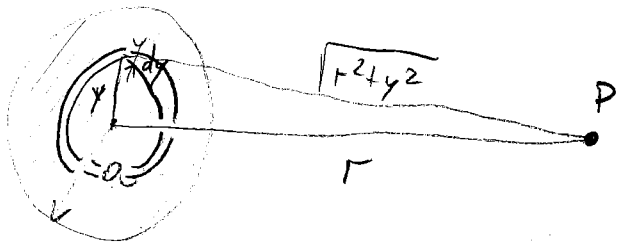
E14

Kes. tel. str. 630

Poišči izraz za električni potencial na osi enakomerno nabitega tankega okroglega diska, ki ima površinsko gostoto naboja $\sigma = 10^{-2}\,As/m^2$ in radij $a = 1\,m$?

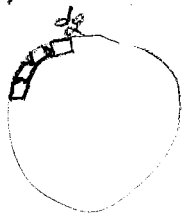
$a = 1\,m$

$\sigma = 10^{-2}\,\frac{As}{m^2}$



$$r' = \sqrt{r^2+y^2}$$

$$\sum dq = \sigma \cdot 2\pi y\, dy$$



$$d\varphi = \frac{1}{4\pi\varepsilon_0}\frac{\sum dq}{r'} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi y\, dy}{\sqrt{r^2+y^2}}$$

$$\varphi = \int d\varphi = \int_0^a \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi y\, dy}{\sqrt{r^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^a \frac{y}{\sqrt{r^2+y^2}}\,dy =$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{r^2+a^2} - r\right]$$

$$\int \frac{y}{\sqrt{r^2+y^2}}\,dy = \int \frac{y}{\sqrt{u}}\frac{du}{2y} = \frac{1}{2}\int \frac{du}{\sqrt{u}}$$

$\boxed{r^2+y^2=u}$ $\quad 2y\,dy = du$

$$= \frac{1}{2}\cdot 2\cdot u^{1/2} = u^{1/2}$$

torej:

$$\int \frac{y}{\sqrt{r^2+y^2}}\,dy = \sqrt{r^2+y^2}$$

1. Nabita kroglica z maso 0,1 g in nabojem -10^{-5} As se lahko giblje vzdolž geometrijske osi zelo tankega ploščatega kolobarja z zunanjim radijem 10 cm in notranjim radijem 5 cm, po katerem je enakomerno razmazan naboj s površinsko gostoto 10^{-5} As/m^2. Kolikšen je nihajni čas kroglice, če jo za malenkost izmaknemo iz ravnovesne lege v središču kolobarja ?

A-L

$m = 0{,}1\ g$

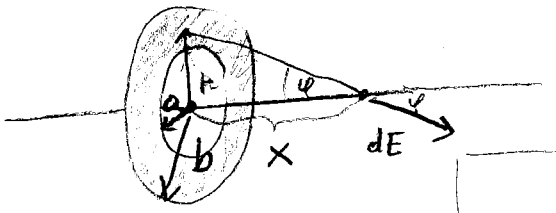
$e = -10^{-5}\ As$

$b = 0{,}1\ m$

$a = 0{,}05\ m$

$\sigma = 10^{-5}\ As/m^2$

$t_0 = ?$



$$\boxed{\cos\varphi = \frac{x}{\sqrt{r^2+x^2}}}$$

$$E = \int_a^b \frac{\sigma\, 2\pi r\, dr}{4\pi\varepsilon_0 (r^2+x^2)} \frac{x}{\sqrt{r^2+x^2}} = \frac{\sigma x}{2\varepsilon_0}\int_a^b \frac{r\,dr}{(r^2+x^2)^{3/2}} =$$

$r^2 + x^2 = u$

$2r\,dr = du \Rightarrow dr = \frac{du}{2r}$

$$= \frac{\sigma x}{4\varepsilon_0}\int_{(a^2+x^2)}^{(b^2+x^2)} \frac{du}{u^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{(a^2+x^2)^{1/2}} - \frac{1}{(b^2+x^2)^{1/2}}\right] = E(x)$$

$$x \ll 0 : \boxed{E(x) \simeq \frac{\sigma x}{2\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]}$$

$ma = eE \quad (e<0)!$

$$m\ddot{x} = \frac{e\sigma}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)x \Rightarrow (2\pi\nu)^2 = \frac{e\cdot\sigma}{2m\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{e\cdot\sigma}{2m\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)} = 119{,}6\ s^{-1} \Rightarrow t_0 = 8{,}36\cdot 10^{-3}\ s}$$

$$\frac{1}{2\pi}\left(\frac{10^{-10}(0{,}05)}{2\cdot 10^{-4}\cdot 8{,}85\cdot 10^{-12}(0{,}005)}\right)^{1/2} = \frac{1}{2\pi}\left(\frac{1}{2\cdot 8{,}85}\cdot\frac{10^{-12}}{10^{-19}}\right)^{1/2} = \frac{1}{2\pi}\left(\frac{10}{2\cdot 8{,}85}\right)^{1/2}\cdot 10^{3}$$

5. Kroglica mase 3 g in naboja 8 μAs se približuje tankemu obroču s polmerom 3 cm, ki ima po svoji površini enakomerno porazdeljen naboj 0,1 μAs. Kroglica se giblje vzdolž geometrijske osi obroča. Hitrost kroglice na veliki oddaljenosti od obroča je 30 km/h. Do katere najmanjše razdalje se kroglica približa središču obroča, če je le ta fiksiran v prostoru ?

$m = 3 \cdot 10^{-3}\ kg$

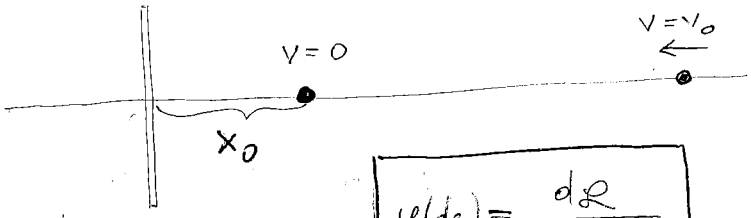
$e = 8\ \mu As$

$a = 3 \cdot 10^{-2}\ m$

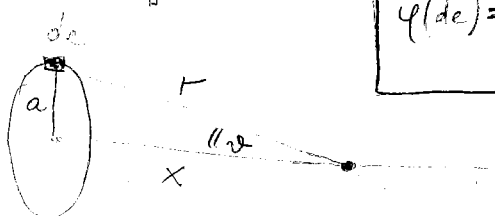
$v_0 = 30\ km/h = 8.33\ \frac{m}{s}$

$Q = 0{,}1\ \mu As$

$x_0 = ?$



$$\boxed{\varphi(de) = \frac{dQ}{4\pi\varepsilon_0 r}}$$



$$\varphi(x) = \int d\varphi = \frac{Q}{4\pi\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 (a^2 + x^2)^{1/2}}$$

$$\boxed{\frac{m v_0^2}{2} = e \cdot \varphi(x_0)}$$ ohranitev energije

$$\Downarrow$$

$$\frac{m v_0^2}{2} = \frac{e \cdot Q}{4\pi\varepsilon_0 (a^2 + x_0^2)^{1/2}}$$

$$r_0 = (a^2 + x_0^2)^{1/2} = \frac{e \cdot Q}{2\pi\varepsilon_0 \cdot m \cdot v_0^2} = \frac{8 \cdot 0{,}1 \cdot \cancel{10^{-12}}}{2\pi \cdot 8{,}85 \cdot \cancel{10^{-12}} \cdot 3 \cdot 10^{-3} \cdot (8{,}33)^2} = 0{,}069\ m$$

$$r_0^2 = a^2 + x_0^2$$

$$x_0 = (r_0^2 - a^2)^{1/2} = 6{,}22\ cm$$

E24

1994

M-Ž

2. Po toplotno izolirani bakreni žici s presekom 1 mm^2 pošljemo tok 10 A, ki traja samo 1 sekundo. Za koliko se žica segreje ? (ρ_{Cu} = 8,9·10^3 kg/m^3, c_p = 3,78·10^2 J/kgK, ζ = 1,7·10^{-8} Ωm)

