$$

$$\boxed{\int \vec{E}\,d\vec{s} = U_i = E\,2\pi r}$$

Max. eq.: $\oint \vec{E}\,d\vec{s} = -\int \left(\frac{\partial \vec{B}}{\partial t}\right) d\vec{S}$

$$\boxed{\int E\,2\pi r\,dt = BS}$$

$$\boxed{\frac{J\omega}{r\,e_T} = \frac{BS}{2\pi r}} \Rightarrow$$

$$\Rightarrow \omega = \frac{e_T B S}{2\pi J} \Rightarrow 2\pi\nu = \frac{e_T B \cdot \pi r^2}{2\pi m r^2} \Rightarrow$$

$$\Rightarrow \boxed{\nu = \frac{e_T B}{4\pi m}} = \underline{0{,}08\ s^{-1}}$$

$$= \frac{10^{-2} \cdot 1}{\ldots\ 10^{-2}} = \frac{1}{4\pi}$$

4. Kondenzator s kapaciteto C_2=1 μF je nabit na napetost U_o=200 V. Ob času t=0 ga priključimo na nedušen nihajni krog, ki ima lastno frekvenco ν_1=200 Hz in kondenzator s kapaciteto C_1=1,5 μF. Kolikšen tok bo odtekal s kondenzatorja C_1 ob času t=0,01 s po priključitvi?

M-Ž

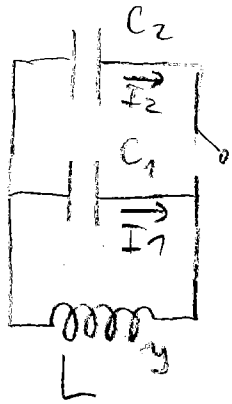
$C_2 = 1\ \mu F$

$C_1 = 1{,}5\ \mu F$

$\nu_1 = 200\ Hz$

$t = 0{,}01$

$U_0 = 200\ V$



od t=0 (naboj se ohranja)

$$\boxed{e_0 = e_{01} + e_{02}}$$

$$C_2 U_0 = C_2 U_{20} + C_1 U_{10}, \quad \boxed{U_{20} = U_{10}}$$

$$C_2 U_0 = U_{01}(C_1 + C_2) \Rightarrow \boxed{U_{01} = U_0 C_2/(C_1+C_2)}$$

od vsakem t: $U_2 = U_1$

$$\frac{e_2}{C_2} = \frac{e_1}{C_1} \Rightarrow \boxed{\frac{I_2}{C_2} = \frac{I_1}{C_1}}$$

$$\Downarrow$$

$$I_2 = \frac{C_2}{C_1} I_1$$

$$\boxed{J = J_1 + J_2}$$

$$-\frac{e_1}{C_1} - L\dot{J} = 0 \quad / \frac{d}{dt}$$

$$-\frac{I_1}{C_1} - L(\ddot{J}_1 + \ddot{J}_2) = 0$$

$$-\frac{I_1}{C_1} - L\left(1 + \frac{C_2}{C_1}\right)\ddot{J}_1 = 0$$

$$\ddot{J}_1 = -\frac{1}{L(C_1+C_2)} J_1 \quad / \quad \boxed{\omega_0 = \frac{1}{\sqrt{L(C_1+C_2)}}} = \underline{973{,}6\ s^{-1}}$$

$$(2\pi\nu_1)^2 = \frac{1}{LC_1} \Rightarrow L = \frac{1}{C_1(2\pi\nu_1)^2}$$

$L = 0{,}422$

$$\boxed{J_1 = J_{10} \sin(\omega_0 t)}$$

U_{01}

$$U_{C1} = -U_L = L\dot{J} = L(\dot{J}_1 + \dot{J}_2) = L\left(1 + \frac{C_2}{C_1}\right)\dot{J}_1 = \boxed{L\left(1 + \frac{C_2}{C_1}\right) J_{10}\, \omega_0 \cos(\omega_0 t)}$$

$$\boxed{U_{01} = L\left(1 + \frac{C_2}{C_1}\right) J_{10}\, \omega_0} \Rightarrow J_{10} = \frac{U_{01}}{L\left(1 + \frac{C_2}{C_1}\right)\omega_0} = \frac{U_0 C_2}{(C_1+C_2) L \left(1 + \frac{C_2}{C_1}\right)\omega_0}$$

$$\boxed{J_{01} = \frac{U_0 C_2 C_1 \sqrt{L(C_1+C_2)}}{(C_1+C_2)^2 L} = \frac{U_0 C_1 C_2}{C_1 + C_2} \cdot \frac{1}{\sqrt{L(C_1+C_2)}}} \quad \boxed{J_{01} = \frac{U_0 C_1 C_2}{(C_1+C_2)} \omega_0}$$

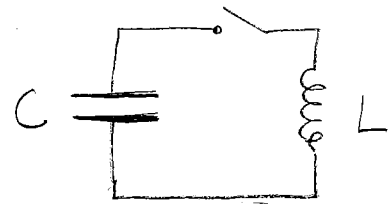
Idealni

E57

Nihajni krog sestavljata kondenzator s kapaciteto $1\,\mu F$ in tuljava z induktivnostjo 10^{-3} Vs/A. Kondenzator nabijemo in nato s stikalom sklenemo nihajni krog. Po kolikšnem času odda kondenzator polovico svoje začetne energije tuljavi?

$C = 1\,\mu F$

$L = 10^{-3}$ Vs/A , $\boxed{U_c(t=0) = U_0}$



$\boxed{e = C\,U_c}$, $\boxed{\phi = L\,y}$

$\boxed{U_c = \frac{e}{C}}$, $\boxed{U_L = \frac{d\phi}{dt} = L\,\dot{y}}$

$$\boxed{U_L + U_c = -L\frac{dy}{dt} - \frac{e}{C} = 0} \quad \Big/ \frac{d}{dt}$$

$$-L\frac{d^2y}{dt^2} - \frac{1}{C}y = 0$$

$$\frac{d^2y}{dt^2} + \frac{1}{LC}y = 0$$

$$\boxed{\omega_0 = (LC)^{-1/2}}$$

$$\left\{\begin{aligned} &\boxed{y = y_0 \sin\omega_0 t} \\ &\dot{y} = +y_0\omega_0\cos\omega_0 t \\ &\ddot{y} = -y_0\omega_0^2\sin\omega_0 t \\ &\boxed{\ddot{y} = -\omega_0^2 y} \end{aligned}\right\}$$

$\boxed{W_e = \frac{1}{2}CU^2}$

$\boxed{W_m = \frac{1}{2}Ly^2}$

$$\boxed{U_L = -L\frac{dy}{dt} = -Ly_0\omega_0\cos(\omega_0 t)}$$

$$U_c = -U_L = +\underbrace{Ly_0\omega_0}_{U_0}\cos\omega_0 t$$

$\boxed{U_0 = Ly_0\omega_0}$

energija tuljave:

$$A = \int U\,de = -\int U_L\,y\,dt =$$

$$= \int L\frac{dy}{dt}\,y\,dt =$$

$$= \int L\,y\,dy = \frac{1}{2}Ly^2$$

$$\boxed{W_e = \frac{1}{2}CU^2 = \frac{1}{2}CU_0^2\cos^2\omega_0 t}$$

$$\cos^2\omega_0 t_{1/2} = \frac{1}{2}$$

$$\cos[\omega_0 t_{1/2}] = \frac{1}{\sqrt{2}}$$

$$\omega_0 t_{1/2} = \arccos\left(\frac{1}{\sqrt{2}}\right)$$

2. Kondenzator s kapaciteto C_2=1,3 μF je nabit na napetost U_o.
Ob času t=0 ga priključimo na nedušen nihajni krog, ki ima lastno frekvenco ν_1=180 Hz in kondenzator s kapaciteto C_1=1,2 μF. Ob času t=0,01 s po priključitvi odteka s kondenzatorja C_2 tok 0,1 A.
Kolikšna je napetost U_o?

A-L

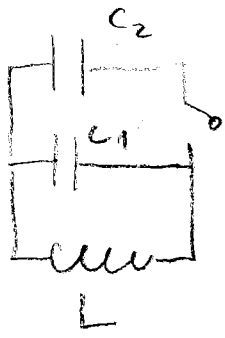
$C_2 = 1{,}3\,\mu F$

$C_1 = 1{,}2\,\mu F$

$t = 0{,}01\ s$

$J_2(t=0{,}01\ s) = 0{,}1\ A$

$\nu_1 = 180\ Hz$



$$\boxed{\omega_0 = \frac{1}{\sqrt{L(C_1+C_2)}}}$$

$$\boxed{J_2 = J_{20}\sin(\omega_0 t)}$$

$$\boxed{U_{02} = U_0 C_2 / (C_1+C_2)}$$

$$U_{C2} = -U_L = L\dot{J} = L(\dot{J}_1+\dot{J}_2) = L\left(1+\frac{C_1}{C_2}\right)\dot{J}_2 = \overbrace{L\left(1+\frac{C_1}{C_2}\right)J_{20}\omega_0}^{U_{20}}\cos(\omega_0 t)$$

$$\boxed{U_{20} = L\left(1+\frac{C_1}{C_2}\right)J_{20}\omega_0}$$

$$J_{20} = \frac{U_{20}}{L\left(1+\frac{C_1}{C_2}\right)\omega_0} = \frac{U_0 C_2\sqrt{L(C_1+C_2)}}{(C_1+C_2)L\left(1+\frac{C_1}{C_2}\right)} = \frac{U_0 C_2^2}{(C_1+C_2)}\omega_0$$

$$\boxed{J_{20} = \frac{U_0 C_2^2}{(C_1+C_2)}\omega_0}$$

$$(2\pi\nu_1)^2 = \frac{1}{LC_1} \Rightarrow L = \frac{1}{C_1(2\pi\nu_1)^2} = 0{,}651$$

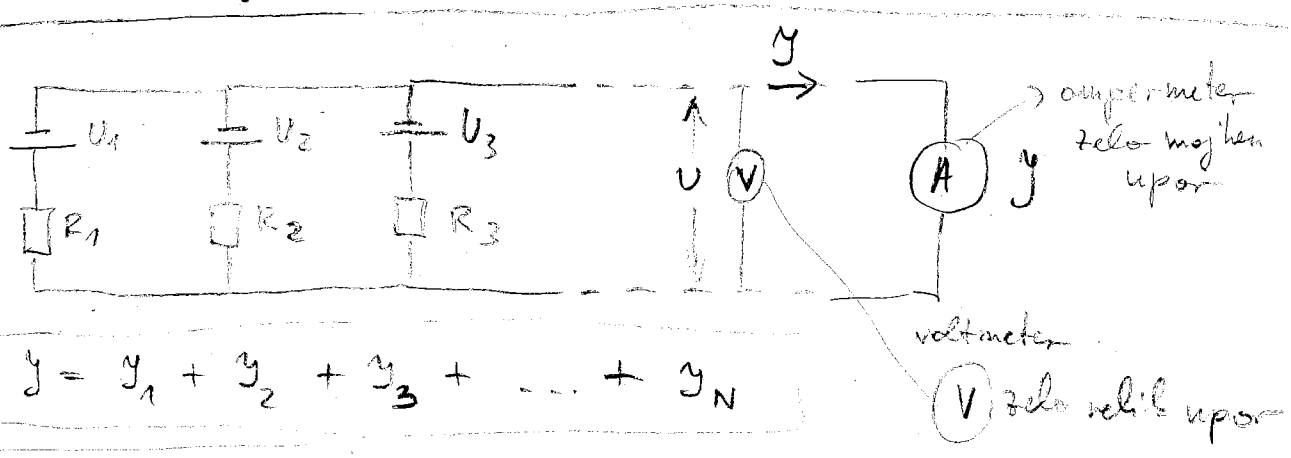
$$\omega_0 = 783{,}9\ s^{-1}$$

$$J_2 = \frac{U_0 C_2^2}{(C_1+C_2)}\omega_0 \cdot \sin(\omega_0 t) \Longrightarrow \boxed{U_0 = \frac{J_2(C_1+C_2)}{C_2^2\cdot\omega_0\cdot\sin\omega_0 t}}$$

$$\boxed{U_0 = 188{,}7\ V}$$

KL /24/117

N različnih galvanskih členov zvežemo vzporedno. Kolikšna sta gonilna napetost (U_g) in notranji upor (R_n) nastale baterije?



$$I = I_1 + I_2 + I_3 + \dots + I_N$$

$$U_1 - I_1 R_1 = U \Rightarrow I_1 = (U_1 - U)/R_1$$

$$U_2 - I_2 R_2 = U \Rightarrow I_2 = (U_2 - U)/R_2$$

$$\vdots$$

$$U_N - I_N R_N = U \Rightarrow I_N = (U_N - U)/R_N$$

$$\Downarrow$$

$$I = \sum_{i=1}^{N} I_i = \sum_{i=1}^{N} \frac{U_i}{R_i} - U \sum_{i=1}^{N} \frac{1}{R_i}$$

PRIMERJAJ

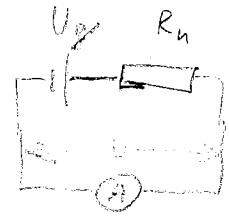
$$\frac{U_g}{R_n} = \sum_{i=1}^{N} \frac{U_i}{R_i}$$

$$\boxed{\frac{1}{R_n} = \sum_{i=1}^{N} \frac{1}{R_i}}$$ notranji upor

$$\Downarrow$$

$$U = \left(\sum_{i=1}^{N} \frac{U_i}{R_i}\right) \Big/ \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)$$ gonilna

vodovarstvena vezja:



(A) ima velik upor R

$$U_g = I R_n + U$$

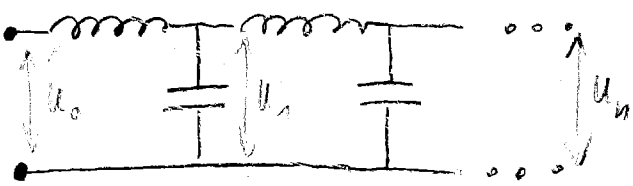
ali

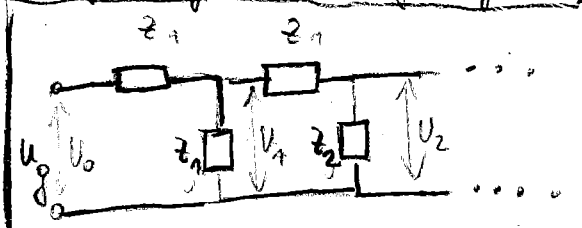
$$U = U_g - I R_n$$

$$\Downarrow$$

$$I = \frac{U_g}{R_n} - \frac{U}{R_n}$$

Zgled (neskončno dolga veriga) Kako se spreminja U_n kot funkcija ω? E64



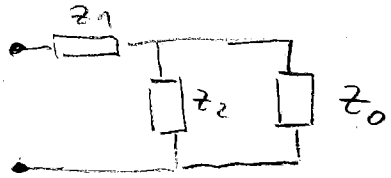


priključeno na oscilirajočo napetost: $\boxed{U_g = U_0 e^{i\omega t}}$

$\boxed{Z_1 = i\omega L}$ $\boxed{Z_2 = -i\frac{1}{\omega C}}$

impendande

$\Downarrow$ nadomestni upor



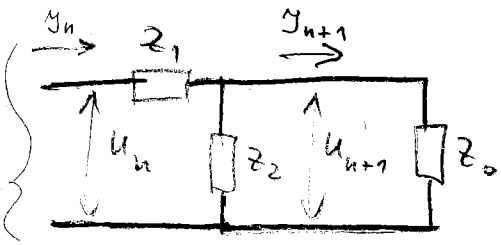
$U_1 \neq U_2 \neq U_g$ ker padec napetosti tudi na Z_1 !

$$\boxed{Z_1 + \frac{Z_2 \cdot Z_0}{Z_2 + Z_0} = Z_0}$$

$$Z_0 = \frac{Z_1 \pm \sqrt{Z_1^2 + 4Z_1Z_2}}{2}$$

$$Z_1(Z_2 + Z_0) + Z_2 \cdot Z_0 = Z_0 \cdot (Z_2 + Z_0) \Rightarrow \boxed{Z_0^2 - Z_0 Z_1 - Z_1 Z_2 = 0}$$

$$\boxed{Z_0 = \frac{i\omega L}{2} + \sqrt{\frac{L}{C} - \left(\frac{\omega L}{2}\right)^2}}$$



$$\boxed{U_n = Z_1 y_n + U_{n+1}} \Rightarrow$$

$$\Rightarrow U_{n+1} = U_n - Z_1 y_n \quad / U_n$$

$$\boxed{\frac{U_{n+1}}{U_n} = 1 - Z_1 \frac{y_n}{U_n}}$$

$$\boxed{y_{n+1} \cdot Z_0 = U_{n+1}}$$

$$\frac{y_{n+1}}{U_{n+1}} = \frac{1}{Z_0} = \frac{y_n}{U_n}$$

lahko bi postavili nadomestni upor za U_{n-1} in bi to dobili

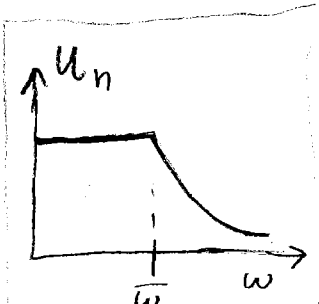
$\Downarrow$

$$\boxed{\frac{U_{n+1}}{U_n} = 1 - Z_1 \cdot \frac{1}{Z_0} = \frac{Z_0 - Z_1}{Z_0} = \alpha} \Rightarrow \boxed{U_{n+1} = \alpha \cdot U_n} \Rightarrow \begin{array}{l} U_1 = \alpha U_0 \\ U_2 = \alpha U_1 = \alpha^2 U_0 \end{array}$$

$$\boxed{U_n = \alpha^n U_0}$$

$$\boxed{\alpha = \frac{\sqrt{\frac{L}{C} - \left(\frac{\omega L}{2}\right)^2} - \frac{i\omega L}{2}}{\sqrt{\frac{L}{C} - \left(\frac{\omega L}{2}\right)^2} + \frac{i\omega L}{2}} = \frac{A - Bi}{A + Bi}}$$

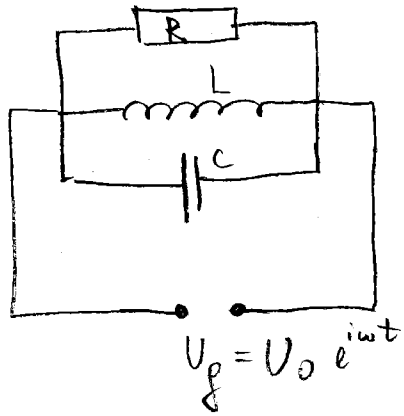
$A, B \in \mathbb{R}$



če $\left[\frac{L}{C} - \left(\frac{\omega L}{2}\right)^2\right] < 0 \Rightarrow \alpha \in \mathbb{R}$, $\alpha = \frac{A - B}{A + B} < 1$ $(\omega > \omega_0)$

če $\left[\frac{L}{C} - \left(\frac{\omega L}{2}\right)^2\right] > 0 \Rightarrow \alpha \in \mathbb{C}$, $\alpha = \frac{A - Bi}{A + Bi} \Rightarrow |\alpha| = 1$

$\{|y_0|, \delta\}$?

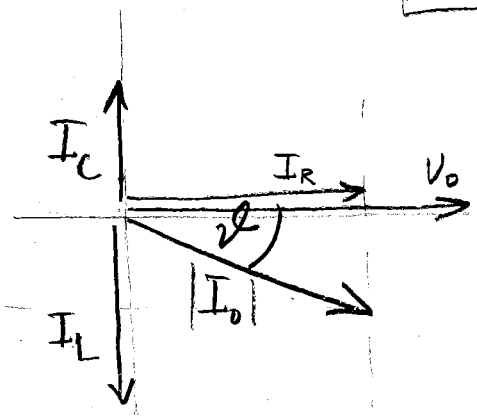


IMPENDANCE:

$$\boxed{z_R = R}$$

$$\boxed{z_L = i\omega L}$$

$$\boxed{z_C = -i\,\frac{1}{\omega C}}$$



Na uporniku, tuljavi in kondenzatorju je enaka napetost.
Ugodno, če U_0 realen in I_0 kompleksen

$$\boxed{U_0 = z \cdot I_0}$$

$$\boxed{\frac{1}{z} = \frac{1}{R} + \frac{1}{i\omega L} - \frac{\omega C}{i} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C}$$

$$\boxed{I_0 = \frac{1}{z} U_0 = U_0\left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C\right)} \Longrightarrow$$

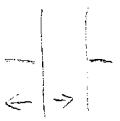
$$\boxed{I_0 = U_0\left[\frac{1}{R} + i\cdot\left(\omega C - \frac{1}{\omega L}\right)\right]}$$

$$|I_0|^2 = I_0 \cdot I_0^* = U_0^2 \cdot \left[\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2\right]$$

$$\boxed{|I_0| = U_0 \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

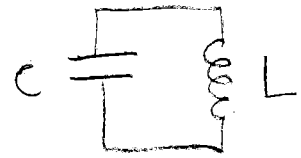
$$\boxed{\operatorname{tg}\delta = \frac{\operatorname{Im}(I_0)}{\operatorname{Re}(I_0)} = \left(\omega C - \frac{1}{\omega L}\right)\cdot R}$$

Nihajni krog sestavljata ploščati kondenzator s ploščino plošč po 100 cm² v razmiku 1 mm in tuljava s 100 ovoji, dolžino 10 cm in radijem 1 cm. V nihajnem krogu vzbudimo lastno nihanje z energijo 10^{-6} J. Kolikšna je amplituda napetosti na kondenzatorju in amplituda toka skozi tuljavo?



$2D_1 S = e$, $D = 2D_1 = \frac{e}{S} = \varepsilon_0 E$

$\frac{e}{S} = \varepsilon_0 \frac{U}{d}$, $e = \left(\varepsilon_0 \frac{S}{d}\right) U$ $\boxed{C = \varepsilon_0 \frac{S}{d}}$



$S = 100\ cm^2$, $d = 1\ mm$ } kondenzator

$N = 100$, $l = 10\ cm$, $r = 1\ cm$ } tuljava

$W = 10^{-6}\ J$ } energija

$H l = N I$, $B = \mu_0 \frac{N I}{l}$

$\Phi_m = N B S = N S \mu_0 \frac{N I}{l} = \mu_0 \frac{N^2 S}{l} \cdot I$

$\boxed{L = \mu_0 \frac{N^2 \cdot S}{l}}$

$U_L + U_C = -L \frac{dI}{dt} - \frac{e}{C} = 0 \quad / \frac{d}{dt}$

$\boxed{I = I_0 \sin \omega_0 t}$

$\dot{I} = +I_0 \omega_0 \cos \omega_0 t$

$\ddot{I} = -I_0 \omega_0^2 \sin \omega_0 t$

$\ddot{I} = -\omega_0^2 I$

$-\frac{d^2 I}{dt^2} = \frac{1}{LC} I \Rightarrow \boxed{\omega_0 = (LC)^{-1/2}}$

$U_L = -L I_0 \omega_0 \cos(\omega_0 t) = -L \frac{dI}{dt}$

$U_C = -U_L = +L I_0 \omega_0 \cos(\omega_0 t) \Rightarrow \boxed{U_0 = L I_0 \omega_0}$

$\boxed{W_e = \frac{1}{2} C U^2}$

$\boxed{W_m = \frac{1}{2} L I^2}$

$W_m = \frac{1}{2} L I^2 = \frac{1}{2} L I_0^2 \sin^2(\omega_0 t)$

$W_e = \frac{1}{2} C U^2 = \frac{1}{2} C U_0^2 \cos^2(\omega_0 t)$

$\frac{1}{2} C U_0^2 = \frac{1}{2} C L^2 I_0^2 \omega_0^2 = \frac{1}{2} C L^2 I_0^2 \frac{1}{LC} = \frac{1}{2} L I_0^2$

$W = W_m + W_e = \frac{1}{2} L I_0^2 \sin^2(\omega_0 t) + \frac{1}{2} C U_0^2 \cos^2(\omega_0 t)$

$= \frac{1}{2} L I_0^2 (\sin^2 + \cos^2) = \frac{1}{2} L I_0^2 = \frac{1}{2} C U_0^2$

$\boxed{W = \frac{1}{2} L I_0^2}$ } $\Rightarrow U_0 =$ $\Leftarrow$ Iz: $W = \frac{1}{2} L I_0^2$, $W = \frac{1}{2} C U^2$

3. Dve enaki kovinski zanki s polmerom 5 cm imata središči na isti osi. V prvi zanki teče tok 2 A. V začetku sta ravnini v katerih ležita obe zanki vzporedni in oddaljeni 0,5 m. Oceni kolikšna je maksimalna inducirana napetost v drugi zanki, če se le ta začne vrteti s kotno hitrostjo 5 s^{-1} okoli osi, ki leži v ravni zanke in gre skozi središče zanke ?

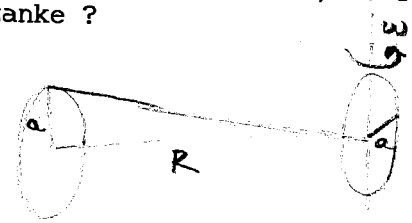
24.9.

$a = 5\ cm$

$I = 2\ A$

$R = 0{,}5\ m$

$\omega = 5\ s^{-1}$



$\vec{dB}$

$$\vec{dB} = \frac{\mu_0 I}{4\pi}\left(\frac{\vec{r} \times \vec{dl}}{r^3}\right)$$

r R

$$B_R = \frac{\mu_0 I}{4\pi}\,\frac{r\,2\pi a}{r^3}\,\cos\vartheta$$

dl ϑ a

kot projekcija

$$\boxed{B_R = \frac{\mu_0 I}{2}\,\frac{a^2}{(a^2+R^2)^{3/2}}}$$

$$\approx \frac{\mu_0 I}{2}\,\frac{a^2}{R^3}$$

$$\Phi = B\pi a^2 \cdot \cos(\omega t)$$

$$U_i = -\frac{d\Phi}{dt} = \underbrace{\frac{\mu_0 I}{2}\,\frac{a^2}{R^3}\,\pi a^2\,\omega \cdot \sin(\omega t)}$$

$$U_0 = \frac{\mu_0 I \omega a^4 \cdot \pi}{2 \cdot R^3} = \underline{9{,}87 \cdot 10^{-10}\ V}$$

izpit 1991

E65

1. Plošcati kondenzator sestavljata kvadratni navpični plošči z višino 10 cm v razdalji 2 cm. Ob zgornjem robu pozitivno nabite plošče spustimo kapljico z maso 0,1 g in nabojem 10^{-8} As. Razdalja med spodnjim delom plošč in tlemi je 30 cm. Napetost med ploščama kondenzatorja je 90 V. Kako daleč od mesta navpično pod točko, v kateri smo kapljico spustili, le ta pade na tla ?

$h = 0{,}1\ m$

$d = 0{,}02\ m$

$m = 10^{-4}\ kg$

$e = 10^{-8}\ As$

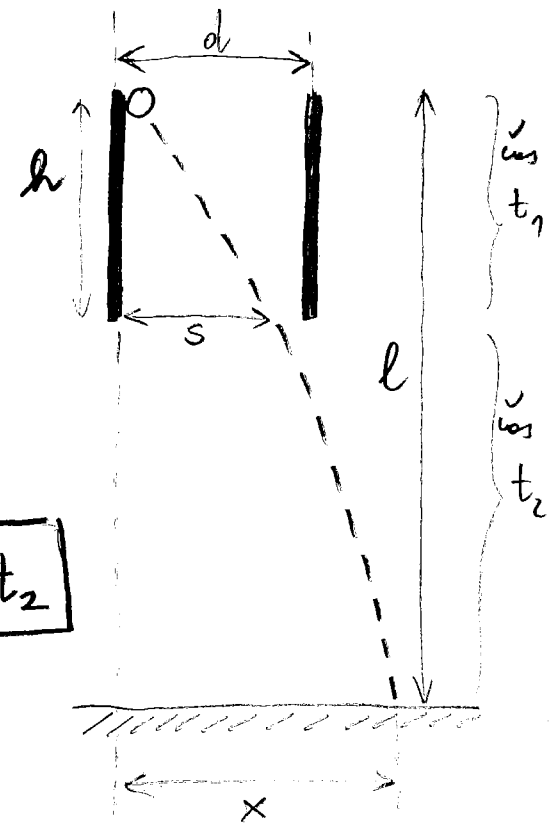
$l = 0{,}4\ m$

$U = 80\ V$

$$F = ma = lE = e\frac{U}{d}$$

$$\Downarrow$$

$$\boxed{a = eU/(d \cdot m)}$$



$$\boxed{h = g t_1^2/2} \qquad \boxed{s = a \cdot t_1^2/2}$$

$$t = t_1: \quad \boxed{v_x = a t_1 = \frac{eU}{d \cdot m} \cdot t_1} \qquad \boxed{x = s + v_x \cdot t_2}$$

$$\boxed{l = g(t_1 + t_2)^2/2}$$

$$\rightarrow t_1 = (2h/g)^{1/2}$$

$$\rightarrow t_2 = \left(\frac{2l}{g}\right)^{1/2} - t_1 = \left(\frac{2l}{g}\right)^{1/2} - \left(\frac{2h}{g}\right)^{1/2}$$

$$x = \frac{eU}{d \cdot m} \cdot \frac{2h}{g \cdot 2} + \frac{eU}{d \cdot m} \cdot \left(\frac{2h}{g}\right)^{1/2} \left[\left(\frac{2l}{g}\right)^{1/2} - \left(\frac{2h}{g}\right)^{1/2}\right] \Rightarrow$$

$$\boxed{x = \frac{eUh}{mgd} \cdot \left[2 \cdot \left(\frac{l}{h}\right)^{1/2} - 1\right] = \underline{1{,}38\ cm}}$$

FOR

1. Na zelo lahki nitki dolžine 1 cm je obešena majhna nabita kroglica z pozitivnim nabojem $20 \cdot 10^{-9}$ As in maso 2 g. Kroglica se nahaja med dvema vodoravno ležečima in vzporednima ter nasprotno nabitima kovinskima ploščama. S kolikšnim nihajnim časom zaniha kroglica, če jo za malenkost izmaknemo iz ravnovesne lege ? Površinska gostota naboja na ploščah je +/- 5 $\mu As/m^2$. Spodnja plošča je pozitivno nabita. 1892

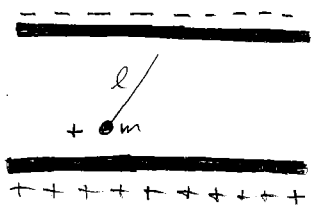
$g = 10\ ms^{-2}$

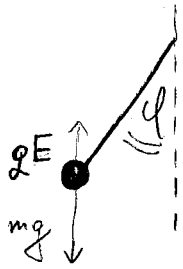
$l = 0{,}01\ m$

$q = 20 \cdot 10^{-9}\ As$

$m = 2 \cdot 10^{-3}\ kg$

$\sigma = \pm 5 \cdot 10^{-6}\ As/m^2$





$\varepsilon_0 E_{1/2} 2S = q$

$E = \frac{1}{\varepsilon_0} \frac{q}{S} = \frac{\sigma}{\varepsilon_0}$

$\sin\varphi \approx \varphi$

$$l \cdot (mg - qE)\varphi = -ml^2 \cdot \ddot{\varphi}$$

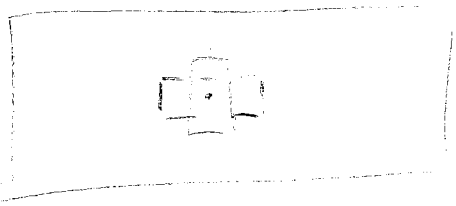
$$\boxed{T = 2\pi \cdot \sqrt{\frac{ml}{mg - q\frac{\sigma}{\varepsilon_0}}}} = \underline{\underline{0.3\ s}}$$

7. Po tuljavi z dolžino 1m in z 200 ovoji teče tok 25 A. V tuljavi je merilna tuljavica s premerom 10 cm, 75 ovoji in z uporom 10 Ω. Spočetka sta geometrijski osi tuljave in tuljavice vzporedni. Nato hitro zasučemo merilno tuljavico za 90° okoli osi, ki je pravokotna na njeno geometrijsko os. Kolikšen tokovni sunek steče skozi balistični galvanometer, ki je priključen na merilno tuljavico ? 1892

Upor galvanometra zanemarimo.