$\boxed{A = eU}$ $\boxed{U = R \cdot I}$

$\boxed{P = \frac{dA}{dt}} = \frac{de}{dt} \cdot U = I \cdot U = I^2 \cdot R$

$S = 1\ mm^2$

$I = 10\ A$

$t = 1\ s$

$\rho_{Cu} = 8.9 \cdot 10^3\ kg/m^3$

$c_p = 3.78 \cdot 10^2\ J/kg\,K$

spec. upor (zeta) $\zeta = 1.7 \cdot 10^{-8}\ \Omega\,m$

$\Delta W = \boxed{I^2 R t = c_p m \Delta T}$

$\boxed{R = \frac{\zeta \cdot l}{S}}$

$$\frac{I^2 \zeta \cdot l t}{S} = c_p m \Delta T$$

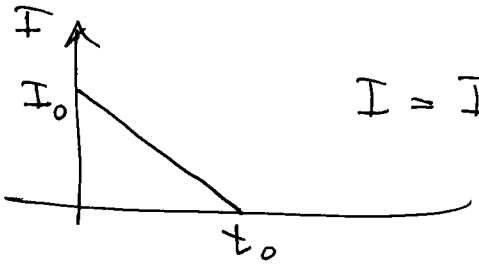
$$\Delta T = \frac{I^2 \zeta \cdot l t}{S \cdot c_p \cdot m} \cdot \frac{S}{S} = \frac{I^2 \cdot \zeta \cdot t}{S^2 \cdot c_p \rho_{Cu}} = 0.5\ K$$

3. Skozi upornik 8 Ω steče ~~v zelo dolgem času~~ naboj 30 As. Koliko toplote se sprosti v uporniku, če pada tok linearno proti 0 tako, da pade na nič v 60 s ?

$R = 8\,\Omega$

$e_0 = 30\,As$

$t_0 = 60\,s$



$$I = I_0 - \frac{I_0}{t_0}\cdot t$$

$$e_0 = \int_0^{t_0} I\,dt = \int_0^{t_0}\left(I_0 - \frac{I_0}{t_0}t\right)dt = I_0 t_0 - \frac{I_0 t_0}{2} = \frac{I_0 t_0}{2}$$

$$\boxed{I_0 = \frac{2e_0}{t_0}}$$

$$Q = \int U\,de = \int IR\,de = \int_0^{t_0} I^2 R\,dt = R\int_0^{t_0}\left[I_0^2 - \frac{2I_0^2}{t_0}t + \frac{I_0^2}{t_0^2}t^2\right]dt =$$

$$= RI_0^2 t_0 - 2R\frac{I_0^2 t_0^2}{t_0\,2} + R\frac{I_0^2 t_0^3}{t_0^2\,3} =$$

$$= RI_0^2 t_0 - \frac{2R\cdot I_0^2 t_0}{2} + \frac{RI_0^2 t_0}{3} =$$

$$= RI_0^2 t_0\left(1 - 1 + \frac{1}{3}\right) = \frac{RI_0^2 t_0}{3} = \frac{R t_0\,4e_0^2}{3\,t_0^2} =$$

$$= \frac{4Re_0^2}{3t_0} = 160\,J$$

$$\frac{4\cdot 8\cdot 900}{3\cdot 60}$$

FOR

4. Polkroglasta lupina radija 2 cm je enakomerno nabita z nabojem 10^{-9} As. V središče odprtega prereza lupine postavimo proton. Kolikšna je hitrost protona na veliki oddaljenosti od lupine ?

M-7

$R = 0{,}02\ m$

$e = 10^{-9}\ As$, $e_0 = 1.6 \cdot 10^{-19}\ As$

$m_p = 1{,}67 \cdot 10^{-27}\ kg$

$$\varphi = \int d\varphi = \int \frac{de}{4\pi\varepsilon_0 R} = \frac{e}{4\pi\varepsilon_0 R}$$

$$\boxed{e_0 \varphi = \frac{m_p v^2}{2}} = 7.17 \cdot 10^{-17}\ J$$

$$v = \left(\frac{2 e_0 \varphi}{m_p}\right)^{1/2} = \left(\frac{2 e_0 \cdot e}{m_p \cdot 4\pi\varepsilon_0 R}\right)^{1/2} = \boxed{0{,}293 \cdot 10^{6}\ \frac{m}{s}}$$

$$\frac{2 \cdot 1{,}6 \cdot 10^{-19} \cdot 10^{-9}}{1{,}67 \cdot 10^{-27} \cdot 4 \cdot \pi \cdot 8{,}85 \cdot 10^{-12} \cdot 2 \cdot 10^{-2}} = W_k = 7.17 \cdot 10^{-17}\ J$$

$$\frac{3{,}2 \cdot 10^{-28}}{1{,}67 \cdot 4 \cdot \pi \cdot 8{,}85 \cdot 2 \cdot 10^{-41}} = 0{,}293 \cdot 10^{6}$$

Fiz. II

A-L

2. V kovinski mreži pride na en atom kovine v povprečju en prevodniški elektron. Skozi žico iz te kovine, katere presek je krog s premerom 1 mm, teče tok 1 A. Oceni povprečno hitrost s katero se prevodniški elektroni gibljejo skozi žico ?
(M_{kovine} = 64 kg/kmol, ρ_{kovine} = 9 g/cm^3)

$2r = 1\ mm$

$I = 1\ A$

$M_{k.} = 64\ kg/kmol$

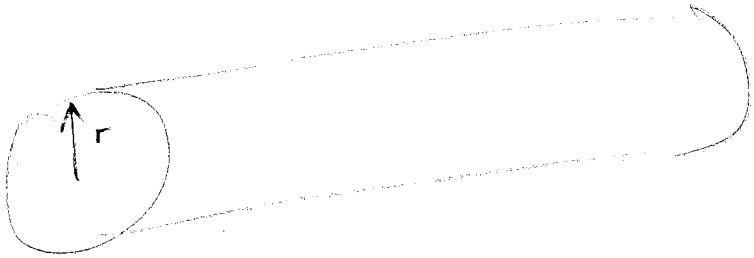
$\rho_{k} = 9\ g/cm^3$, $N_A = 6 \cdot 10^{26}/kmol$

$S = \pi r^2$

$$j = \boxed{\frac{I}{S} = n_e e_0 \bar{v}} \Rightarrow \boxed{\bar{v} = \frac{I}{S n_e e_0}} = \frac{I \cdot M}{S \rho N_A \cdot e_0}$$

$$\boxed{n_e = \frac{N}{V} = \frac{m_1 N \cdot N_A}{m_1 V \cdot N_A} = \frac{m}{V}\frac{N_A}{M} = \frac{\rho N_A}{M}}$$

$$\boxed{\bar{v} = \frac{I \cdot M}{\pi r^2 \cdot \rho \cdot N_A \cdot e_0}} = 9{,}43 \cdot 10^{-5}\ \frac{m}{s}$$



$$\frac{1 \cdot 64}{\pi \cdot 0{,}25 \cdot 10^{-6}\ 8000 \cdot 6 \cdot 10^{26} \cdot 1{,}6 \cdot 10^{-19}} = \frac{64}{\pi \cdot 0{,}25 \cdot 8000 \cdot 6 \cdot 1{,}6 \cdot 10}$$

M-Ž

1. Tanka kovinska žica, ki je zvita v polkrog polmera R = 0.1 m , je enakomerna nabita tako, da je naboj na dolžinsko enoto enak 10^{-4} C/m. V točki, ki je enako oddaljena od vseh delov žice, se nahaja pozitiven točkasti naboj q = 10^{-9} C. Določite smer in velikost trenutne sile, ki deluje na ta naboj ?

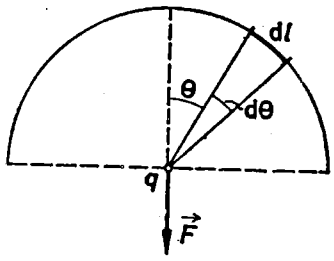
$\lambda = 10^{-4}$ C/m

▶ U situaciji kao na skici horizontalne komponente električnog polja simetričnih elemenata dužine niti se poništavaju, a vertikalne se zbrajaju. Rezultantno polje niti djeluje vertikalno prema dolje. To je ujedno i smjer rezultantne sile na naboj q.