$N = 200$, $l = 1\,m$, $I = 25\,A$ } velika tuljava

$2r' = 10\,cm$, $N' = 75$, $R' = 10\,\Omega$ } mala tuljava



velika tuljava: $B = \mu_0 \frac{I}{l}$

$\varphi_0 = 1°$, ce $\int I\,dt = 10^{-5}\,As$

$\int H\,ds = NI$

$Hl = NI \Rightarrow B = \frac{\mu_0 N I}{l}$

MALA TULJAVA:

$U_i = \frac{d\Phi}{dt} \Rightarrow \int U_i\,dt = \Delta\Phi' = \Phi'$

$S' = \pi r'^2$

$\int U_i\,dt = N' B S'$

$U = IR$

$R' \cdot \int I\,dt = N' B \cdot S'$

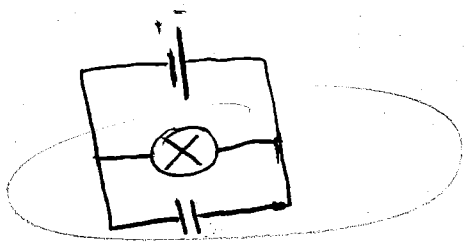
$\Downarrow$

$$\int I\,dt = \frac{N' B \cdot S'}{R'} = \frac{N' \mu_0 N \cdot I \cdot \pi \cdot r'^2}{l \cdot R'} = 3{,}7 \cdot 10^{-4}\,As$$

E28

100 W žarnica je priklopljena ~~priklopimo~~ na 110 V istosmerne napetosti. Kakšen kondenzator moramo priključiti in kako, da bo žarnica potem ko tok prekinemo, svetila še 10 sekund. ~~Če vemo, da je~~ Najmanjša napetost pri kateri žarnica še sveti je 20 V.

$$e = C U_c = \frac{e}{C}$$



$$U_c - IR = 0$$

$$\frac{dU_c}{dt} - \frac{dI}{dt} R = 0$$

$$-\frac{1}{C}\frac{de}{dt} - \frac{dI}{dt} R = 0$$

$$\frac{1}{C} \cdot I - \frac{dI}{dt} R =$$

$$\frac{dI}{I} = -\frac{1}{RC} dt \Rightarrow I = I_0 e^{-\frac{t}{RC}}$$

$$U = RI = RI_0 e^{-t/RC}$$

$$P = U \cdot I = \frac{U^2}{R}$$

$$R = \frac{U^2}{P_1}$$

$$U = U_0 \cdot e^{-\frac{t}{RC}}$$

$$\ln \frac{U}{U_0} = -\frac{t}{RC}$$

$$\ln \frac{U_0}{U} = \frac{t}{RC}$$

$$C = \frac{t}{\ln \frac{U_0}{U} \cdot R} \Rightarrow C = \frac{t \, P}{\ln \frac{U_0}{U} \cdot U_0^2} =$$

$$= \frac{10\,s \cdot 100\,VA}{\ln \frac{110}{20} \cdot 110^2\,V^2} = \frac{1000\,VAs}{1{,}7 \cdot 1{,}2 \cdot 10^4\,V^2} = 49 \cdot 10^{-3} \frac{As}{V}$$

$$= \underline{\underline{49\,mF}}$$

4. Skozi pretočni bojler teče vodni tok $\Phi = 9$ l/min. Voda se v bojlerju segreje od 15° C do 70° C. Kolikšna je amplituda izmeničnega toka skozi grelec, ki je priključen na efektivno izmenično napetost 220 V, če je izkoristek grelca 85 %?

$$\phi_V = 9\ \ell/\text{min} = \frac{9\cdot 10^{-3}\ m^3}{60\ s} = 1{,}5\cdot 10^{-4}\ m^3/s$$

$T_1 = 15\ °C$

$T_2 = 70\ °C$

$U_{ef} = 220\ V$

$\eta = 0{,}85$

$$u = U_0 \cos(\omega t),\quad I = I_0 \cos(\omega t + \delta)$$

$$\overline{P} = I_0 U_0 \left[\cos\omega t \cos\delta - \sin\omega t \sin\delta\right] \cos\omega t$$

$$= I_0 U_0 \left[\underbrace{\overline{\cos^2\omega t}}_{1/2} \cos\delta - \underbrace{\overline{\cos\omega t \sin\omega t}}_{\frac{1}{2}\sin(2\omega t) = 0} \sin\delta\right]$$

$$= I_0 U_0 \frac{1}{2} \cos\delta$$

$$\boxed{\overline{P} = \underbrace{\frac{I_0}{\sqrt{2}}}_{I_{ef}} \cdot \underbrace{\frac{U_0}{\sqrt{2}}}_{U_{ef}} \cdot \cos\delta}$$

$$\frac{Q}{t} = \frac{m\, c_p (T_2 - T_1)}{t} = \eta P$$

$$\phi_m \cdot c_p (T_2 - T_1) = \eta \frac{I_0}{\sqrt{2}} \cdot U_{ef}$$

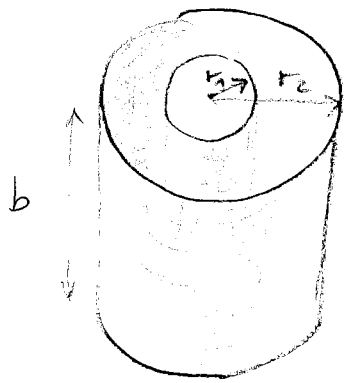
$$\boxed{\frac{\phi_V \cdot \rho \cdot c_p (T_2 - T_1) \cdot \sqrt{2}}{\eta \cdot U_{ef}} = I_0 = \underline{\underline{262\ A}}}$$

(nasprotno nabitima)

E10

1. Med dvema valjastima ploščama, ki imata skupno geometrijsko os, je napetost 5 kV. Kolikšna je električna poljska jakost na razdalji 6 cm od skupne geometrijske osi, če je radij prvega valja 4 cm, radij drugega valja pa 8 cm ?

Hl. 25/45 | Slovad str. 333



$$\boxed{D = \varepsilon_0 E}$$

$r_1 = 0.04$ m

$r_2 = 0.08$ m

$r = 0.06$ m

$|\Delta U| = 5$ kV

$$\int D\,dS = e \qquad E = -\operatorname{grad}\varphi = -\frac{d\varphi}{dr}$$

$$D\,2\pi r b = e$$

$$\varphi(r_2) - \varphi(r_1) = d\varphi\Big|_{r_1}^{r_2} = -\int_{r_1}^{r_2} E\,dr = -\int_{r_1}^{r_2} \frac{e}{2\pi\varepsilon_0 b}\frac{dr}{r} = \frac{-e}{2\pi\varepsilon_0 b}\ln\frac{r_2}{r_1}$$

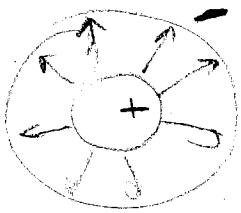
$$\boxed{E = \frac{e}{2\pi r \varepsilon_0 b}}$$

$$\boxed{|\Delta\varphi| = \frac{e}{2\pi\varepsilon_0 b}\ln\frac{r_2}{r_1}}$$

$$\frac{e}{2\pi\varepsilon_0 b} = \frac{\Delta\varphi}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{5\cdot 10^3}{\ln 2} = 7.213\cdot 10^3\ \mathrm{V}$$

$$E = \frac{e}{2\pi\varepsilon_0 b}\cdot\frac{1}{r} = 7.213\cdot 10^3\ \mathrm{V}\cdot\frac{1}{0.06\ \mathrm{m}} =$$

$$= 120{,}2\cdot 10^3\ \frac{\mathrm{V}}{\mathrm{m}}\ \mathrm{V}$$



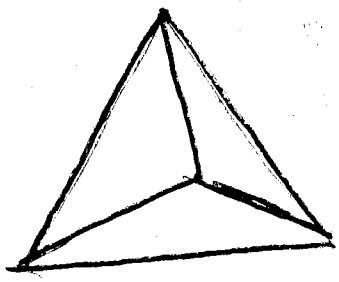
februar 1991

Fizika II

N-Ž

Izdelovi 80/81

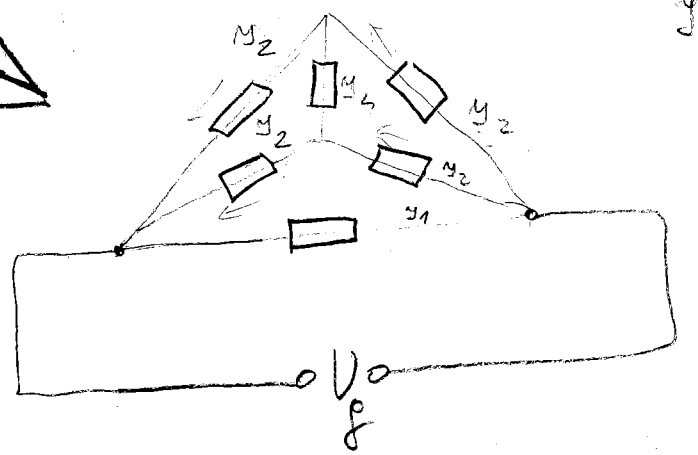
4. 6 enakih uporov R = 100 Ω je vezanih tako, da tvorijo tetraeder. Kolikšen tok teče med dvema ogljiščema tetraedra, če ~~nanju~~ med njima priključimo napetost 30 V?



$R = 100\ \Omega$

$U = 30\ V$

$y_{celotna} = ?$

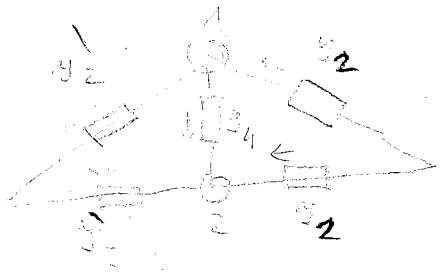


$\boxed{y_4 = 0}$

$y_1, y_2 = ?$

$$\left.\begin{aligned} U_g &= y_1 R \\ U_g &= y_2 2R \end{aligned}\right\} \Rightarrow \begin{aligned} y_1 &= U_g / R \\ y_2 &= U_g / 2R \end{aligned}$$

$$y = y_1 + 2y_2 =$$



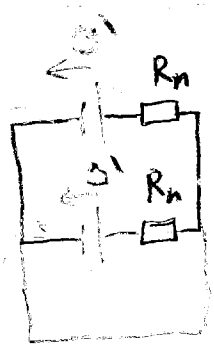
$$\text{①}: \quad y_2 = y_4 + y_2' \Rightarrow y_2 = y_2' + y_4$$

$$\text{②}: \quad y_2' = y_4 + y_2 \Rightarrow y_2 = y_2' - y_4$$

$y_4 = 0$

Kolokvij 90/91

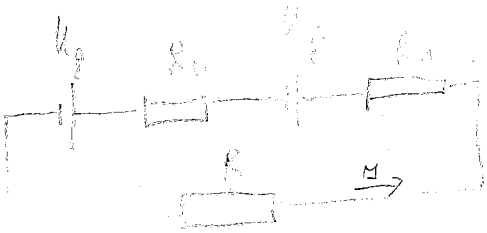
5. Če 2 vzporedno vezana akumulatorja ($U_g = 12$ V) kratko sklenemo, teče tok 80 A. Nato akumulatorja zvežemo zaporedno in obremenimo z uporom 2 Ω. Kolikšna je napetost med sponkama upora in kolikšen tok teče po njem?



$$I = 2I'$$

$$I' R_n = U_g \Rightarrow \frac{I}{2} \cdot R_n = U_g$$

$$R_n = \frac{2U_g}{I}$$



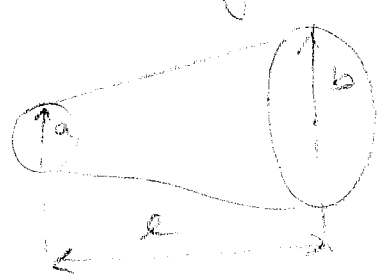
$$2U_g = I(2R_n + R) \Rightarrow I = \frac{2U_g}{(2R_n + R)}$$

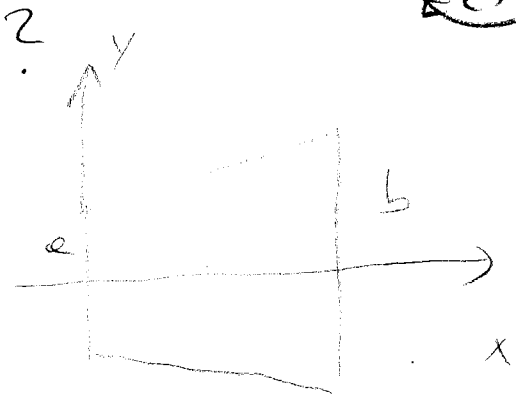
$$U_R = I \cdot R$$

Res. Hall. 609/21

E27

Kolikrát je vyšší vyšší?





$$\boxed{R = \frac{\xi \cdot l}{S}}$$

$$y = a + \frac{b-a}{l} \cdot x$$

$$S = \pi y^2$$

$$\boxed{dR = \frac{\xi \cdot dx}{\pi y^2}}$$

$$R = \int_0^l \frac{\xi \, dx}{\pi \left(a + \frac{b-a}{l} x\right)^2} = \frac{\xi}{\pi} \int_a^b \frac{du \cdot l}{(b-a)\, u^2} = \frac{\xi \cdot l}{\pi (b-a)} \int_a^b \frac{1}{u^2} \, du$$

$$a + \frac{b-a}{l} x = u$$

$$\frac{b-a}{l} dx = du \Rightarrow dx = du \cdot l / b-a$$

$$= \frac{\xi \cdot l}{\pi (b-a)} \left(-\frac{1}{u}\right)\Bigg|_a^b = \frac{\xi \cdot l}{\pi (b-a)} \left(\frac{1}{a} - \frac{1}{b}\right) =$$

$$= \frac{\xi \cdot l}{\pi (b-a)} \frac{b-a}{ab} = \underline{\underline{\frac{\xi\, l}{\pi a b}}}$$

E33

Skozi bakreno žico, katere presek je krog s premerom 1 mm, teče tok 1 A. Oceni poprečno hitrost s katero se elektroni gibljejo skozi žico, če je na 1 m^3 bakra 8.10^{28} prostih elektronov.

41

$e_0 = 1{,}6 \cdot 10^{-19}$ As

$n = 8 \cdot 10^{28} / m^3$

$2r = 1 \cdot 10^{-3}$ m

$I = 1$ A

$$\frac{1}{S}\frac{de}{dt} = \frac{I}{S} = j = \frac{(S \cdot dx \cdot n)\, e_0}{S\, dt} = n e_0 v$$

$$n = \frac{N}{V}$$

$$j = \frac{I}{S} = n e_0 \bar{v}$$

$$\bar{v} = \frac{I}{S \cdot n \cdot e_0} \cong 1 \cdot 10^{-4} \text{ m/s}$$

premer?

$ma = eE - \mu \bar{v}$

$a = 0$

$\Downarrow$

$\bar{v} = \frac{e}{\mu} E$

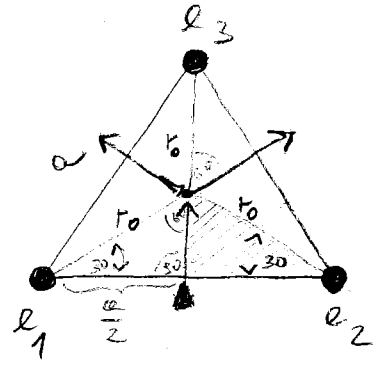
$\bar{v} = \beta E$

$S = 3{,}14 \, mm^2$

$$\frac{1 \quad 10^{19}}{\pi \cdot 10^{-6} \, 8 \cdot 10^{28} \, 1{,}6} = 2{,}486 \cdot 10^{-5} \frac{m}{s}$$

1. Na ogljiščih enakostraničnega trikotnika s stranico $a = 5$ cm so razpostavljeni 3 naboji: $e_1 = e_2 = 2 \cdot 10^{-6}$ As, $e_3 = -5 \cdot 10^{-6}$ As. Kolikšna je električna poljska jakost v težišču trikotnika? Nariši sliko!

1E3



$e_1 = e_2 = 2 \cdot 10^{-6}$ As

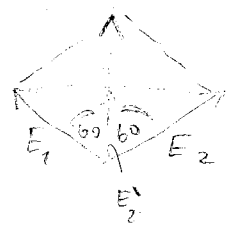
$e_3 = -5 \cdot 10^{-6}$ As

$a = 5$ cm

$$\boxed{E = \frac{e}{4\pi\varepsilon_0 r_0^2}}$$

$$\cos 30^\circ = \frac{a}{2r_0} \Rightarrow 2r_0 = \frac{a}{\cos 30^\circ} \Rightarrow \boxed{r_0 = \frac{a}{2 \cdot \cos 30^\circ}}$$

$$\vec{E} = \sum \vec{E}_i =$$



$$\cos 60 = \frac{E_2'}{E_2}$$

$$E_2' = E_2 \cdot \cos 60^\circ$$

$$\boxed{E_1 + E_2 = 2E_1 \cos 60}$$

$$E = E_3 + 2E_1 \cos 60 =$$

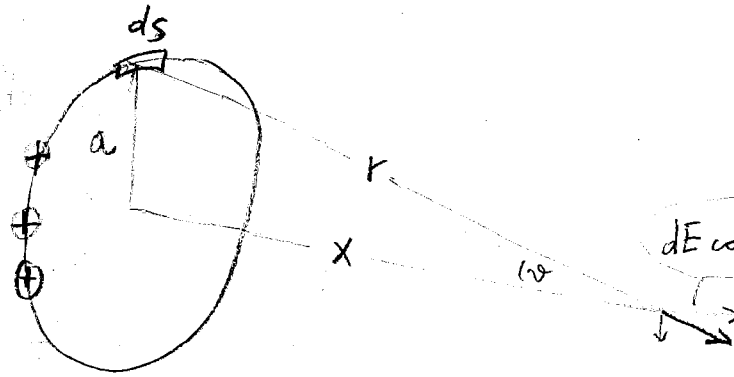
$$= \frac{|e_3|}{4\pi\varepsilon_0 r_0^2} + \frac{2e_1 \cdot \cos 60}{4\pi\varepsilon_0 r_0^2} = \frac{1}{4\pi\varepsilon_0 r_0^2}\left[|e_3| + 2e_1 \cos 60^\circ\right] \Rightarrow$$

$$\boxed{E = \frac{1 \cdot 4 \cdot \cos^2 30^\circ}{4\pi\varepsilon_0 \cdot a^2}\left[|e_3| + 2e_1 \cdot \cos 60^\circ\right]}$$

Resnick, Halliday str. 518:

E11

Tanek obroč z radijem $a = 1\,m$ ima homogeno porazdeljen naboj 10^{-7} As. Izračunaj jakost električnega polja $\vec{E}$ na osi obroča 3 m od njegovega centra?



$a = 1\,m$

$Q = 10^{-8}\,As$

$x = 3\,m$

$$dQ = Q\frac{ds}{2\pi a}$$

$$\boxed{\vec{E} = \int d\vec{E}}$$

$$dE = \frac{dQ}{4\pi\varepsilon_0 r^2}$$

$$\boxed{r^2 = x^2 + a^2}$$

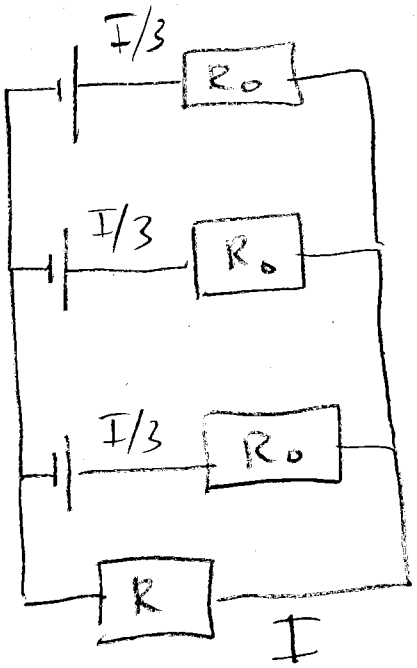
$$\boxed{dE = \frac{1}{4\pi\varepsilon_0}\cdot dQ\,\frac{1}{r^2} = \frac{1}{4\pi\varepsilon_0}(dQ)\frac{1}{x^2+a^2}}, \quad \boxed{\cos\vartheta = \frac{x}{\sqrt{a^2+x^2}}}$$

$$E = \int dE\cos\vartheta = \int \frac{1}{4\pi\varepsilon_0}\cdot dQ\,\frac{x}{(a^2+x^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{x}{(a^2+x^2)^{3/2}}\int dQ \Rightarrow$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\,\frac{Q\cdot x}{(a^2+x^2)^{3/2}}}$$

E20

1. Trije galvanski členi z gonilno napetostjo 12 V (vsak) so vzporedno priključeni na upor R = 10 Ω. Skozi upor teče tok 1 A. Kolikšna je notranja upornost posameznega galvanskega člena?



$U_0 = 12\,V$

$R = 10\,\Omega$

$I = 1\,A$

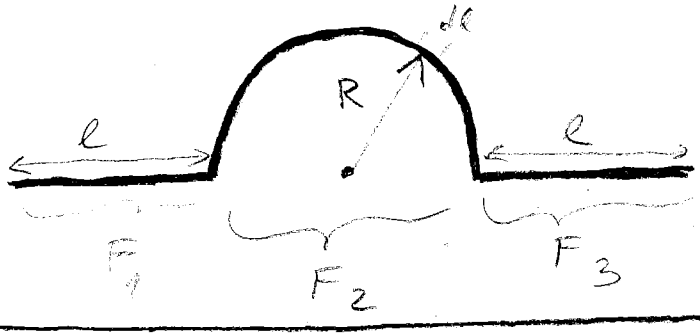
$R_0 = ?$

$$\boxed{U_0 = \frac{I}{3} \cdot R_0 + I \cdot R}$$

$$3U_0 = IR_0 + 3IR$$

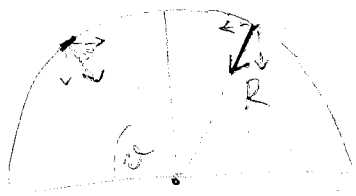
$$3(U_0 - IR) = IR_0 \Rightarrow \boxed{R_0 = \frac{3(U_0 - IR)}{I} = 6\,\Omega}$$

Kakšna sila deluje na žico prikazane oblike, če teče po njej tok I, magnetno polje B pa je pravokotno na ravnino žice?



$$\boxed{\vec{F} = I \int d\vec{l} \times \vec{B}}$$

$$\boxed{F_1 = F_3 = I l B}$$



$$dF = I\,dl\,B = I B R\,d\vartheta$$

$$F_2 = \int_0^{\pi} dF \sin\vartheta = \int_0^{\pi} I B R \sin\vartheta\, d\vartheta = I B R\,(-\cos\vartheta)\Big|_0^{\pi} = 2 I B R$$

$$F = F_1 + F_2 + F_3 = 2 I l B + 2 I B R = 2 I B (l + R)$$

izpit, 1991

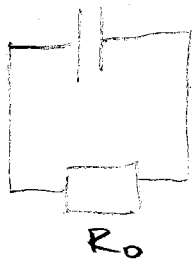
2. Kondenzator s kapaciteto 25 μF nabijemo na napetost 60 V. Zaradi slabe izolacije pade napetost na kondenzatorju po 10 milisekundah na 50 V. Do katere napetosti se bo izpraznil kondenzator v istem času, če priključimo nanj upor 500 ohmov?

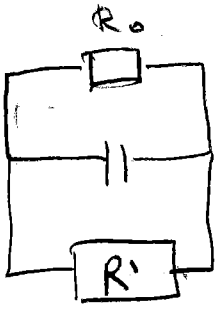
$C = 25\,\mu F$

$U_0 = 60\,V$

$U(t = 10\,ms) = 50\,V$

$R' = 500\,\Omega$





$$\boxed{U = U_0 e^{-t/R_0C}}$$

$$\ln\frac{U}{U_0} = -\frac{t}{R_0C}, \quad \ln\frac{U_0}{U} = \frac{t}{R_0C}$$

$$R_0 = \frac{1}{C}\cdot\frac{t}{\ln\frac{U_0}{U}} = \underline{2194\ \Omega}$$

$$\frac{1}{R} = \frac{1}{R_0} + \frac{1}{R'} \Rightarrow R =$$

$$U = U_0 e^{-t/RC} =$$

$t = 10$ ms

$U_0 = 60$ V

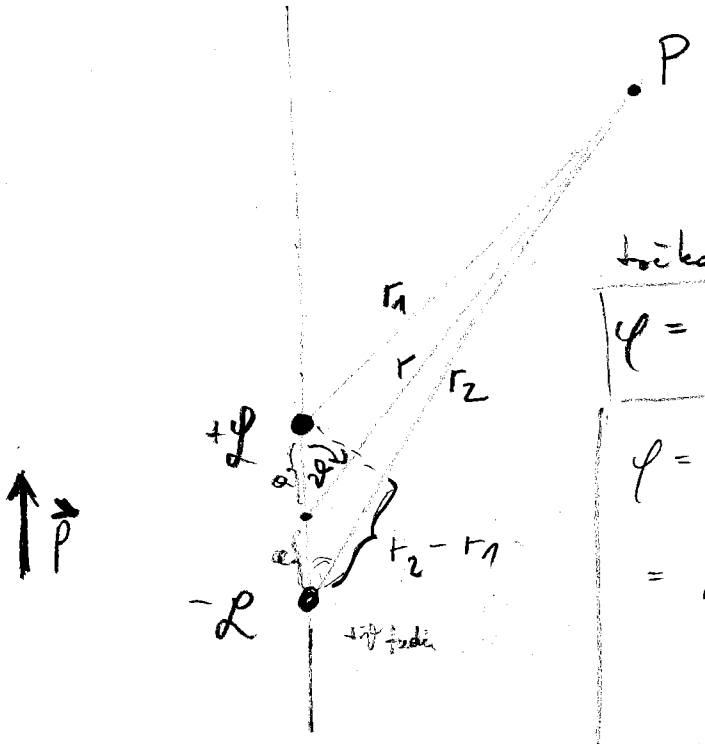
$R =$

$C = 25\,\mu F$

Res. Hell. Ar. 630

E15

Električni dipol se sestoji iz dveh nasprotno enakih nabojev ($q = 2\cdot 10^{-6}$ As), ki sta oddaljena 1 cm. Izračunaj potencial dipola 2 m nad dipolom v osi dipola in 3 m od dipola v smeri pravokotno na os dipola?



$q = 2\cdot 10^{-6}$ As

$2a = 2$ cm

$r_0 = 3$ m, $\vartheta = 0°$

$r_0 = 3$ m, $\vartheta = 90°$

točkasti naboj:

$$\boxed{\varphi = \frac{q}{4\pi\varepsilon_0 r}}$$

$$\varphi = \varphi_1 + \varphi_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right] =$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{2a\cos\vartheta}{r^2}$$

DIPOLNI MOMENT $p = q\cdot 2a$

$$\Downarrow$$

$$\boxed{\varphi = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cdot\cos\vartheta}{r^2}}$$

$$\left\{\begin{array}{l} \underline{\text{približki za } r \gg a} \\ r_2 - r_1 = 2a\cos\vartheta \\ r_1 r_2 = r^2 \end{array}\right\}$$

$\vartheta = 0°$: $\varphi = \frac{1}{4\pi\varepsilon_0}\frac{p}{r_0^2}$

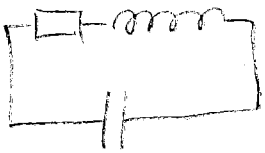
$\vartheta = 90°$: $\varphi = 0$

Str/82/3 $\boxed{e^{iz} = \cos z + i \sin z}$ ← iz rozvoja za $\cos x$, e^x

E59

Nihajni krog sestavljata kondenzator z $0{,}5\,\mu F$ in tuljava z induktivnostjo $0{,}001\ Vs/A$ in uporom $1.5\,\Omega$. Izračunajte nihajni čas in koeficient dušenja tega kroga? (NI GENERATORJA)

$C = 0{,}5\,\mu F$, $L = 0{,}001\ Vs/A$, $R = 1{,}5\,\Omega$



$$\boxed{U_L + U_R + U_C = -L\frac{dy}{dt} - Ry - \frac{e}{C} = 0} \quad d/dt$$

$$\boxed{\frac{d^2y}{dt^2} + \frac{R}{L}\frac{dy}{dt} + \frac{1}{LC}y = 0} \qquad \boxed{\ddot{y} + \frac{R}{L}\dot{y} + \omega_0^2 y = 0}$$

$$\boxed{\omega_0 = (LC)^{-1/2}}$$

$y = y_0 \cos(\omega_0 t)$
$\dot{y} = -y_0\omega_0 \sin(\omega_0 t)$
$\ddot{y} = -\omega_0^2 y_0 \cos(\omega_0 t)$
$\ddot{y} = -\omega_0^2 y$

nenadeni nih. krog

nastavek:

$$\boxed{y = y_0 e^{-\beta t}\cos(\omega_0' t)} \quad \text{ali} \quad \boxed{y = y_0 e^{-\beta t} e^{i\omega_0' t}}$$

$$\dot{y} = -y_0\beta e^{-\beta t} e^{i\omega_0' t} + y_0 e^{-\beta t} i\omega_0' e^{i\omega_0' t}$$

$$\ddot{y} = y_0\beta^2 e^{-\beta t} e^{i\omega_0' t} - y_0\beta e^{-\beta t} i\omega_0' e^{i\omega_0' t} - y_0\beta e^{-\beta t} i\omega_0' e^{i\omega_0' t} - y_0 e^{-\beta t}\omega_0'^2 e^{i\omega_0' t}$$

torej:

$$y_0\beta^2 e^{-\beta t} e^{i\omega_0' t} - y_0 e^{-\beta t}\omega_0'^2 e^{i\omega_0' t} - 2y_0\beta i\omega_0' e^{-\beta t} e^{i\omega_0' t}$$

$$+ \frac{R}{L}\left(-y_0\beta e^{-\beta t} e^{i\omega_0' t} + y_0 e^{-\beta t} i\omega_0' e^{i\omega_0' t}\right) + \frac{1}{LC} y_0 e^{-\beta t} e^{i\omega_0' t} = 0$$

upoštevaš: $y(t) = y_0 e^{-\beta t} e^{i\omega_0' t}$

$$\left\{\beta^2 - \omega_0'^2 - \beta\frac{R}{L} + \underbrace{i\omega_0'\left[-2\beta + \frac{R}{L}\right]}_{\text{imaginarni del}} + \overbrace{\frac{1}{LC}}^{\omega_0^2}\right\} y(t) = 0$$

v splošnem ni nič

$$\Downarrow$$

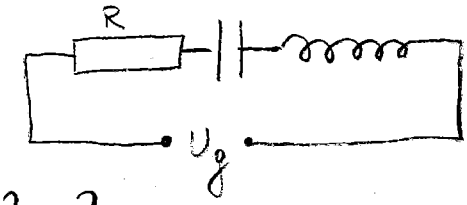
$$\boxed{\beta^2 - \omega_0'^2 + \omega_0^2 - \overbrace{\beta\frac{R}{L}}^{-2\beta^2} + i\omega_0'\left[-2\beta + \frac{R}{L}\right] = 0}$$

imag. del enak nič: $-2\beta + \frac{R}{L} = 0 \Rightarrow \boxed{\beta = \frac{1}{2}\frac{R}{L}}$

realni del enak nič: $-\beta^2 - \omega_0'^2 + \omega_0^2 = 0 \Rightarrow \boxed{\omega_0' = \sqrt{\omega_0^2 - \beta^2}}$

$\boxed{\text{če } \beta \geq \omega_0^2} \Rightarrow \boxed{\omega_0' = i\alpha}$

Zgledek: Strnad str. 333 438, 145



$\boxed{\text{vsiljeno nihanje}: \; U_g = U_0 e^{i\omega t}}$

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člen zaradi lastnega nihanja pojema kot $e^{-\beta t}$ in izgine po zadosti dolgem času

$\boxed{e^{i\omega t} = \cos\omega t + i \sin\omega t}$

$\{ J_0, \delta \}$?

$$\boxed{U_g + U_C + U_L + U_R = 0}$$

$$U_g - \frac{e}{C} - L\dot{J} - JR = 0 \;\Big/ \frac{d}{dt} \Rightarrow$$

$$\Rightarrow \boxed{\dot{U}_g - \frac{J}{C} - L\ddot{J} - R\dot{J} = 0}$$

nastavek: $\boxed{J = J_0 e^{i\omega t}}$

vsiljeno nihanje: $\boxed{U_g = U_0 e^{i\omega t}}$

$$i\omega U_0 e^{i\omega t} - \frac{1}{C} J_0 e^{i\omega t} + L\omega^2 J_0 e^{i\omega t} - Ri\omega J_0 e^{i\omega t} = 0 \Rightarrow$$

$$\boxed{i\omega U_0 - \frac{1}{C} J_0 + L\omega^2 J_0 - Ri\omega J_0 = 0} \Rightarrow$$

$$\boxed{U_0 = \left(-\frac{\omega L}{i} + R + \frac{1}{i\omega C}\right) J_0 = \left(R + i\omega L - i\frac{1}{\omega C}\right) J_0}$$