Dakle, sumiramo vertikalne komponente sila svih elemenata dužine niti:

$$F = qE = q\frac{1}{4\pi\varepsilon_0}\int\frac{\lambda\,dl}{R^2}\cos\Theta = \frac{q\lambda}{4\pi\varepsilon_0 R}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos\Theta\,d\Theta,$$

$$F = \frac{2q\lambda}{4\pi\varepsilon_0 R} = 18\cdot 10^{-3}\,\mathrm{N}.$$



$\int \cos x = \sin x$

A-L

1895

1. Štirje identični točkasti (pozitivni) in kakšen naboji (vsak po 1 As) se nahajajo v ogljiščih kvadrata. ~~Kje~~ Kam moramo postaviti peti točkasti naboj, da bo sistem v ravnotežju, in kolikšna mora biti velikost tega naboja ?

$e = 1$ As

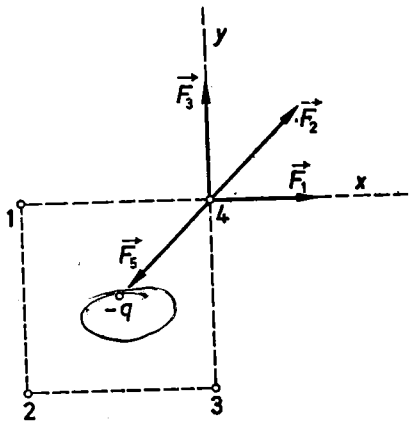
$$x\text{-os: } F_1 + F_2\cos\alpha - F_5\cos\alpha = 0$$

$$y\text{-os: } F_3 + F_2\sin\alpha - F_5\sin\alpha = 0$$

$$x\text{-os: } k\frac{e^2}{a^2} + \left[k\frac{e^2}{(\sqrt{2}a)^2} - k\frac{eq}{(\sqrt{2}a/2)^2}\right]\frac{1}{\sqrt{2}} = 0$$

Iz toga slijedi:

$$q = e\left(\frac{\sqrt{2}}{2} + \frac{1}{4}\right) = 0{,}96\,e.$$



$q = 0{,}96$ As

5. Bakreno žico s presekom 0,5 mm^2 in dolžino 20 m priključimo na akumulator z gonilno napetostjo 1,5 V. Za koliko miliamperov se zmanjša tok po žici vsako sekundo, če je žica toplotno izolirana? Gostota bakra je 8,9 g/cm^3, specifična toplota 390 J/kgK in specifični upor 0,017 $ohm.mm^2/m$. Specifični upor se poveča za 0,4 %, če se temperatura poveča za 1 stopinjo.

$U_0 = 1.5\ V$

$l = 20\ m$

$S_0 = 0.5\ mm^2$

$\rho =$

$c_p =$

$\xi = 0.017\ \Omega mm^2/m$

$\alpha = 0.004\ K^{-1}$

$\frac{dy}{dt} = ?$

$$R(T + dT) = R(T) + \alpha\, dT$$

$$(1)\quad \boxed{\frac{dR}{R} = \alpha\, dT}$$

$$P\,dt = dQ = c_p m\, dT$$

$$\Downarrow$$

$$(2)\quad \boxed{dT = \frac{P\,dt}{c_p m}}$$

$$y = \frac{U_0}{R}$$

$$\boxed{dy = U_0\left(-\frac{1}{R^2}\right) dR} \quad (3)$$

(1) vstavi v (3) : $dy = -\frac{U_0}{R^2} R\alpha\, dT$

upoštevaj (2) : $dy = -\frac{U_0}{R^2} R\alpha \frac{P\,dt}{c_p m} \Rightarrow \boxed{\frac{dy}{dt} = -\frac{U_0 \alpha P}{R c_p m}}$

$R = \frac{\xi l}{S_0}$ $\quad m = \rho l S$ $\quad P = \frac{U_0^2}{R} = \frac{U_0^2 S_0}{\xi l}$

v telo dolgem času

4. 9.

7. Skozi upornik 8 Ω steče naboj 30 As. Koliko toplote se sprosti v uporniku, če pada tok (eksponentno) proti 0 tako, da se vsakih 24 sekund zmanjša za polovico ?

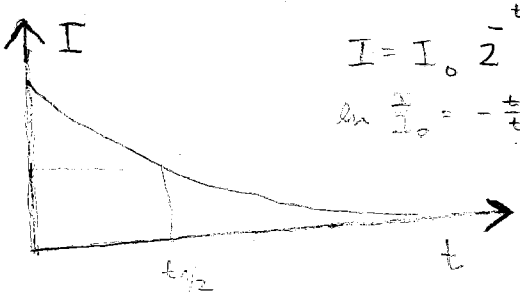
$R = 8\,\Omega$

$e_0 = 30\,As$

$t_{1/2} = 24\,s$

$Q = ?$

$I = \frac{de}{dt}$



$I = I_0\, 2^{-t/t_{1/2}}$

$\ln \frac{I}{I_0} = -\frac{t}{t_{1/2}} \ln 2$

$I = I_0\, e^{-\frac{\ln 2}{t_{1/2}} \cdot t}$

$$Q = A = \int U\,de = \int JR\,de = \int_0^\infty R I^2\,dt$$

$$e_0 = \int de = \int I\,dt = \int_0^\infty I_0\, e^{-\frac{\ln 2}{t_{1/2}} \cdot t} = -\frac{t_{1/2}}{\ln 2} I_0\, e^{-\frac{\ln 2}{t_{1/2}} \cdot t}\Big|_0^\infty = I_0 \cdot \frac{t_{1/2}}{\ln 2}$$

$$\boxed{I_0 = \frac{e_0 \cdot \ln 2}{t_{1/2}}}$$

$$Q = \int_0^\infty R \cdot \left[\frac{e_0 \cdot \ln 2}{t_{1/2}}\right]^2 e^{-\frac{2 \cdot \ln 2}{t_{1/2}} \cdot t} = -R \left[\frac{e_0 \cdot \ln 2}{t_{1/2}}\right]^2 \cdot \frac{t_{1/2}}{2 \ln 2}\, e^{-\frac{2 \ln 2}{t_{1/2}} \cdot t}\Big|_0^\infty \Rightarrow$$

$$\boxed{Q = R \cdot \frac{e_0^2 \cdot \ln 2}{2 \cdot t_{1/2}} = \underline{104\ J}}$$

E22

18.) Če potopimo plošči s površino 5 cm^2 v raztopino KCl 2,5 cm narazen in vzpostavimo med ploščicama razliko napetosti 5o V, teče tok 1,2 mA. Izračunaj specifično prevodnost elektrolitske raztopine? Kakšna je gibljivost K in Cl ionov, če koncentracija KCl 100 mmol/l. Predpostavimo, da je gibljivost K in Cl ionov enaka.

$l = 2{,}5\ cm$, $S = 5\ cm^2$

$V = 50\ V$

$I = 1{,}2\ mA$

$\oplus \rightarrow$ $\leftarrow \ominus$

$KCl \rightarrow K^+ \ Cl^-$

a) $R = \frac{U}{I} = 41 \cdot 10^3\ \Omega$

b) $\sigma = \frac{1}{R}\frac{l}{S} = 1.2 \cdot 10^{-3}\ \Omega^{-1} m^{-1} = \frac{Il}{US}$

$\sigma = n e_0 (\beta^- + \beta^+) = 2 n e_0 \beta$

$\sigma = 2 n e_0 \beta \Rightarrow \boxed{\beta = \frac{\sigma}{2 n e_0}}$

Kolšen je n?