IMPENDANCE:

$\boxed{Z_R = R}$

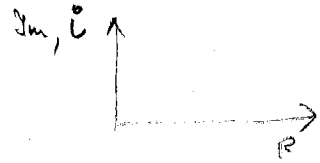
$\boxed{Z_L = i\omega L}$

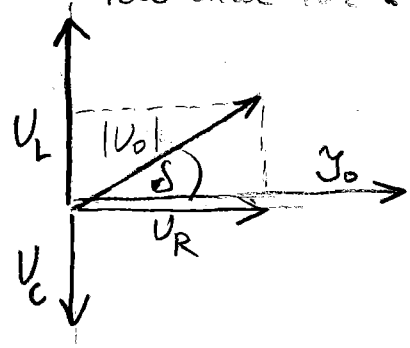
$\boxed{Z_C = -i\frac{1}{\omega C}}$

Skozi upornik, tuljavo in kondenzator, ki so vezani zaporedno, teče enak tok:

J_0 je realen

U_0 je kompleksen





$$\Rightarrow |U_0|^2 = U_0 \cdot U_0^* = J_0^2 \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

$$\Downarrow$$

$$\boxed{J_0 = \frac{|U_0|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

Če J realen:

$\Rightarrow (J_0)_{max}$, če $\omega L - \frac{1}{\omega C} = 0$

torej $\boxed{\omega^2 = \frac{1}{LC} = \omega_0^2}$

$$\Rightarrow U_0 = J_0 \left[R + i\left(\omega L - \frac{1}{\omega C}\right)\right] \Rightarrow \boxed{\operatorname{tg}\delta = \left(\omega L - \frac{1}{\omega C}\right)/R}$$

fazni premik gonilne napetosti glede na tok

Nihajni krog rabi moč in jo oddaja pri T=konst. kot Joulov toplotni tok

$$\bar{P} = \frac{1}{2} J U_0 \cos\delta = \frac{1}{2} J_0 U_R = \frac{1}{2} J_0 R J_0 = \frac{1}{2} R J^2$$

(seveda kroge zanemarimo ker $P \propto \omega^4$)

p. test

2. Dva izvora EM valovanja [redacted] antenni na konicah dveh enako visokih stolpov sevata z močmi 4 kW in 1 kW. Koliksna je izotropna gostota energijskega toka v točki, ki je 3 km oddaljena od prvega izvora in 5 km od drugega, če sta v tej točki vala obeh izvorov vedno premaknjena v fazi za $\pi/4$? Opazovalna točka je na isti visini kot oba izvora.

$\delta = \frac{\pi}{4}$

$$j = \frac{wV}{St} = \frac{wSx}{St} = w \cdot c$$

$$w = \varepsilon_0 E^2$$

Str (K) / 476 447

$P_1 = 4\ kW$

$P_2 = 1\ kW$

$r_1 = 3000\ m$

$r_2 = 5000\ m$

$\delta = \pi/4$

$$w = w_e + w_m = 2w_m = 2w_e = 2 \cdot \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E^2 \Rightarrow$$

$$\overline{w} = 2 \cdot \frac{1}{2}\varepsilon_0 E_0^2 \underbrace{\cos^2(\omega t - kx)}_{\frac{1}{2}\ \text{povprečje}} = \frac{1}{2}\varepsilon_0 E_0^2$$

$$j_1 = \frac{1}{2}\varepsilon_0 E_{01}^2 c_0 = \frac{P_1}{4\pi r_1^2} \Rightarrow E_{01} = \sqrt{\frac{P_1}{4\pi r_1^2} \cdot \frac{2}{\varepsilon_0 c_0}} = \sqrt{\frac{P_1}{2\pi \varepsilon_0 c_0 r_1^2}}$$

$$E_{01} = \sqrt{\frac{4 \cdot 10^3}{2 \cdot \pi \cdot 8{,}85 \cdot 10^{-12} \cdot 3 \cdot 10^8 \cdot 9 \cdot 10^6}} = 0{,}163\ \frac{V}{m}$$

$$E_{02} = \sqrt{\frac{10^3}{2\pi \cdot 8{,}85 \cdot 10^{-12} \cdot 3 \cdot 10^8 \cdot 25 \cdot 10^6}} = 0{,}048\ \frac{V}{m}$$

$j_1 = 35{,}3 \cdot 10^{-6}\ \frac{\mu W}{m^2}$

$j_2 = 3{,}18 \cdot 10^{-6}\ \frac{\mu W}{m^2}$

$j_1 + j_2 = 38{,}45 \cdot 10^{-6}\ \frac{W}{m^2}$

$$w = \varepsilon_0 E^2 = \varepsilon_0 \left[E_{01}\cos\left(\varphi - \frac{\delta}{2}\right) + E_{02}\cos\left(\varphi + \frac{\delta}{2}\right)\right]^2 =$$

$$= \varepsilon_0 \left[E_{01}^2 \cos^2\left(\varphi - \frac{\delta}{2}\right) + E_{02}^2 \cos^2\left(\varphi + \frac{\delta}{2}\right) + 2E_{01}E_{02}\cos\left(\varphi - \frac{\delta}{2}\right)\cos\left(\varphi + \frac{\delta}{2}\right)\right] =$$

$$\cos\left(\varphi - \frac{\delta}{2}\right)\cos\left(\varphi + \frac{\delta}{2}\right) = \left[\cos\varphi\cos\frac{\delta}{2} + \sin\varphi\sin\frac{\delta}{2}\right]\left[\cos\varphi\cos\frac{\delta}{2} - \sin\varphi\sin\frac{\delta}{2}\right] =$$

$$= \cos^2\varphi\cos^2\frac{\delta}{2} - \sin^2\varphi\sin^2\frac{\delta}{2}$$

$$= \varepsilon_0 \left[E_{01}^2 \cos^2\left(\varphi - \frac{\delta}{2}\right) + E_{02}^2 \cos^2\left(\varphi + \frac{\delta}{2}\right) + 2E_{01}E_{02}\left(\cos^2\varphi\cos^2\frac{\delta}{2} - \sin^2\varphi\sin^2\frac{\delta}{2}\right)\right] \Rightarrow$$

$$\overline{w} = \varepsilon_0 \left[E_{01}^2 \cdot \frac{1}{2} + E_{02}^2 \cdot \frac{1}{2} + E_{01}E_{02}\left(\cos^2\frac{\delta}{2} - \sin^2\frac{\delta}{2}\right)\right] = \varepsilon_0 \left[\frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + E_{01} \cdot E_{02}\cos\delta\right]$$

$$\overline{j} = \overline{w} \cdot c = \varepsilon_0 c \left(\frac{E_{01}^2}{\ } + \frac{E_{02}^2}{\ }\right) \quad 38{,}45\ \frac{\mu W}{\ }$$

4. Tanek kovinski obroč radija 10 cm se nahaja v homogenem magnetnem polju (B = 2,5 T), katerega smer je pravokotna na ravnino obroča. Na obroč je postavljena tanka kovinska palica, ki se giblje po ravnini obroča s konstantno hitrostjo 0,2 m/s proti središču obroča tako, da ves čas tvori njegovo tetivo. Kolikšna napetost se inducira čez 0,5 sekunde po začetku gibanja palice v zanki, ki jo tvori palica in obroč ? Palica na začetku tvori tangento na obroč ?

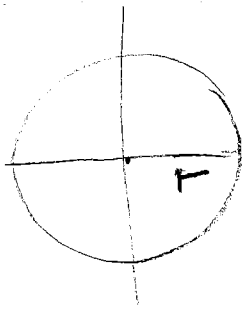
A - L

$r = 0{,}1\ m$

$B = 2{,}5\ T$

$v = 0{,}2\ m/s$

$t = 0{,}5\ s$



$s = vt = 0{,}1\ m$

$U_i = Blv = B2rv = 0{,}1\ V$

E55

Str. (Hrg) 35: Pravokotna tuljava s presekom $S = 100\ cm^2$ in z $N = 100$ ovoji se vrti s konstantno kotno hitrostjo $\omega = 30\ s^{-1}$ okoli osi, ki je pravokotna na geometrijsko os tuljave in na smer magnetnega polja $B = 0.1\ T$. Kolikšna napetost se inducira v tuljavi?

$S = 100\ cm^2$

$N = 100$

$\omega = 30\ s^{-1}$

$B = 0.1\ T$

$U_i = ?$

$$\boxed{\Phi = NBS\cos\varphi}$$

$$\boxed{\varphi = \omega t}$$

$$\Phi = NBS\cos\omega t$$

$$U_i = \frac{d\Phi}{dt} = \underbrace{-NBS\omega}\sin\omega t$$

$$U_{ef} = NBS\omega \cdot \frac{1}{\sqrt{2}}$$

M-Č

1995

4. Električna kuhalna plošča ima dve spirali. Če je vključena ena, voda zavre v 10 minutah, če je vklučena druga, pa zavre voda v 20 minutah. V kolikšnem času zavre voda, če sta vključeni obe spirali: (a) zaporedno , (b) vzporedno .
Začetna temperatura, količina vode, izkoristek ter napetost so enake v vseh primerih.

$$P = I \cdot U = \frac{U^2}{R}$$

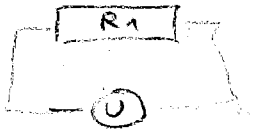
$U = 220\,V$

$t_1 = 10$ minut

$t_2 = 30$ minut

$$P \cdot t = m c_p \Delta T \Rightarrow \boxed{\frac{U^2}{R} \cdot t = m \cdot c_p \cdot \Delta T}$$

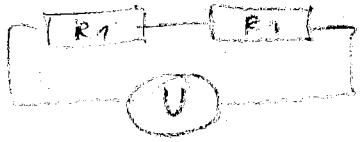
VSAK POSEBEJ:



$$\eta \cdot \frac{U^2}{R_1} t_1 = m c_p \Delta T \qquad \eta \frac{U^2}{R_2} t_2 = m c_p \Delta T$$

$$\Rightarrow R_1 = \frac{\eta U^2 t_1}{m c_p \Delta T} \qquad \Rightarrow R_2 = \frac{\eta U^2 t_2}{m c_p \Delta T}$$

a)



$$R_a = R_1 + R_2 \qquad \boxed{\eta \frac{U^2}{R_a} \cdot t_a = m c_p \Delta T} \Rightarrow$$

$$t_a = \frac{m c_p \Delta T}{\eta U^2} \cdot R_a = \frac{m c_p \Delta T}{\eta \cdot U^2}(R_1 + R_2) = \frac{m c_p \Delta T}{\eta U^2}\left(\frac{\eta U^2 t_1}{m c_p \Delta T} + \frac{\eta U^2 t_2}{m c_p \Delta T}\right) =$$

$$= t_1 + t_2 = 30 \text{ minut}$$

b)

$$\frac{1}{R_b} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \boxed{\eta \frac{U^2}{R_b} \cdot t_b = m c_p \Delta T} \Rightarrow$$

$$R_b = R_1 R_2 / (R_1 + R_2)$$

$$\eta \frac{U^2}{t_b}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = m c_p \Delta T \Rightarrow$$

$$\eta U^2 t_b \left(\frac{m c_p \Delta T}{\eta U^2 t_1} + \frac{m c_p \Delta T}{\eta U^2 t_2}\right) = m c_p \Delta T$$

$$t_b \left(\frac{1}{t_1} + \frac{1}{t_2}\right) = 1 \Rightarrow \boxed{t_b = \frac{t_1 t_2}{t_1 + t_2} = 6.7 \text{ min}}$$

4. V akvarij nalijemo 60 litrov vode s temperaturo 15°C in jo z ~~1~~ V OMREŽJE PRIKLJUČENIM A-L električnim grelcem v 12 urah segrejemo na 24°C (izgube zaradi TOPL. TOKA V OKOLICO ~~prevajanja toplo~~te zanemarimo). Kolikšen efektivni tok teče skozi grelec ? Hkrati si želimo skuhati še kavo. Električni kuhalnik (2 kW) z razdelilcem priključimo v isto vtičnico. Ali bo 10 A varovalka vzdržala ? (odgovor argumentirajte z ustreznimi izračuni)

1PQS

omrežje: $P = \frac{1}{2} U_0 I_0 = U_{ef} \cdot I_{ef}$, $\boxed{U_{ef} = \frac{U_0}{\sqrt{2}} = 220\,V}$, $I_{ef} = \frac{I_0}{\sqrt{2}}$

$m = 60\,kg$

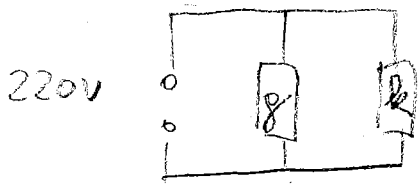
$T_z = 15\,°C$, $T_k = 24\,°C$, $\Delta T = T_k - T_z$

$t = 12\,h$

a) $m c_p \Delta T = P \cdot t = I_{gef} \cdot U_{ef} \cdot t$

$$\underline{I_{gef}} = \frac{m c_p \Delta T}{U_{ef} \cdot t} = \underline{0{,}24\,A}$$

b)



$\boxed{P = I_{ef} \cdot U_{ef}}$, $\boxed{U_{ef} = 220\,V}$

$$\underline{I_{ef}} = I_{1ef} + I_{2ef} = \frac{P_g}{U_{ef}} + \frac{P_k}{U_{ef}} = \underline{I_{gef} + \frac{P_k}{U_{ef}}} = \underline{9{,}33\,A}$$

$U = I_g R_g$, $U = I_k R_k$

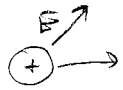
$I = I_g + I_k$

$I = U\left(\frac{1}{R_g} + \frac{1}{R_k}\right)$

$\frac{1}{R} = \frac{1}{R_g} + \frac{1}{R_k}$

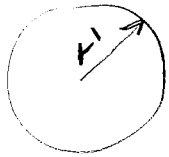
$I_{ef} = \frac{I_0}{\sqrt{2}} \Rightarrow \underline{I_0} = I_{ef} \cdot \sqrt{2} = \underline{13.2\,A}$

NAL. Nariši potek potenciala in jakosti električnega polja zunaj in znotraj krogle z enakomerno porazdeljenim fiksiranim nabojem (a) po površini krogle (b) in po prostornini. Naboj krogle je 10^{-10} As, radij krogle pa je 1 mm. (E9)



$D = \varepsilon_0 E$

a)



$\boxed{\int D\,dS = e}$

$r' = 10^{-3}$ m

$e = 10^{-10}$ As

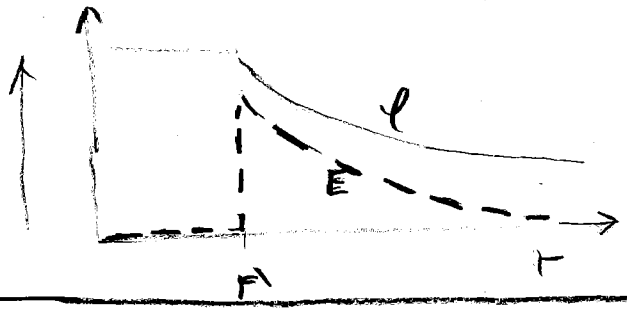
$\varepsilon = 1$ povsod

$\varepsilon_0 E\, 4\pi r^2 = e \Rightarrow \boxed{E = \frac{e}{4\pi\varepsilon_0 r^2}}$ (1)

$\boxed{E = -\operatorname{grad}\varphi} \Rightarrow \boxed{d\varphi = -E\,dr}$ $\quad \varphi\Big|_{\infty}^{r} = -\int_{\infty}^{r} E\,dr = -\int_{\infty}^{r} \frac{e}{4\pi\varepsilon_0 r^2}\,dr$

$\varphi(r) - \varphi(\infty) = \frac{e}{4\pi\varepsilon_0 r}\Big|_{\infty}^{r} = +\frac{e}{4\pi\varepsilon_0 r} \Rightarrow$ ie $\varphi(\infty) = 0$

$\boxed{\varphi(r) = \frac{e}{4\pi\varepsilon_0 r}}$ (2)



notranjost $\int D\,dS = 0 \Rightarrow D = \varepsilon_0 E = 0$

potencial znižan

b) $\varepsilon = 1$, $r > r'$: enako kot prej $\quad \rho\frac{4\pi r'^3}{3} = e \Rightarrow \boxed{\rho = \frac{3e}{4\pi r'^3}}$

$\varepsilon = 1$, $r \leq r'$: $\boxed{\int D\,dS = \rho\frac{4\pi r^3}{3}} \Rightarrow \boxed{\varepsilon_0 E\, 4\pi r^2 = \rho\frac{4\pi r^3}{3}}$

$\boxed{E = \left(\frac{e}{4\pi\varepsilon_0 r'^3}\right)\cdot r}$ (3)

$d\varphi = -\int_{r}^{r'} E\,dr$

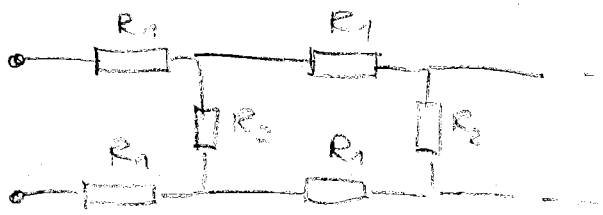
$\varphi\Big|_{r}^{r'} = -\int_{r}^{r'} \frac{e}{4\pi\varepsilon_0 r'^3}\, r\,dr \Rightarrow \varphi(r') - \varphi(r) = -\frac{e}{4\pi\varepsilon_0 r'^3}\frac{r^2}{2}\Big|_{r}^{r'} = -\frac{e}{8\pi\varepsilon_0 r'^3}\left(\frac{r'^2}{2} - \frac{r^2}{2}\right)$

$\boxed{\varphi(r) = \varphi(r') - \left(\frac{e}{8\pi\varepsilon_0 r'^3}\right) r^2 + \frac{e}{8\pi\varepsilon_0 r'^3}\cdot r'^2 = -\left(\frac{e}{8\pi\varepsilon_0 r'^3}\right) r^2 + C}$ (4)

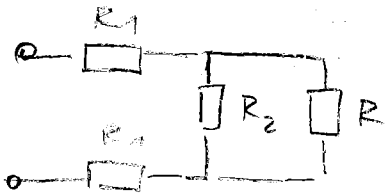
potencial v $r = r'$ znižan $\Rightarrow -\left(\frac{e}{8\pi\varepsilon_0 r'^3}\right) r'^2 + C = \frac{e}{4\pi\varepsilon_0 r'} \Rightarrow C = \left(\frac{e}{8\pi\varepsilon_0 r'^3}\right)\cdot 3r'^2 \Rightarrow$

Kl./67/vd.108

Izračunaj nadomestni upor R neskončno dolge verige uporniko~~v~~ $R_1 = 10\,\Omega$ in $R_2 = 20\,\Omega$, ki so vezani kot kaže slika.



originalna



nadomestni upor

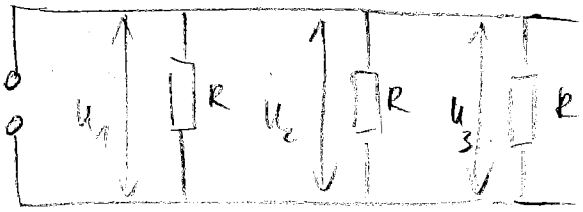
$$\Downarrow$$

$$\boxed{R = 2R_1 + \frac{R_2 R}{(R + R_2)}}$$

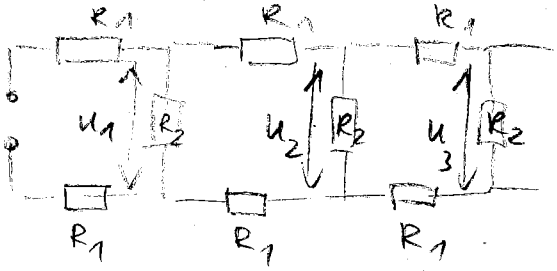
$$\Downarrow$$

$$R^2 - 2R_1 R - 2R_1 R_2 = 0 \quad \text{(kvadratna enačba)}$$

$$\boxed{R = \frac{2R_1 + \sqrt{4R_1^2 + 8R_1 R_2}}{2}}$$



$U_1 = U_2 = U_3 =$



$U_1 \neq U_2 \neq U_3 =$

kar pade napetosti na uporih R_1

(dec.) 1992

6. Po eni izmed osnovnih ploskev dielektričnega valja z radijem 6 cm je enakomerno prazdeljen naboj 10^{-5} As. Valj se enakomerno vrti okoli geometrijske osi s frekvenco 100 s^{-1}. Kolikšna je gostota magnetnega polja na nabiti ploskvi ob osi valja ?

Str 73/3

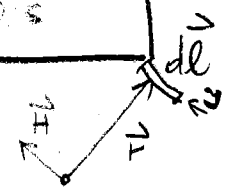
$r_0 = 0.05$ m
$e_T = 10^{-5}$ As
$\nu = 80\ s^{-1}$

okrogla žička:

Biot - Savart

$$\vec{B} = \mu_0 \vec{H}$$

$$\boxed{\vec{H} = \frac{I}{4\pi}\int \vec{r} \times d\vec{l}/r^3}$$



$$\rightarrow H = \frac{I}{4\pi}\frac{r\,2\pi r}{r^3} = \frac{I}{2}\frac{1}{r} = \frac{I}{2r} \Rightarrow \boxed{H = \frac{I}{2r}}$$

VALJ

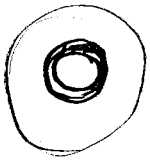
$$\sigma = \frac{de}{dS} \rightarrow de = \sigma \cdot 2\pi r\, dr \qquad \sigma = \frac{e_T}{\pi r_0^2}$$

$$\boxed{dJ = \frac{de}{t_0} = \frac{\sigma 2\pi r\, dr}{t_0} = \nu \sigma 2\pi r\, dr}$$

$$B = \mu_0 H = \mu_0 \int_0^{r_0} \frac{dJ}{2r} = \mu_0 \int_0^{r_0} \frac{\nu \sigma 2\pi r\, dr}{2r} = \mu_0 \nu \pi \sigma r_0 = \mu_0 \nu \pi \frac{e_T}{\pi r_0^2} r_0 =$$

$$= \frac{\mu \nu e_T}{r_0}$$

$$\boxed{dJ = \frac{de}{t_0}}$$



$$\boxed{B = \mu_0 \cdot \nu \cdot e_T / r_0}$$

$$\underline{\underline{B}} = \frac{4\pi \cdot 10^{-7} \cdot 100 \cdot 10^{-5}}{6 \cdot 10^{-2}} = \frac{4\pi \cdot 100 \cdot 10^{-12}}{6 \cdot 10^{-2}} = \frac{4\pi \cdot 100}{6} \cdot 10^{-10} =$$

$$\underline{\underline{= 2{,}09 \cdot 10^{-8}\ T}}$$

3. Kvadratna žičnata zanka z obsegom l= 20 cm in skupnim uporom R = 10 Ω ter maso m = 0,1 kg pada v prostoru tako, da je normala na ravnino zanke vzporedna s horizontalno ravnino. V prostoru padanja zanke obstaja magnetno polje z gostoto B= 0,1 T. Smer F2 magnetnega polja je pravokotna na ravnino zanke. Kolikšna je hitrost zanke v stacionarnem stanju? Sila upora zraka je sorazmerna kvadratu hitrosti zanke, sorazmernostna konstanta je 0,01 $N \cdot s^2/m^2$.

$k = 0{,}01 \ Ns^2/m^2$

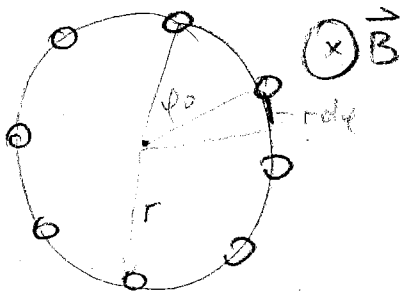
$m = 0{,}1 \ kg$

$$mg = kv^2$$

$$v = \sqrt{\frac{mg}{k}} = \sqrt{\frac{0{,}1 \cdot 10}{0{,}01}} = \underline{\underline{10 \ m/s}}$$

3. 10 nabitih kroglic z naboji po $+10^{-3}$ As je pritrjenih v enakih razmikih po obodu plošče z radijem 10 cm. Plošča je vrtljiva okoli geometrijske osi in ima skupni vztrajnostni moment 10^{-4} kg·m^2. Ploščo damo v homogeno magnetno polje z gostoto 0,5 T, tako da je smer magnetnega polja vzporedna z geometrijsko osjo. S kolikšno kotno hitrostjo se začne vrteti plošča, ko izključimo magnetno polje ?

1993



$e = +10^{-3}$ As, $r = 10$ cm

$\varphi_0 = 2\pi/10$, $N = 10$, $J = 10^{-4}$ kg m^2

$B = 0.5$ T

$v = ?$

$\boxed{F_e = eE}$

$M = NrF_e = NreE$

$\boxed{\int M\,dt = J\omega}$

$\Downarrow$

$\boxed{\int rNeE\,dt = J\omega}$

$\phi_m = \oint \vec{B}\,d\vec{S}$

$-\int \vec{E}\,d\vec{s} = \frac{d\phi_m}{dt}$

$\boxed{-E2\pi r = \frac{d\phi_m}{dt}}$

$\Downarrow$

$\int E2\pi r\,dt = -\int d\phi_m = -\Delta\phi_m$

$\boxed{-\Delta\phi_m = \phi_{m,zač} = B\pi r^2}$

$\Downarrow$

$\int E2\pi r\,dt = B\pi r^2$

$2\pi r\int E\,dt = B\pi r^2$

$\boxed{\int E\,dt = \frac{B\cdot r}{2}}$

$rNe\int E\,dt = J\omega$

$\Downarrow$

$rNe\cdot\frac{B\cdot r}{2} = J\omega$

$\Downarrow$

$\boxed{\omega = \frac{NeBr^2}{2J} = 0.25\ s^{-1}}$

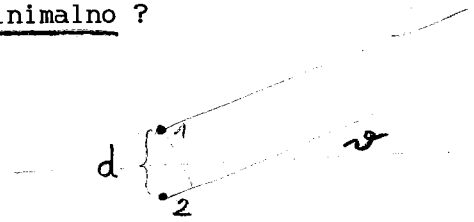
Max. enačb $\oint \vec{E}\,d\vec{s} = -\oint\left(\frac{\partial\vec{B}}{\partial t}\right)d\vec{S} = -\frac{\partial}{\partial t}\oint\vec{B}\,d\vec{S}$

5. Dve ~~antene~~ izvora EM valovanja, ki sta oddaljeni 100 m, v fazi oddajata ~~radijske~~ EM valove z valovno dolžino $\lambda = 200$ m. Intenziteti emisije obeh ~~anten~~ izvorov sta enaki v horizontalni smeri. V kateri smeri (v horizontalni ravnini) glede na simetralo med obema ~~antenama~~ izvoroma je skupno žarčenje obeh ~~anten~~ izvorov maksimalno in v kateri smeri minimalno ? 24.9.

1992

$d = 100$ m

$\lambda = 200$ m



$j = w \cdot c$

$w = w_e + w_m = \varepsilon_0 E^2$

$\bar{w} = \frac{1}{2}\varepsilon_0 E_0^2$

$$w_{12} = \varepsilon_0 E^2 = \varepsilon_0 \left[E_0 \cos\left(\varphi - \frac{\delta}{2}\right) + E_0 \cos\left(\varphi + \frac{\delta}{2}\right)\right]^2 =$$

$$= \varepsilon_0 E_0^2 \left[\cos^2\left(\varphi - \frac{\delta}{2}\right) + \cos^2\left(\varphi + \frac{\delta}{2}\right) + 2\underbrace{\cos\left(\varphi - \frac{\delta}{2}\right)\cos\left(\varphi + \frac{\delta}{2}\right)}_{\cos\delta}\right]$$

$$\left(\cos\varphi \cos\frac{\delta}{2} + \sin\varphi \sin\frac{\delta}{2}\right)\left(\cos\varphi \cos\frac{\delta}{2} - \sin\varphi \sin\frac{\delta}{2}\right) =$$

$$= \cos^2\varphi \cos^2\frac{\delta}{2} - \sin^2\varphi \sin^2\frac{\delta}{2}$$

$$\bar{w}_{12} = \varepsilon_0 E_0^2 \left[1 + \left(\cos^2\frac{\delta}{2} - \sin^2\frac{\delta}{2}\right)\right] = \varepsilon_0 E_0^2 (1 + \cos\delta)$$

$$\boxed{j_{12} = \bar{w}_{12} \cdot c = \varepsilon_0 E_0^2 c (1 + \cos\delta) = 2 \cdot j \cdot (1 + \cos\delta)}$$

$$\Delta x = d \cdot \sin\varphi \Rightarrow \delta = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi d \sin\vartheta}{\lambda} = \pi \cdot \sin\vartheta$$

$$\boxed{\delta = \pi \cdot \sin\vartheta}$$

$$\boxed{j_{12} = 2 \cdot j \cdot \left[1 + \cos(\pi \cdot \sin\vartheta)\right]}$$

ekstrem:

$$\frac{dj_{12}}{d\vartheta} = -2j \sin(\pi \cdot \sin\vartheta)\pi \cos\vartheta = 0$$

$$\sin(\pi \cdot \sin\vartheta)\cos\vartheta = 0 \Rightarrow \boxed{\begin{array}{ll} \vartheta = 0 & \text{(MAKSIMUM)} \\ \vartheta = \frac{\pi}{2} & \text{(MINIMUM)} \end{array}}$$

3. Magnetna igla niha v zemeljskem magnetnem polju s frekvenco $0{,}7\ s^{-1}$. Če damo iglo 10 cm nad žico, ki je napeta v smeri vzhod-zahod in po kateri teče tok od vzhoda proti zahodu, začne nihati s frekvenco $7\ s^{-1}$. Kolikšen tok teče po žici ? Vodoravna komponenta zemeljskega magnetnega polja ima gostoto $2{,}1\cdot 10^{-4}$ T.

M-8

1994

žica: $\int \vec{H}_2 d\vec{s} = I \Rightarrow H_2 2\pi r = I \Rightarrow H_2 = \frac{I}{2\pi r} \Rightarrow \boxed{B_2 = \frac{\mu_0 I}{2\pi r}}$

NIHANJE MAG. IGLE:

$\vec{M} = \vec{p}_m \times \vec{B}$

$M = p_m B \sin\varphi \cong p_m B \varphi$

$M = p_m B \varphi = -J \cdot \alpha$

$\alpha = -\frac{p_m B}{J} \cdot \alpha \Rightarrow \boxed{\omega_0^2 = \frac{p_m B}{J}}$

NIHANJE:

$\varphi = \varphi_0 \cos\omega_0 t$

$\dot{\varphi} = -\varphi_0 \omega_0 \sin\omega_0 t$

$\boxed{\ddot{\varphi} = -\omega_0^2 \varphi}$

$\boxed{\omega_0 = 2\pi\nu}$

$\boxed{\frac{B_1}{B_2} = \frac{\nu_1^2}{\nu_2^2}}$

zemeljsko polje: $B_1 = 2{,}1\cdot 10^{-4}$ T

$\nu_1 = 0{,}7\ s^{-1}$

nad žico: $B_2 = ?$

$\nu_2 = 7\ s^{-1}$

$r = 10$ cm

$B_2 = B_1\left(\frac{\nu_2}{\nu_1}\right)^1 = \frac{\mu_0 I}{2\pi r}$

$\boxed{I = (2\pi r)\frac{B_1}{\mu_0}\left(\frac{\nu_2}{\nu_1}\right)^2} = 10500\ A$

Če upoštevamo še polje zemlje v 2. primeru

4. V brezteženem prostoru se nahajata dve nabiti kroglici mas $m_1 = 5$ g in $m_2 = 15$ g, ki nosita naboj $q_1 = 8 \cdot 10^{-8}$ As in $q_2 = -2 \cdot 10^{-8}$ As. Kroglici zadržujemo na razdalji $l_0 = 20$ cm. Nato sprostimo drugo kroglico, ki se zaradi električne privlačne sile začne približevati prvi mirujoči kroglici . Kolikšna bo relativna hitrost prve kroglice, ko sta kroglici na razdalji 8 cm ? Gravitacijsko silo zanemarimo.