$c = \frac{N}{N_A V} = \frac{n}{N_A}$

$n = \frac{100 \cdot 10^{-3}\ mol}{10^{-3}\ m^3} = \frac{100 \cdot 10^{-3} \cdot 6 \cdot 10^{23}}{10^{-3}}$

$\boxed{N_A = 6 \cdot 10^{26}/kmol}$

$R = \frac{\zeta l}{S}$, ζ = specifični upor

$R = \frac{l}{\sigma S}$, σ = specifična prevodnost

$\boxed{E = \frac{U}{l}}$ $\quad I = \frac{dq}{dt}$, $\boxed{j = \frac{I}{S}}$

$\boxed{j = \sigma E \ / \cdot S}$

$I = j \cdot S = \sigma \cdot S \cdot E = \sigma \cdot S \frac{U}{l}$

$I = U \cdot \sigma \frac{S}{l} = U/R \Rightarrow \boxed{R = \frac{1}{\sigma}\frac{l}{S}}$

enačba gibanja: $\boxed{m\bar{a} = eE - \gamma \bar{v}}$

stacionarno stanje: $a = 0 \Rightarrow \bar{v} = \frac{e}{\gamma}\bar{E}$

$\boxed{\bar{v} = \beta \cdot \bar{E}}$ $\longrightarrow$

gibljivost

$j = \frac{I}{S} = \frac{dq}{S\,dt} = \frac{(S \cdot dx \cdot n \cdot) e_0}{S\,dt} = n e_0 v$

elektrolit ($\oplus$ in $\ominus$)

$j = n^-(-e_0)(-\langle v_E^- \rangle) + n^+ e_0 \langle v_E^+ \rangle =$

$= n^- e_0 \langle v_E^- \rangle + n^+ e_0 \langle v_E^+ \rangle$, $\boxed{n^- = n^+}$

$j = n^+ e_0 (\beta^- + \beta^+) E$

$\boxed{\sigma = n \cdot e_0 (\beta^- + \beta^+)}$

$n = \frac{N}{V}$

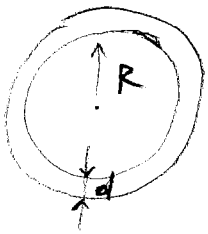
2. Kovinska krogla polmera R = 10 cm nosi naboj 0,1 μC. Krogla je obdana s slojem gume debeline 1 cm z dielektričnostjo ε = 2,2. Določite energijo električnega polja znotraj omenjenega sloja gume! F2

$R = 0{,}1\ m$

$e = 0{,}1\ \mu C$

$d = 0{,}01\ m$

$\varepsilon = 2{,}2$



$$w = \varepsilon_0 \varepsilon \frac{E^2}{2}$$

$$E = \frac{e}{S \varepsilon \varepsilon_0} = \frac{e}{\varepsilon_0 \cdot \varepsilon \cdot 4\pi r^2}$$

$$W = \int_R^{R+d} w\, dV = \int_R^{R+d} w\, 4\pi r^2 dr = \int_R^{R+d} \varepsilon_0 \varepsilon \frac{1}{2} \frac{e^2 \cdot 4\pi r^2 dr}{\varepsilon^2 \varepsilon_0^2\, 16\pi^2 r^4} =$$

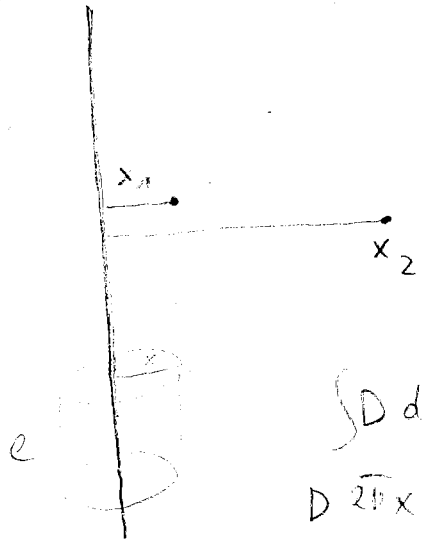
$$= \int_R^{R+d} \frac{e^2}{8\pi \varepsilon \cdot \varepsilon_0} \cdot \frac{dr}{r^2} = \frac{-e^2}{8\pi \varepsilon \varepsilon_0} \left. \frac{1}{r} \right|_R^{R+d} = \frac{e^2}{8\pi \varepsilon \varepsilon_0} \cdot \frac{d}{R(R+d)} \Rightarrow$$

$$\boxed{W = \frac{e^2}{8\pi \varepsilon \varepsilon_0} \frac{d}{R(R+d)} = 1{,}86 \cdot 10^{-5}\ J}$$

FOR kolokvij 90/91

2. 1 cm od enakomerno nabite, neskončno dolge žice $\mu = +10^{-7}$ As/m spustimo proton (z začetno hitrostjo 0 m/s). Kolikšna je njegova hitrost 5 cm od žice? V kateri smeri se giblje? $m_p = 1{,}7 \cdot 10^{-27}$ kg

IE2



$\mu = \frac{e_i}{l} = 10^{-7}$ As

$x_1 = 1$ cm

$x_2 = 5$ cm

$m_p = 1{,}7 \cdot 10^{-27}$ kg

$$\int D\, dS = e_i$$

$$D\, 2\pi x\, l = e_i$$

$$D = \frac{e_i}{2\pi x \cdot l} = \frac{\mu}{2\pi x}, \quad D = \varepsilon_0 E$$

$$\boxed{E = \frac{\varepsilon_0 \mu}{2\pi x}}$$

$$E = -\operatorname{grad} \varphi = -\frac{d\varphi}{dx}$$

$$\Delta\varphi = -\int E\, dx, \quad F = eE, \quad \Delta W = e\Delta\varphi$$

$$\Delta W = +\int_{x_1}^{x_2} F\, dx = +e\int_{x_1}^{x_2} E\, dx = +\frac{e\,\varepsilon_0 \mu}{2\pi}\int_{x_1}^{x_2}\frac{dx}{x} = \frac{e\,\varepsilon_0 \mu}{2}\ln\frac{x_2}{x_1}$$

$$\Delta W = \frac{m_p v^2}{2} \Rightarrow v = \sqrt{\frac{2\Delta W}{m_p}} =$$

7. Ploščati kondenzator s površino plošč 150 cm^2 in začetnim razmikom med ploščama 1 mm je priključen na akumulator z napetostjo 12 V. Kolikšen tok odteka s kondenzatorja 1ms potem, ko začnemo oddaljevati plošči kondenzatorja s stalno hitrostjo 1 m/s ?

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}$ As/Vm

$S = 150\ cm^2 = 1{,}5 \cdot 10^{-2}\ m^2$

$d_0 = 1\ mm = 0.001\ m$

$U_0 = 12\ V,\ v = 1\ m/s$

$I(t = 0{,}001\ s) = ?$

$$\boxed{e = C U_0}$$

$$\boxed{C = \varepsilon_0 \frac{S}{d} = \varepsilon_0 \frac{S}{(d_0 + vt)}}$$

$$I = -\frac{de}{dt} = -\frac{d(CU_0)}{dt} = +U_0 \cdot \varepsilon_0 \cdot S \cdot \frac{v}{(d_0 + vt)^2} = \underline{0{,}4\ \mu A}$$

Vrtljivi kondenzator ima kapaciteto, ki se lahko spreminja od 0,1 μF do 0,5 $.10^{-2}$ μF. Pri maksimalni vredmosti kapacitete kondenzator nabijemo tako, da je potencialna razlika 50 V. Nato kondenzator odklopimo od električnega izvira. Kakšna (je končne potencialna razlika med ploščama in) kakšno delo moramo opraviti med vrtenjem plošč, če plošči zasučemo tako, da je kapaciteta minimalna ?

$C_2 = 0.5 \cdot 10^{-2}\,\mu F$, $C_1 = 0{,}1\,\mu F$

pri $C_1 = 0.1\,\mu F$ je $U_1 = 50\,V$

a) končna potencialna razlika :

$$e = C_1 U_1 = C_2 U_2 \Rightarrow U_2 = \frac{C_1 U_1}{C_2} = \underline{1000\,V}$$

b) opravljeno delo :

$$A = \frac{1}{2} C_2 U_2^2 - \frac{1}{2} C_1 U_1^2 = 2.5 \cdot 10^{-3}\,J - 0.125 \cdot 10^{-3}\,J = \underline{2.375 \cdot 10^{-3}\,J}$$

$$\left(A = \int U\,dQ = \int CU\,dU = \frac{1}{2} CU^2 \right)$$

Elektron z kinetično energijo 10 eV kroži v ravnini, ki je pravokotna na smer magnetnega polja $B = 10^{-4}\,T$. Kakšen je radij kroga po katerem kroži elektron in v kakšnem času naredi en obhod?

$$\vec{F} = e\,\vec{v} \times \vec{B}$$

$$m a_r = m \frac{v^2}{r} = |e| v B$$

$$\rightarrow m v = |e| r B$$

$$r = \frac{m v}{|e| B} = 11\,cm$$

$$W_k = \frac{m v^2}{2} = 10\,eV$$

$$\Downarrow$$

$$v = \sqrt{\frac{2 W_k}{m}} = 1.8 \cdot 10^{6}\,\frac{m}{s}$$

$$\boxed{\text{ki } m_e = 9.1 \cdot 10^{-31}\,kg}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m v}{e B \cdot v} = \frac{2\pi m}{e \cdot B} = 3.6 \cdot 10^{-7}\,s$$

4. ~~Magnetnico postavimo v smer magnetnega polja, jo malo zasučemo in spustimo.~~ (le pričnemo postote mag. polja) Magnetnica zaniha s frekvenco 0,8 s^{-1}. Kolikšna je gostota magnetnega polja ? V zemeljskem magnetnem polju z vodoravno komponentno gostote magnetnega polja $2 \cdot 10^{-4}$ T niha magnetnica okoli navpične osi s frekvenco 0,02 s^{-1}.