1994

M-8

$$k\frac{q_1 q_2}{l_0} = k\frac{q_1 q_2}{l_1} + \frac{1}{2} m_2 v_2^2$$

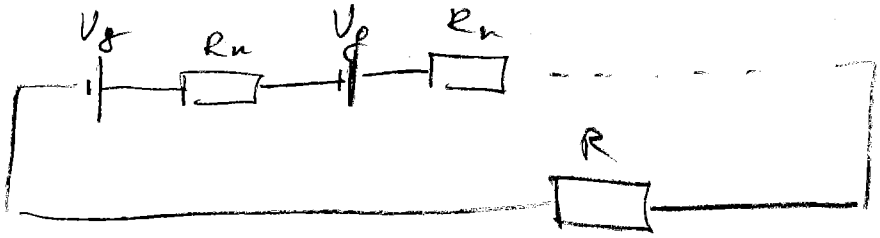
$$v_2 = \frac{1}{\sqrt{m_2}}\sqrt{\frac{2k\, q_1 q_2 (l_1 - l_0)}{l_0 l_1}} = \boxed{0{,}12\ \mathrm{m\,s^{-1}}}$$

$$v = v_2 - v_1 = 0{,}12 - 0 = \underline{\underline{0{,}12\ \mathrm{m\,s^{-1}}}}.$$

$$v_2 = \sqrt{\frac{q_1 q_2 (l_1 - l_0)}{2\pi m_2 \varepsilon_0 l_0 l_1}}$$

kolokvij 95/96

3. Na več zaporedno vezanih baterij, vsaka ima gonilno napetost 2 V in notranjo upornost 1 ohm, vežemo zaporedno upor 5 ohmov. Koliko baterij potrebujemo, da teče skozi ta upor tok 1 A ?



$U_g = 2V$

$R_n = 1\,\Omega$

$R = 5\,\Omega$

$n = (?)$ da $I = 1\,A$

$$n U_g = I \cdot n \cdot R_n + I R$$

$$n(U_g - I R_n) = I R$$

$$n = \frac{I R}{U_g - I R_n} = \underline{\underline{5}}$$ ✓

Repit 1991 [E64]

3. Po volumnu zelo tankega valjastega diska je enakomerno porazdeljen pozitiven naboj $0{,}8\cdot10^{-5}$ As. Disk se enakomerno vrti okoli geometrijske osi s frekvenco $60\ s^{-1}$. Gostota magnetnega polja v geometrijski osi diska tik ob njegovi površini je $2\cdot10^{-8}$ T. Kolikšen je radij diska ?

$\nu = 60\ s^{-1}$

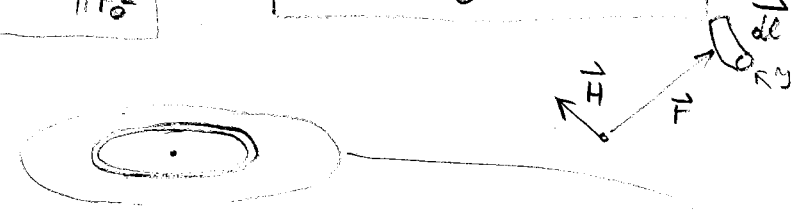
$B = 2\cdot10^{-8}\ T$

$e_T = 0{,}8\cdot10^{-5}\ As$

$$\sigma = \frac{e_T}{\pi r_0^2}$$

Biot-Savart

$$\vec{H} = \frac{y}{4\pi}\int \vec{r}\times\vec{dl}/r^3$$



$$dy = \frac{de}{t_0} = \frac{\sigma\cdot 2\pi r\,dr}{t_0} = \nu\cdot\frac{e_T}{\pi r_0^2}\cdot 2\pi r\,dr = \frac{\nu\cdot e_T}{r_0^2}\,2r\,dr$$

zanka:

$$\delta H = \frac{dy}{4\pi}\cdot\frac{r\,2\pi r}{r^3} = \frac{dy}{2r}$$

$$B = \mu_0 H = \mu_0\int_0^{r_0}\frac{dy}{2r} = \mu_0\int_0^{r_0}\frac{\nu\cdot e_T}{r_0^2}\,dr = \mu_0\frac{\nu\cdot e_T\cdot r_0}{r_0^2} = \frac{\mu_0\cdot\nu\cdot e_T}{r_0}$$

$$B = \mu_0\cdot\nu\cdot e_T/r_0$$

$$\Downarrow$$

$$r_0 = \frac{\mu_0\cdot\nu\cdot e_T}{B} = \underline{3\ cm}$$

$$\frac{4\pi\cdot10^{-7}\cdot 60\cdot 0{,}8\cdot10^{-5}}{2\cdot10^{-8}} =$$

$$= \frac{240\cdot\pi\cdot 8\cdot10^{-5}}{3} = 0{,}03\ m$$

Res. Hal 663/9 $\quad P = \varepsilon_0(\varepsilon-1)E \quad D = \varepsilon_0 E + P$

E16

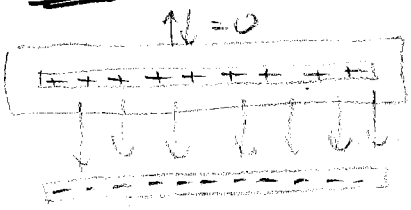
Med ploščama ploščatega kondenzatorja s površino posamezne plošče 100 cm² in razdaljo med njima 1 cm je napetost 100 V. Med ploščama je v začetku samo zrak. Nato vtaknemo v sredino med plošči kondenzatorja dielektrično ploščo debeline 0.5 cm in z diel. konst. 7. Kolikšna je kapaciteta kondenzatorja, naboj na ploščah, jakost el. polja med ploščama in napetost med ploščama po tem? $\varepsilon_0 = 8.85 \cdot 10^{-12}$ As/Vm

PREJ:

$V_0 = 100$ V

$S = 100$ cm²

$d = 1$ cm



ena plošča: $\int D_p dS = \varepsilon_0 E_p 2 \cdot S = e \Rightarrow$

$\Rightarrow E_p = \frac{e}{2\varepsilon_0 S}$, $E = 2E_p = \frac{e}{\varepsilon_0 S}$ $\quad \boxed{E = \frac{e}{\varepsilon_0 S}}$

$\Delta\varphi = \int_d^0 E\,dx = \frac{e}{S\varepsilon_0} d \Rightarrow \boxed{U = \frac{e}{\varepsilon_0 S} d}$

$\boxed{E = \frac{U}{d}}$ $\quad \boxed{e = \varepsilon_0 \frac{S}{d} U}$ $\quad \boxed{C = \varepsilon_0 \frac{S}{d}}$

a) $C_0 = \frac{\varepsilon_0 S}{d} = 8.9 \cdot 10^{-12}\,F = 8.9\,pF$

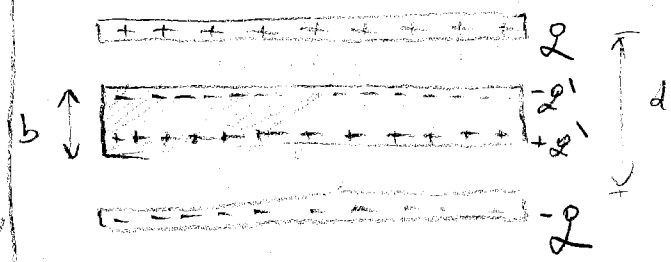
b) $e = C_0 \cdot V_0 = 8{,}9 \cdot 10^{-10}$ As

c) $E_0 = \frac{e}{\varepsilon_0 S} = 1 \cdot 10^4$ V/m

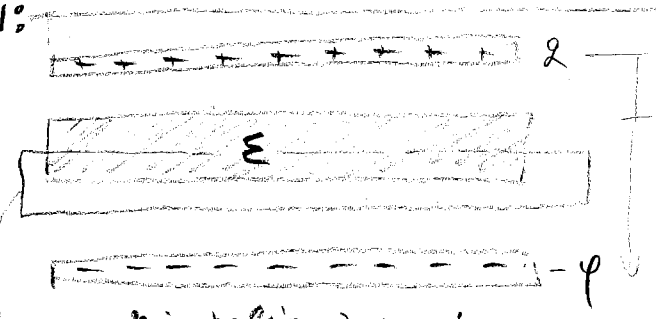
POTEM:

$b = 0.5$ cm

$\varepsilon = 7$



ALI:



ni polja znaj

$D_1 S - D_0 S = 0$

$\varepsilon\varepsilon_0 E_1 = \varepsilon_0 E_0 \Rightarrow \boxed{E_1 = \frac{E_0}{\varepsilon}}$

$E_1 = \frac{e}{\varepsilon\varepsilon_0 S} = 0{,}14 \cdot 10^4 \frac{V}{m}$

$\Delta\varphi = U_1 = \int_d^0 E\,dx = \int_d^{d-\frac{d-b}{2}} E_0\,dx + \int_{d-\frac{d-b}{2}}^{\frac{d-b}{2}} E_1\,dx + \int_{\frac{d-b}{2}}^{0} E_0\,dx = E_0(d-b) + E_1 b = 57\,V$

$\boxed{e = C_1 U_1} \Rightarrow C_1 = \frac{e}{U_1} = 16\,pF$

Izpit 1996

4. Kvadraten okvir iz žice s specifičnim uporom 0,06 $\Omega mm^2/m$ in presekom 1 mm^2 ima dolžino osnovne stranice 4 cm . Okvir vrtimo okoli simetrale kvadrata, ki je vzporedna dvema stranicama in pravokotna na magnetno polje z gostoto 0,1 T. Kolikšen povprečen navor je potreben, da se okvir zavrti 300 krat v minuti ? FII

$\rho_k = 0{,}06\ \Omega mm^2/m$

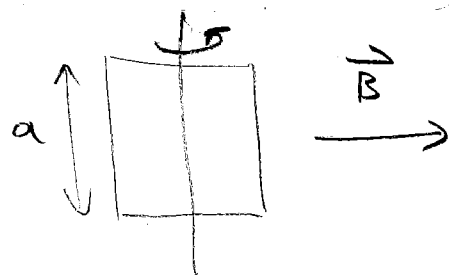
$S_0 = 1\ mm^2$

$a = 0.04\ m$

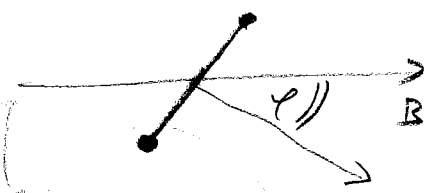
$B = 0{,}1\ T$

$\nu = 5\ s^{-1}$

$\boxed{\omega = 2\pi\nu}$



TLORIS :



$\langle M \rangle = ?$

$\boxed{\phi = B \cdot S \cdot \cos\varphi}$, $\boxed{\varphi = \omega t}$

$$\boxed{U_i = -\frac{d\phi}{dt} = B \cdot S \cdot \omega \sin \omega t}$$

$$\boxed{R = \frac{\rho_k \cdot 4a}{S_0}}$$

$$\boxed{I = \frac{U_i}{R} = \frac{B \cdot S \cdot \omega \cdot \sin \omega t \cdot S_0}{\rho_k \cdot 4a}}$$

$$\vec{M} = \vec{p_m} \times \vec{B} \Rightarrow \boxed{M = p_m B \sin\varphi = p_m \cdot B \cdot \sin(\omega t)}$$

$$\boxed{p_m = I \cdot S}$$

$$M = I \cdot S \cdot B \cdot \sin(\omega t) = \frac{B^2 \cdot S^2 \omega \cdot S_0 \cdot \sin^2(\omega t)}{\rho_k \cdot 4a} \Rightarrow$$

$$\boxed{\langle M \rangle = \frac{B^2 \cdot a^3 \cdot \omega \cdot S_0}{8 \cdot \rho_k}} = \cancel{0{,}026\ Nm}$$

$$\boxed{\langle \sin^2(\omega t) \rangle = \frac{1}{2}}$$

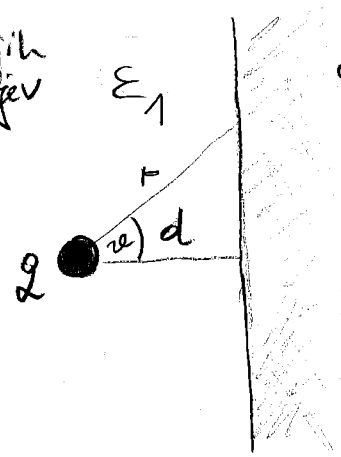
Kakšna sila deluje na proton na oddaljenosti d od biološke membrane in v biološki membrani z dielektričnostjo 2? Dielektričnost vode je 80.

$$\oint \vec{D}\, d\vec{A} = \int \rho\, dV = \oint \operatorname{div} \vec{D}\, d\vec{A}$$

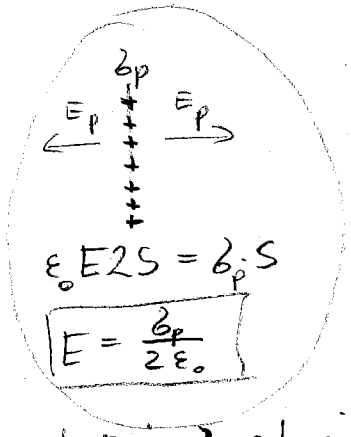
ceni zunanjih nabojev

$$\Rightarrow -\varepsilon_1 E_{1\perp}\, dA + \varepsilon_2 E_{2\perp}\, dA = 0$$

$$\boxed{-\varepsilon_1 E_{1\perp} + \varepsilon_2 E_{2\perp} = 0}$$



$$\boxed{\cos\vartheta = \frac{d}{r}}$$



$$E_p \equiv \boxed{\frac{\sigma_p}{2\varepsilon_0} = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\, \frac{1}{4\pi\varepsilon_0\varepsilon_1} \cdot \frac{q}{r^2} \cdot \frac{d}{r}}$$

(b)

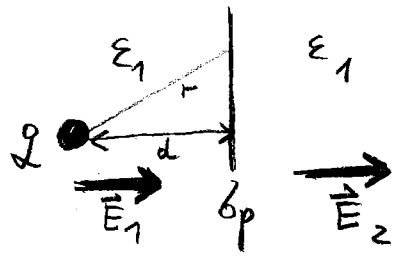
(c)

kakor polj od naboja

$$\boxed{q' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\, q}$$ v mediju $\varepsilon = \varepsilon_1$

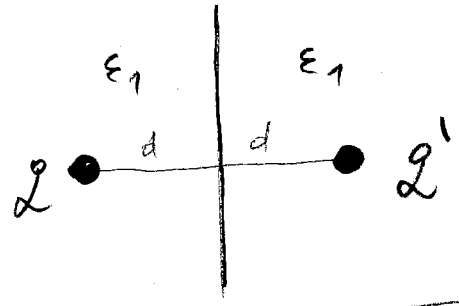
(A) VSTAVIMO NAZAJ

če predstavimo, da tudi na desni $\varepsilon \equiv \varepsilon_1$, razliko v ε-ih pa upoštevamo z površinskim nabojem desnega področja σ_p; torej



$$\boxed{\begin{aligned} E_{1\perp} &= \frac{q}{4\pi\varepsilon_0\varepsilon_1}\, \frac{1}{r^2}\, \frac{d}{r} - \frac{\sigma_p}{2\varepsilon_0} \\ E_{2\perp} &= \frac{q}{4\pi\varepsilon_0\varepsilon_1}\, \frac{1}{r^2}\, \frac{d}{r} + \frac{\sigma_p}{2\varepsilon_0} \end{aligned}}$$ vrednost polja na stični ploskvi

na desni strani (z $\varepsilon = \varepsilon_1$)



(D)

sila med q in q' v mediju ε_1:

$$\boxed{F = \frac{1}{4\pi\varepsilon_0\varepsilon_1} \left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\right) \frac{q^2}{4d^2}}$$

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(1)  $\varepsilon_1 = 80$, $\varepsilon_2 = 2$: $F > 0 \Rightarrow$ odbojna sila stran od mejne ploskve

(2)  $\varepsilon_1 = 2$: $F < 0 \Rightarrow$ privlačna sila proti mejni ploskvi

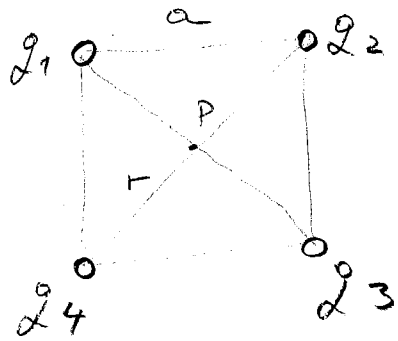
Ka. Hll. str. 629

E13

Kakšen je električni potencial v centru okvirja, ki ima v ogljiščih naboje 10^{-8} As, $-2\cdot10^{-8}$ As, $3\cdot10^{-8}$ As, $2\cdot10^{-8}$ As? Stranica okvirja ima 1 m.