(2)

(1)

$$\boxed{\vec{M} = \vec{p}_m \times \vec{B}}$$

$$\boxed{M = J \cdot \alpha}$$

$$-p_m B \sin\varphi = J \cdot \alpha \quad , \sin\varphi \approx \varphi$$

$$-\varphi \frac{p_m B}{J} = \alpha$$

$$\varphi = \varphi_0 \sin(2\pi\nu t)$$

$$\dot{\varphi} = \varphi_0 (2\pi\nu) \cos(2\pi\nu t)$$

$$\ddot{\varphi} = -(2\pi\nu)^2 \underbrace{\varphi_0 \sin(2\pi\nu t)}_{\varphi}$$

$$\alpha = \ddot{\varphi} = -(2\pi\nu)^2 \varphi$$

$$\Downarrow$$

$$\boxed{(2\pi\nu)^2 = \frac{p_m \cdot B}{J}}$$

$B_2 = 2 \cdot 10^{-4}$ T , $\nu_2 = 0.02\ s^{-1}$

$B_1 = ?$, $\nu_1 = 0.8\ s^{-1}$

$$\downarrow$$

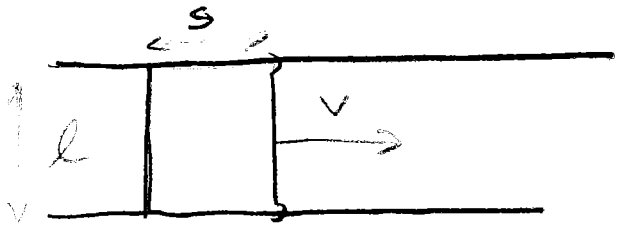
$$\frac{\nu_1^2}{\nu_2^2} = \frac{B_1}{B_2} \Rightarrow \boxed{B_1 = B_2 \frac{\nu_1^2}{\nu_2^2}} = 0.$$

0.32 T

2. Dva dolga vzporedna vodnika v razmiku 5 cm sta na prvih krajiščih povezana s prečko, medtem ko druga prečka drsi po njiju. Oba vodnika in obe prečki imajo enak presek in enak specifični upor. Pravokotno na ravnino vodnikov je homogeno magnetno polje z gostoto 1 T. Premična prečka, ki je na začetku ob prečki na krajišču, se začne gibati enakomerno pospešeno. Kolikšna je razdalja ~~od~~ med prečkama ~~do krajišča~~ v trenutku, ko je inducirani tok največji ?

$l = 5\,cm$

$B = 1\,T$



$v = a \cdot t$

$s = \int v\,dt = \frac{at^2}{2}$

$U_i = \frac{d\Phi}{dt} = \frac{d(B \cdot S)}{dt} = B\frac{dS}{dt} = B\frac{d(l \cdot x)}{dt} = B \cdot l \cdot \frac{dx}{dt} = B \cdot l \cdot a \cdot t$ ($\frac{dx}{dt} = a \cdot t$)

ali $U_i = B l v = B l a t$

$R = \frac{\rho(2l + 2x)}{S} = \frac{\rho(2l + \frac{at^2}{2})}{S}$

$U = I \cdot R$

$I = U/R = \frac{B \cdot l \cdot a t \cdot S}{\rho(2l + 2 \cdot \frac{at^2}{2})} = \frac{B \cdot l \cdot a \cdot S \cdot t}{\rho(2l + at^2)} =$

$\frac{dI}{dt} \propto \frac{1}{2l + at^2} - \frac{t(2at)}{(2l + at^2)^2} = 0$

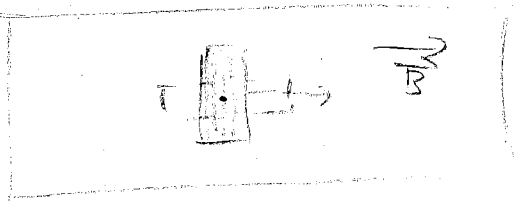
$2l + at^2 = 2at^2$

$l = \frac{at^2}{2} = s = 5\,cm$

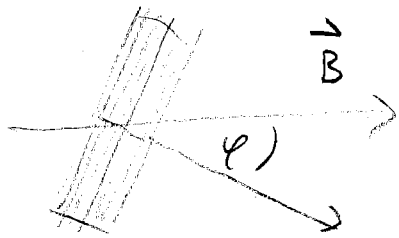
1. Po tuljavi z dolžino 0,5 m in z 300 ovoji teče tok 20 A. V tuljavi je merilna tuljavica s polmerom 3 cm, 50 ovoji in z uporom 5 ohmov. Spočetka sta geometrijski osi tuljave in tuljavice vzporedni. Nato hitro zasučemo tuljavico za 30^0 okoli osi, ki je pravokotna na njeno geometrijsko os. Kolikšen sunek toka pokaže balistični galvanometer, ki je priključen na merilno tuljavico. Galvanometer ima upor 15 ohmov.

$\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am$

$\left.\begin{array}{l} l = 0.5\ m \\ N = 300 \\ I = 20A \end{array}\right\}$ velika tuljava



$\left.\begin{array}{l} r' = 3\ cm \\ N' = 50 \\ R' = 5\ \Omega \end{array}\right\}$ mala tuljavica



$\varphi = 30^0$

$R_{\emptyset} = 15\ \Omega$

$$\boxed{\int I\,dt = ?}$$

$$U_i = \frac{d\phi}{dt} \Rightarrow \underbrace{\int I\,dt} = \Delta\phi' = BS'\cos\varphi - BS' = \underbrace{BS'(\cos\varphi - 1)}$$

$$\boxed{R \rightarrow R' + R_{\emptyset}} \qquad \boxed{U_i = I \cdot R}$$

velika tuljava: $B = \mu_0 H$

$H \cdot l = N \cdot I$

$$\underbrace{\int U_i\,dt} = R\int I\,dt = \underbrace{(R' + R_{\emptyset})\int I\,dt}$$

$$H = \frac{NI}{l} \Rightarrow \boxed{B = \frac{\mu_0 N I}{l}}$$

torej:

$$(R' + R_{\emptyset})\int I\,dt = N'BS'(\cos\varphi - 1) = \frac{\mu_0 N I}{l} N' \cdot S'(\cos\varphi - 1)$$

$$\int I\,dt = \frac{\mu_0 N I \cdot N' \cdot S' \cdot (\cos\varphi - 1)}{l\,(R' + R_{\emptyset})}$$

$\cos\varphi - 1 = -0.134$

$$\boxed{\int I\,dt = \frac{\mu_0 N \cdot N' \cdot I \cdot \pi r'^2 (\cos\varphi - 1)}{l\,(R' + R_{\emptyset})}} = -1{,}428 \cdot 10^{-5}\ As$$

4. Kvadraten okvir iz žice s specifičnim uporom 0,03 $\Omega mm^2/m$ in presekom 1 mm^2 ima površino 121 cm^2. Okvir vrtimo enakomerno s frekvenco 20 s^{-1} okoli njegove simetrale, ki je pravokotna na magnetno polje z gostoto 1 T. Kolikšna je amplituda toka, ki teče po okvirju ?

FOR 1995

4.9.

$\rho = 0{,}03\ \Omega mm^2/m$

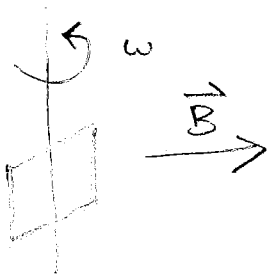
$S = 121\ cm^2 \Rightarrow a = 0{,}11\ m$

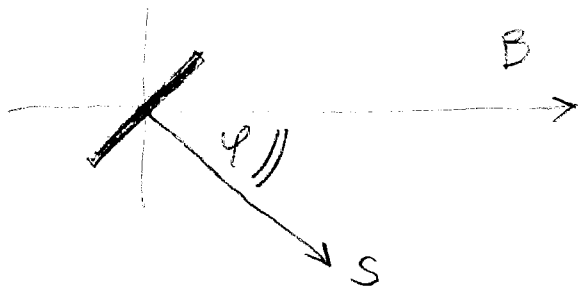
$S_0 = 1\ mm^2$

$B = 1\ T$

$\nu = 20\ s^{-1}$

$\omega = 2\pi\nu$





$$\phi = BS\cos\varphi = BS\cos(\omega t)$$

$$U_i = \left|\frac{d\phi}{dt}\right| = BS\omega\sin\omega t \qquad R = \frac{\rho_k \cdot 4a}{S_0}$$

$$I = \frac{U_i}{R} = \frac{BS\omega\sin\omega t}{R} = \frac{BS\omega\cdot S_0\sin\omega t}{\rho_k\cdot 4a} = I_0\cdot\sin\omega t$$

I_0

$$I_0 = \frac{B\omega S S_0}{\rho_k\cdot 4a} = \frac{B 2\pi\nu S\cdot S_0}{\rho_k\cdot 4a} = 115{,}2\ A$$

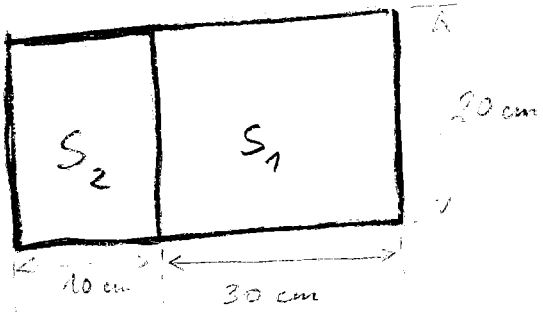
$$R = \frac{0.03\cdot 4\cdot 0{,}11\ m}{1} = 0{,}0132\ \Omega$$

$a = 10\ cm$

$$I_0 = \frac{B a^2 (2\pi\nu)}{0.0132} = \frac{10^{-2}\ 2\pi\cdot 20}{0.0132} = 95{,}2\ A$$

E 51

Iz bakrene žice s prerezom $1\,mm^2$ in specifičnim uporom $0.017\,\Omega\,mm^2/m$ sestavimo ovoja kot kaže slika. Homogeno magnetno polje z gostoto $0{,}4\,T$ je pravokotno na ravnino ovoja. Kolikšen tok teče po srednjem vodniku, če magnetno polje v $10\,s$ enakomerno zmanjšamo na nič?



$$R = \frac{\rho \cdot l}{S}$$

$S = 1\,mm^2$

$\rho_{Cu} = 0.017\,\Omega\,mm^2/m$

$B_0 = 0{,}4\,T$

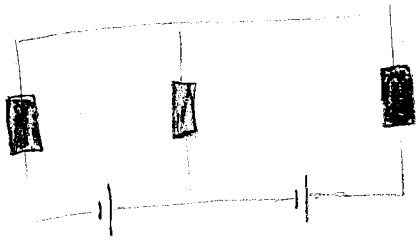
$t_0 = 10\,s$

$$\Phi_1 = B(t) \cdot S_1$$

$$\Phi_2 = B(t) \cdot S_2$$

$$B(t) = B_0 - \frac{B_0}{t_0} \cdot t = B_0\left(1 - \frac{t}{t_0}\right)$$

$$U_1 = \frac{d\Phi_1}{dt} = S_1\left(-\frac{B_0}{t_0}\right), \quad U_2 = \frac{d\Phi_2}{dt} = S_2\left(-\frac{B_0}{t_0}\right)$$



$$B_0 - \frac{\Delta B}{t_0} t$$

[E69]

3. V kondenzator z navpičnima kvadratnima ploščama s stranico 10 cm v razdalji 3 mm nalijemo do polovice tekočino z gostoto 1 kg/dm³ in dielektričnostjo 80. Nato kondenzator nabijemo z napetostjo 800 V in izoliramo. Koliko dela opravimo, ko plošči toliko približamo, da tekočina napolni prostor med ploščama ?

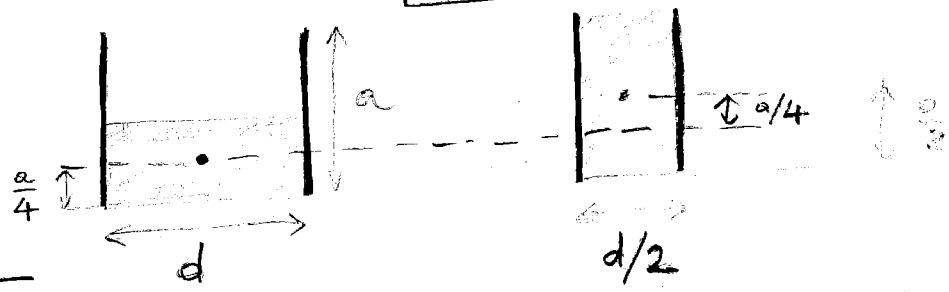
$a = 0{,}1\ m$ $\quad \varepsilon_0 = 8{,}85 \cdot 10^{-12}\ As/Vm$ $\quad \boxed{\Delta h = \frac{a}{4}}$

$d = 0{,}003\ m$

$\rho = 1000\ kg/m^3$

$\varepsilon = 80$

$U_1 = 800\ V$



$$\boxed{A = \Delta W_e + \Delta W_p = \frac{1}{2} C_2 U_2^2 - \frac{1}{2} C_1 U_1^2 + \rho \cdot V \cdot g \cdot \Delta h}$$

$$C_1 = \varepsilon_0 \frac{S}{2 \cdot d} + \varepsilon \varepsilon_0 \frac{S}{2 \cdot d} = \varepsilon_0 S (\varepsilon + 1)/2d \qquad \boxed{S = a^2}$$

$$C_2 = \varepsilon_0 \varepsilon \frac{S \cdot 2}{d}$$

kond. izol.: $e = CU = konst.$ $\Rightarrow C_1 U_1 = C_2 U_2 \Rightarrow \boxed{U_2 = \frac{C_1}{C_2} \cdot U_1}$

$$A = \frac{1}{2} C_2 \cdot \frac{C_1^2}{C_2^2} U_1^2 - \frac{1}{2} C_1 U_1^2 - \rho \cdot g \cdot d \cdot a^3/8 =$$

$$= C_1 U_1^2 \left[\frac{C_1}{C_2} - 1 \right] / 2 - \rho \cdot g \cdot d \cdot a^3/8 =$$

$$= \frac{\varepsilon_0 \cdot a^2 (\varepsilon + 1)}{4d} \left[\frac{\varepsilon_0 a^2 (\varepsilon + 1) \cdot d - 2 d \varepsilon_0 \varepsilon a^2 2}{2d \cdot \varepsilon_0 \cdot \varepsilon \cdot a^2 \cdot 2} \right] - (\ ) \Rightarrow$$

$$\boxed{\frac{-\varepsilon_0 \cdot a^2 \cdot U_1^2}{16 \cdot \varepsilon \cdot d} (\varepsilon + 1)(3\varepsilon - 1) + \rho \cdot g \cdot d \cdot a^3/8 = A}$$ št

$-2{,}86 \cdot 10^{-4} + 3{,}67 \cdot 10^{-3}$

$A = 3{,}38 \cdot 10^{-3}\ J$

FOR N

E2

1. Prosto gibljiv proton damo (ob času t=0) v električno polje, ki se s časom spreminja kot $E(t) = a \cdot t$ ($a = 2\,Vm^{-1}s^{-1}$). Kolikšno hitrost doseže proton v prvi milisekundi v takem polju? Kakšno razdaljo premosti v tem času ?