$Q_1 = 10^{-8}\,As$, $Q_2 = -2\cdot10^{-8}\,As$, $Q_3 = 3\cdot10^{-8}\,As$, $Q_4 = 2\cdot10^{-8}\,As$

$a = 1\,m$



$r^2 + r^2 = a^2$

$2r^2 = a^2$

$r = \frac{a}{\sqrt{2}}$

$$\varphi_0 = \sum_{i=1}^{4} \varphi_i = \frac{1}{4\pi\varepsilon_0}\frac{Q_1+Q_2+Q_3+Q_4}{r}$$

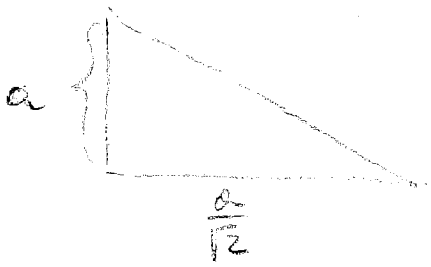
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1+Q_2+Q_3+Q_4}{(a/\sqrt{2})} = \underline{\underline{500\,V}}$$

(2)

Kolikšna je električna napetost med centrom kvadratnega okvirja in točko, ki je 1 m nad centrom. V ogljiščih okvirja so točkasti naboji 10^{-8} As, $-2\cdot10^{-8}$ As, $3\cdot10^{-8}$ As in $2\cdot10^{-8}$ As ? Stranica okvirja ima 1 m.

1996

$$\varphi_1 = \sum_i \varphi_i = \frac{1}{4\pi\varepsilon_0}\frac{Q_1+Q_2+Q_3+Q_4}{\left(\frac{a}{\sqrt{2}}\right)\sqrt{3}} = \underline{\underline{288\,V}}$$



$$R = \sqrt{a^2 + \frac{a^2}{2}} = \sqrt{\frac{3}{2}a^2} = \sqrt{3}\,\frac{a}{\sqrt{2}}$$

$$\Delta\varphi \cong 211\,V$$

$$DS = Q$$

$$\varepsilon_0 E\, 4\pi r^2 = Q$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E = -\frac{d\varphi}{dr}$$

$$\int_r^\infty d\varphi = -\int_r^\infty E\,dr$$

$$\varphi_\infty - \varphi(r) = -\int_r^\infty \frac{Q}{4\pi\varepsilon r^2}\,dr$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\Big|_r^\infty = -\frac{Q}{4\pi\varepsilon_0 r}$$

$$\Downarrow$$

$$\boxed{\varphi(r) = \frac{Q}{4\pi\varepsilon_0 r}}$$

ů $\varphi_\infty = 0$

izpit, 1991

3. V zemeljskem magnetnem polju z vodoravno komponento gostote $2{,}1 \cdot 10^{-4}$ T niha magnetna igla okoli navpične osi s frekvenco $0{,}03\ s^{-1}$ (pri majhnih odklonih). S kakšno frekvenco zaniha ta ista magnetna igla, če jo postavimo v smer magnetnega polja z gostoto 0,4 T, jo malo zasučemo in spustimo ?

$B_1 = 2{,}1 \cdot 10^{-4}\ T, \quad \nu_1 = 0{,}03\ s^{-1}$

$B_2 = 0{,}4\ T, \quad \nu_2 = ?$

$$\boxed{\vec{M} = \vec{p}_m \times \vec{B}}$$

$$\varphi = \varphi_0 \sin(2\pi\nu t)$$

$$\omega = \dot{\varphi} = \varphi_0 (2\pi\nu) \cos(2\pi\nu t)$$

$$\alpha = \ddot{\varphi} = -\varphi_0 (2\pi\nu)^2 \sin(2\pi\nu t)$$

$$\boxed{\alpha = -(2\pi\nu)^2 \cdot \varphi}$$

$$M = J \cdot \alpha$$

$$-p_m \cdot B \cdot \sin\varphi = J \cdot \alpha, \quad \sin\varphi \approx \varphi$$

$$\boxed{-\frac{p_m \cdot B}{J} \cdot \varphi = \alpha}$$

$$\boxed{(2\pi\nu)^2 = \frac{p_m \cdot B}{J}}$$

$$\frac{\nu_1^2}{\nu_2^2} = \frac{B_1}{B_2} \Longrightarrow \boxed{\nu_2 = \nu_1 \cdot \sqrt{\frac{B_2}{B_1}}} = 1.31\ s^{-1}$$

$$0.03 \cdot \sqrt{\frac{0.4}{2.1} \cdot 10^4} = 1{,}3093 \approx 1.31$$

M-8 izpit 1991 ← ODVEČ PODATEK

6. Tanek obroč z radijem 15 cm in maso $2 \cdot 10^{-2}$ kg, ki je prosto vrtljiv okoli geometrijske osi, ima po vsem svojem volumnu enakomerno porazdeljen naboj 10^{-2} As. ~~Ko damo obroč v~~ homogeno magnetno polje, tako da je smer magnetnega polja vzporedna z geometrijsko osjo obroča, se začne le ta vrteti s kotno hitrostjo 1,5 s^{-1}. Kakšna je gostota magnetnega polja? Nosilci naboja v obroču niso gibljivi.

$r = 0{,}15$

$m = 10^{-2}$ kg

$e_T = 10^{-2}$ As

$\omega = 1{,}5 \ s^{-1}$

- Fiksirana os
- vzporedno vektorju B
- v B polju
- ga izkoristiš

$$\boxed{\omega = \frac{e_T B \cdot S}{2\pi m}} = \frac{e_T B \cdot \cancel{\pi r^2}}{2\cancel{\pi} m \cancel{r^2}}$$

$$\Downarrow$$

$$\boxed{B = \frac{2\omega m}{e_T}} = \underline{6\ T} \ ✓$$

$$\frac{2 \cdot 2 \cdot 1{,}5 \cdot 10^{-2}}{10^{-2}} = 6\ T$$

Kolokvij 90/91

3. Skozi toplotno izolirano cev se pretoči vsako sekundo 1 liter vode. Izračunaj, za koliko se segreje voda v cevi, če je vanjo vgrajen grelec z upornostjo 10 ohmov, ki je priključen na istosmerno napetost 220 voltov ?

$c_p = 4200\ J\,kg/K$

$\phi_V = 1\ \frac{l}{s}$

$R = 10\ \Omega$

$U = 220\ V$

$$\boxed{\phi_m = \rho \cdot \phi_V}$$

$U = I \cdot R$

$A = eU$, $P_{el} = YU = \frac{U^2}{R}$

$$\boxed{P_{el} = \phi_m c_p \Delta T}$$

$$\frac{d(m c_p \Delta T)}{dt} = \overbrace{\frac{dm}{dt}}^{\phi_m} c_p \Delta T$$

$$\Downarrow$$

$$\Delta T = \frac{P_{el}}{\phi_m \cdot c_p} = \frac{U^2}{R \cdot \rho \cdot \phi_V \cdot c_p} = \underline{\underline{1{,}15\ K}}$$

$$\eta\ \frac{1}{2} = \underline{\underline{1.84\ K}}$$

Fizika p. Izpit iz Fizike IV (dec. 1992)

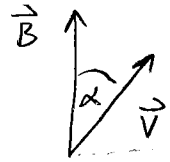
4. Dvakrat ioniziran helijev ion preleti napetost 10^4 V in prileti v magnetno polje z gostoto B = 1,5 T, tako da je kot med vektorjema hitrosti in magnetnega polja $\alpha = 40^\circ$. Kako daleč v smeri polja se premakne ion pri enem obhodu ? $m_{He} = 4 \cdot a.e.m = 6.64 \cdot 10^{-27}$ kg

$U = 10^4$ V

$B = 1{,}5$ T

$\alpha = 40^\circ$

$m_{He} = 6{,}7 \cdot 10^{-27}$ kg , $e_{He} = 2e_0$



$$\frac{\pi \cdot 6{,}7 \cdot 10^{-27}}{1{,}6 \cdot 10^{-19} \cdot 1{,}5} = 8{,}77 \cdot 10^{-8}\ \mathrm{s}$$

$$\boxed{\frac{mv^2}{2} = eU} \Rightarrow v = \sqrt{\frac{2eU}{m}} = \sqrt{\frac{4 \cdot e_0 U}{m}} = \underline{\underline{0{,}98 \cdot 10^6\ \frac{m}{s}}}$$

Newtonov zakon:

$$e(v \sin\alpha)B = \frac{m(v \sin\alpha)^2}{r}$$

$$eB = \frac{m v \sin\alpha}{r}$$

$$2e_0 B = eB = m\omega = m\frac{2\pi}{t_0}$$

$$\boxed{t_0 = \frac{\pi m}{e_0 B} = \underline{\underline{8{,}77 \cdot 10^{-8}\ \mathrm{s}}}}$$

$$s = v \cdot \cos\alpha \cdot t_0 = \underline{\underline{6.6 \cdot 10^{-2}\ \mathrm{m}}}$$

2. Kvadraten okvir iz žice s specifičnim uporom 0,1 $\Omega mm^2/m$ in presekom 1 mm^2 ima stranico 10 cm. Okvir vrtimo okoli njegove simetrale, ki je pravokotna na magnetno polje 0,1 T. Koliko dela porabimo za en obrat okvirja, če ga vrtimo s frekvenco 60 s^{-1} ? Izgube zanemarimo.

$\rho = 0{,}1\ \Omega mm^2/m$

$S_0 = 1\ mm^2$

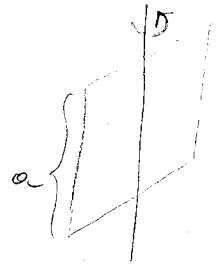
$a = 10\ cm = 10^{-1}\ m$

$B = 0{,}1\ T$

$\nu = 60\ s^{-1}$

$\boxed{\omega = 2\pi\nu}$

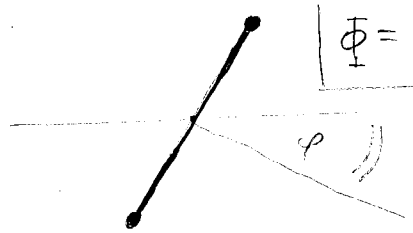
$\boxed{\varphi = \omega \cdot t}$



$\vec{B} \longrightarrow$ $\boxed{S = a^2}$

tlačnis

$\boxed{\Phi = B \cdot S \cdot \cos\varphi}$



$$U_i = \frac{d\phi}{dt} = -BS\omega \sin(\omega t)$$

$$\boxed{I = \frac{U_i}{R} = \frac{B \cdot S \cdot \omega \cdot \sin(\omega t) \cdot S_0}{\rho \cdot 4 \cdot a}}$$

$$R = \frac{\rho \cdot 4 \cdot a}{S_0}$$

$$\vec{M} = \vec{p}_m \times \vec{B} \Rightarrow M = p_m \cdot B \cdot \sin(\omega t)$$

$$\boxed{p_m = I \cdot S}$$

$$\Downarrow$$

$$\boxed{M = I \cdot a^2 \cdot B \cdot \sin(\omega t) = \frac{B^2 \cdot S^2 \cdot \omega \cdot \sin^2(\omega t) \cdot S_0}{\rho \cdot 4a}} = \frac{B^2 \cdot S^2 \cdot \omega \cdot \sin^2(\varphi)}{2} = \frac{B^2 a^2 2\pi \nu S_0}{4\rho} = 4{,}7 \cdot 10^{-3}\ Nm$$

$$\boxed{A = \int_0^{2\pi} M\, d\varphi} = \frac{B^2 \cdot S^2 \cdot \omega \cdot S_0}{\rho \cdot 4 \cdot a} \underbrace{\int_0^{2\pi} \sin^2\varphi\, d\varphi}_{\pi} =$$

$$= \frac{B^2 \cdot a^4\ 2 \cdot \pi \cdot \nu \cdot S_0 \cdot \pi}{\rho \cdot 4 \cdot a} = \frac{B^2 a^3 \cdot \nu \cdot S_0 \cdot \pi^2}{2\rho}$$

$$\boxed{A = \frac{B^2 \cdot a^3 \cdot \nu \cdot S_0 \cdot \pi^2}{2 \cdot \rho} = \Delta W} = \frac{B^2 a^4\ 2\pi\nu \cdot \pi}{R}$$

$$= \frac{10^{-2} \cdot 10^{-3} \cdot 60 \cdot 1 \cdot \pi^2}{2 \cdot 10^{-1}} = 30 \cdot 10^{-4} \cdot \pi^2 = 0{,}0296\ J$$

$$\int \sin^2\varphi\, d\varphi$$

$$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi$$

$$\sin^2\varphi = \cos^2\varphi - \cos 2\varphi$$

$$\int \sin^2\varphi = \int \cos^2\varphi - \int \cos 2\varphi$$

$$= \int (1 - \sin^2\varphi) - \int \cos 2\varphi$$

$$\Downarrow$$

$$2\int \sin^2\varphi = \int d\varphi - \int \cos 2\varphi\, d\varphi$$

$$\Downarrow$$

$$\int \sin^2\varphi = \frac{\varphi}{2} - \frac{1}{4}\sin 2\varphi$$

NAL 3 KOLOKVIJ (1990/1991) $\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am$ B67

V geometrijski osi zanke z radijem 5 cm (4 cm) leži v razdalji 3 cm (4 cm) od ravnine zanke majhna magnetna igla. Začetni tok skozi zanko 10 A (11 A) povečamo za 2%. Kolikšna je sprememba frekvence nihanja magnetne igle pri majhnih odmikih. Magnetni moment igle je $10^{-4}\ Am^2$, vztrajnostni moment igle pa je $10^{-6}\ kg\,m^2$.

$a = 5\ cm\ (4\ cm)$

$h = 3\ cm\ (4\ cm)$

$I = 10\ A\ (11\ A)$

$\Delta I = 2\%\ I$

$p_m = 10^{-4}\ Am^2$

$J = 10^{-6}\ kg\,m^2$

$\varphi = \varphi_0 \sin(2\pi\nu t)$

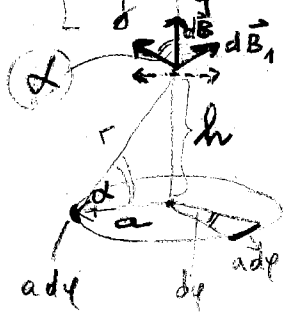
$\alpha = -(2\pi\nu)^2 \varphi$

$-p_m B \sin\varphi = J \cdot \alpha$

$-p_m \cdot B\varphi = J \cdot \alpha$

$-\left[\frac{p_m \cdot B}{J}\right]\varphi = \alpha \Rightarrow \boxed{2\pi\nu = \sqrt{\frac{p_m \cdot B}{J}}}$

$B = \mu_0 H$



$$\boxed{\vec{H} = \frac{I}{4\pi}\int \frac{\vec{r} \times d\vec{l}}{r^3}}$$

$dB = 2\, dB_1 \cos\alpha$

$r^2 = h^2 + a^2$

$\cos\alpha = \frac{a}{r}$

$$dB = 2\left(\frac{\mu_0 I}{4\pi}\,\frac{r \cdot a\,d\varphi}{r^3}\right)\cos\alpha = \left(\frac{\mu_0 I a}{2\pi r^2}\right)\cos\alpha \cdot d\varphi$$

$$B = \int_0^{\pi} dB = \frac{\mu_0 I a \cos\alpha}{2\pi r^2}\int_0^{\pi} d\varphi = \frac{\mu_0 I a \cos\alpha}{2r^2} \Rightarrow$$

$$\boxed{B = \left[\frac{\mu_0 I a^2}{2}\right] \Big/ (a^2 + h^2)^{3/2}} \Rightarrow \boxed{\nu = \frac{1}{2\pi}\left(\frac{p_m/\mu_0 \cdot a^2}{2 \cdot J}\right)^{1/2} \cdot I^{1/2} \Big/ (a^2 + h^2)^{3/4}}$$

A: $a = 5\ cm$, $h = 4\ cm$

$\Delta\nu = (\quad)\frac{1}{2} I^{-1/2}\Delta I = (\quad)\left(\frac{1}{2}\right)\left(I^{1/2}\right)\left(\frac{\Delta I}{I}\right) = \underline{1{,}416 \cdot 10^{-4}\ s^{-1}}$ ✓

B: $a = 4\ cm$, $h = 4\ cm$

$\Delta\nu = (\quad)\frac{1}{2} I^{-\frac{1}{2}}\Delta I = (\quad)\left(\frac{1}{2}\right)\left(I^{1/2}\right)\left(\frac{\Delta I}{I}\right) = \boxed{1{,}244 \cdot 10^{-4}\ s^{-1}}$