A–M

$$F = eE = m_p a = m_p \frac{dv}{dt}$$

$$\boxed{t = 1\,ms}$$

$$eE = m_p \frac{dv}{dt}$$

$$\boxed{m_p = 1{,}67 \cdot 10^{-27}\,kg}$$

$$\frac{eE}{m_p} dt = dv$$

$$\int_0^t \frac{eat}{m_p} dt = \int_0^v dv$$

$$\boxed{\frac{eat^2}{2m_p} = v}$$

$$v = \frac{1.6 \cdot 10^{-19} \cdot 2 \cdot 10^{-6}}{2 \cdot 1{,}67 \cdot 10^{-27}} = \frac{3{,}2 \cdot 10^{-25}}{2 \cdot 1{,}67 \cdot 10^{-27}} = \frac{3.2 \cdot 10^{2}}{2 \cdot 1.67 \cdot} =$$

$$= 95.8\ m/s$$

$$s = \int_0^t v\,dt = \int_0^t \frac{ea}{2m_p} t^2 dt = \frac{ea}{2m_p} \cdot \frac{t^3}{3} = 0{,}032\ m$$

$$\frac{1{,}6 \cdot 10^{-19} \cdot 2 \cdot 1 \cdot 10^{-9}}{2 \cdot 1{,}67 \cdot 10^{-27} \cdot 3} = \frac{1{,}6 \cdot 2 \cdot 10^{-18}}{6 \cdot 1{,}67 \cdot 10^{-27}}$$

februar 1991

Fizika II

A–M

zelo dolgega

2. V ravnini ravnega vodnika, po katerem teče tok 300 A, je žičnati ovoj v obliki kvadrata s stranico 5 cm. Dve stranici ovoja sta vzporedni z vodnikom, bližnja pa je oddaljena od vodnika za 3 cm. Kolikšen tokovni sunek nastane v ovoju, če izključimo tok po vodniku ? Površina preseka žice je 1 mm^2, specifična upornost žice pa je 0,02 $\Omega \cdot mm^2/m$.

M-Ž

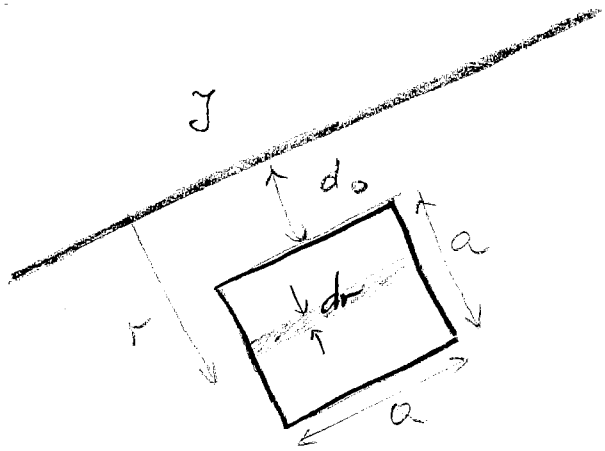
$I = 300\ A$

$a = 5\ cm$

$d_0 = 3\ cm$

$S_0 = 1\ mm^2$

$\xi = 0{,}02\ \Omega \cdot mm^2/m$



$\oint B_0 \cdot 2\pi r = I \mu_0$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$R = \frac{\xi \cdot 4a}{S_0}$$

$\int I\,dt = ?$ $\qquad U_i = I \cdot R$

$$\int U_i\,dt = R\int I\,dt = \Delta\Phi_m = \Phi_m = \int B\,dS \Rightarrow \boxed{\int I\,dt = \frac{1}{R}\int B\,dS}$$

$$\int_{d_0}^{d_0+a} B\,dS = \int_{d_0}^{d_0+a} \frac{\mu_0 I}{2\pi r}\,a\,dr = \frac{\mu_0 I}{2\pi}\,a \ln\frac{d_0+a}{d_0} = \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{d_0}\right)$$

$$\int I\,dt = \frac{S_0}{\xi \cdot 4\cancel{a}} \cdot \frac{\mu_0 I \cancel{a}}{2\pi}\ln\left(1+\frac{a}{d_0}\right) \Rightarrow$$

$$\boxed{\int I\,dt = \frac{\mu_0 I \cdot S_0}{8\pi\xi} \cdot \ln\left(1+\frac{a}{d_0}\right) = \underline{0{,}736\ mAs}}\ \checkmark$$

$$\frac{4\cancel{\pi} \cdot 10^{-7} \cdot 3 \cdot 10^{2} \cdot 1 \cdot \cancel{10^{-6}}}{8\cancel{\pi} \cdot 2 \cdot 10^{-2}\ \cancel{10^{-6}}}\ \ln\left(1+\frac{5}{3}\right) =$$

$$= \frac{3}{4} \cdot 10^{-3} \ln\left(\frac{8}{3}\right) = 0{,}736 \cdot 10^{-3}\ As$$

E 52

Okoli feromagnetnega svitka s povprečno permeabilnostjo $\bar{\mu} = 30$ je navit toroid z $N = 300$ ovoji po katerem teče tok $I = 15$ A. V feromagnetnem svitku s povprečnim radijem $\bar{r} = 10$ cm je reža širine $b = 0.2$ cm. Kolikšni sta jakost in gostota magnetnega polja v reži ter v ostalem delu svitka?

$N = 300$

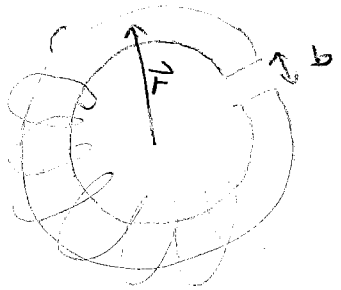
$\bar{\mu} = 30$

$I = 15$ A

$\bar{r} = 10$ cm

$b = 0.2$ cm

$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am



$$\boxed{\oint \vec{H}\, d\vec{s} = NI}$$

$$H(2\pi\bar{r} - b) + H_0 \cdot b = NI$$

$$\boxed{\begin{aligned} B &= \bar{\mu}\mu_0 H \\ B_0 &= \mu_0 H_0 \end{aligned}} \Rightarrow \quad H = \frac{B}{\bar{\mu}\mu_0}, \quad H_0 = \frac{B_0}{\mu_0}$$

$$\boxed{\oint \vec{B}\, d\vec{s} = 0}$$

$$\boxed{B = B_0}$$

$$\frac{B}{\bar{\mu}\mu_0}(2\pi\bar{r} - b) + \frac{B_0}{\mu_0} \cdot b = NI$$

$$\boxed{B_0 = B = \frac{\mu_0 N \cdot I}{\left[\frac{1}{\bar{\mu}}(2\pi\bar{r} - b) + b\right]}}$$

$$H = \frac{B_0}{\bar{\mu}\mu_0}, \quad H_0 = \frac{B_0}{\mu_0}$$

Os vrtenja leži v ravnini obroča in gre skozi njegovo težišče

1995

2. Obroč iz kovine s specifičnim uporom $\xi = 0.1\ \Omega mm^2/m$ ima radij 10 cm in presek žice 1 mm^2. Obroč vrtimo okoli premera, ki je pravokoten na magnetno polje z gostoto 0.1 T. Kolikšen povprečni navor je potreben, da se obroč zavrti 10 krat v sekundi ?

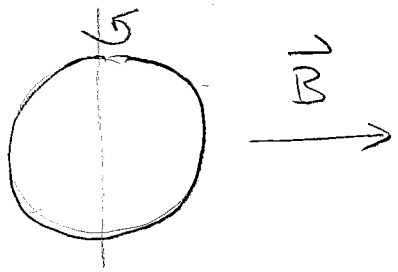
$\rho_k = 0.05\ \Omega mm^2/m$

$r = 5\ cm$

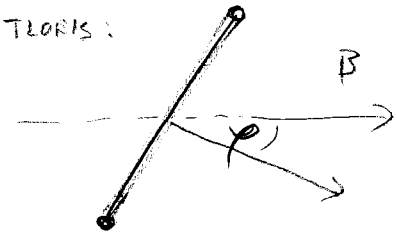
$S_0 = 1\ mm^2$

$B = 0.07\ T$

$\nu = 10\ s^{-1}$, $\boxed{\omega = 2\pi\nu}$



TEORIJA:



$\boxed{\phi = B \cdot S \cdot \cos\varphi}$ $\boxed{\varphi = \omega \cdot t}$

$$\boxed{U_i = \frac{d\phi}{dt} = -B \cdot S \cdot \omega \cdot \sin(\omega t)}$$

$\boxed{U = y \cdot R}$ $\boxed{R = \frac{\rho_k \cdot 2\pi r}{S_0}}$

$$\boxed{y = \frac{U_i}{R} = \frac{B \cdot S \cdot \omega \cdot \sin(\omega t) \cdot S_0}{\rho_k \cdot 2\pi r}}$$

$\vec{M} = \vec{p}_m \times \vec{B}$, $M = p_m B \cdot \sin\varphi = p_m \cdot B \cdot \sin(\omega t)$

$S = \pi r^2$, $S^2 = \pi^2 r^4$

$$\boxed{p_m = y \cdot S}$$

$$M = p_m \cdot B \cdot \sin(\omega t) = y \cdot S \cdot B \cdot \sin(\omega t) = \frac{B^2 S^2 \omega \cdot S_0 \quad \sin^2(\omega t)}{\rho_k \cdot 2\pi r} \Rightarrow$$

$$M = \frac{B^2 \pi r^3 \omega S_0 \quad \sin^2(\omega t)}{\rho_k \cdot 2}$$, $\langle \sin^2(\omega t) \rangle = \frac{1}{2}$

$$\boxed{\langle M \rangle = \frac{1}{4} \cdot \frac{\omega B^2 r^3 S_0 \pi}{\rho_k}}$$

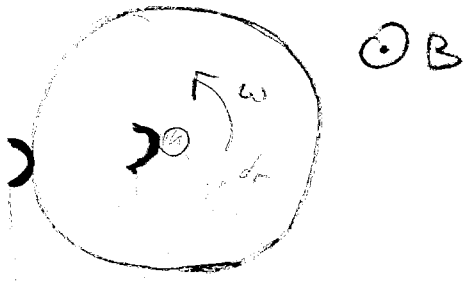
5. Krožna kovinska plošča z radijem 20 cm se enakomerno vrti s kotno hitrostjo 200 s^{-1} okoli svoje geometrijske osi v magnetnem polju B = 0,1 T. Kolikšna je napetost med osjo plošče in njenim obodom ? Magnetno polje je vzporedno z geometrijsko osjo plošče.

$R = 20\ cm = 0{,}2\ m$

$\omega = 200\ s^{-1}$

$B = 0{,}1\ T$

$U_i = ?$



$\Phi = \int \vec{B}\, d\vec{S}$

V

enačba $U_i = \frac{d\Phi}{dt}$ ne velja !

vzeli pa $U_i = \vec{v}\cdot(\vec{B}\times\vec{l})$ ostanemo

$$U_i = \int \vec{v}\cdot(\vec{B}\times d\vec{l})$$

$$U_i = \int_0^R vB\, dr = B\omega \int_0^R r\, dr \Rightarrow$$

$$U_i = \frac{1}{2} B\omega R^2 = 0.4\ V$$

M-7

1994

1. ~~Plastat~~ Tenek toroid ima 100 ovojev s presekom 2 cm^2 in srednjo razdaljo od osi 15 cm. V toroid je vključen balistični galvanometer z občutljivostjo $K = 1{,}67 \cdot 10^4$ delcev/mVs. V osi toroida je zelo dolg vodnik. Kolikšna je amplituda odklona galvanometra (v delcih), ko spustimo po vodniku tok 30 A ?

$\int I\,dt = ?$ Kolikšen je tokovni impulz?

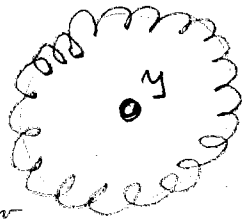
$N = 100$

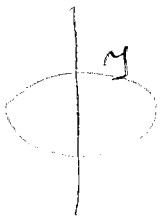
$S = 2\ cm^2$

$r = 15\ cm$

$K = \frac{delci}{\int U_i dt} = 1{,}67 \cdot 10^4 \frac{delcev}{mVs}$

$I = 30\ A$





$\oint \vec{H}\, d\vec{S} = I$

$H\, 2\pi r = I$

$B = \mu_0 H$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$U_i = \frac{d\phi}{dt} \Rightarrow \int U_i\, dt = \Delta\phi = \phi_m = N B S$$

$$\boxed{\int U_i\, dt = \frac{N \mu_0 I \cdot S}{2\pi r}}$$

$$\boxed{[ampl] = K \cdot \int U_i\, dt = K \cdot \frac{N \mu_0 I S}{2\pi r} = 40}$$

$$\frac{1{,}67 \cdot 10^4 \cdot 10^2 \cdot 4 \cdot \cancel{\pi} \cdot 10^{-7} \cdot 30 \cdot 2 \cdot 10^{-4}}{10^{-3} \cdot 2 \cdot \cancel{\pi} \cdot 5 \cdot 10^{-2}} = 40$$

3. Tuljava s 40 ovoji in površino preseka 10 cm^2 se nahaja v homogenem magnetnem polju z gostoto B = 0,8 T. Vrtilna os tuljave je pravokotna na njeno geometrijsko os, obe pa na smer magnetnega polja. S kolikšnim kotnim pospeškom se začne vrteti tuljava, ko steče po njej tok 0,2 A ? Vztrajnostni moment tuljave je 0,0015 $kg \cdot m^2$.

A – L

$N = 40$

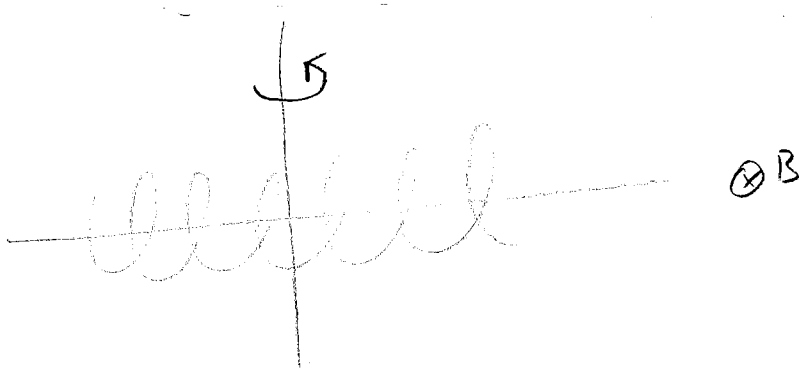
$S = 10\ cm^2$

$B = 0{,}8\ T$

$I = 0.2\ A$

$J = 1{,}5 \cdot 10^{-3}\ kg\,m^2$

$\alpha = ?$



$$p_m = NIS$$

$$\vec{M} = \vec{p}_m \times \vec{B}$$

$\varphi = 90°$

$$p_m B \sin\varphi = p_m B = J \cdot \alpha$$

$$NIS B = J \cdot \alpha \Rightarrow \alpha = \frac{NISB}{J} = 4{,}27\ s^{-2}$$

$$\frac{40 \cdot 0{,}2 \cdot 10 \cdot 10^{-4}\ 0{,}8}{1{,}5 \cdot 10^{-3}}$$

FOR

3. Polkroglasta lupina radija 2 cm je enakomerno nabita z nabojem 10^{-9} As. V središče odprtega prereza lupine postavimo proton. Kolikšna je hitrost protona na veliki oddaljenosti od lupine ?

M-7

$R = 0{,}02\ m$

$e = 10^{-9}\ As$, $e_0 = 1.6 \cdot 10^{-19}\ As$

$m_p = 1{,}67 \cdot 10^{-27}\ kg$

$$\varphi = \int d\varphi = \int \frac{de}{4\pi\varepsilon_0 R} = \frac{e}{4\pi\varepsilon_0 R}$$

$$\boxed{e_0 \varphi = \frac{m_p v^2}{2}}$$

$$\boxed{v = \left(\frac{2 e_0 \varphi}{m_p}\right)^{1/2} = \left(\frac{2 e_0 \cdot e}{m_p \cdot 4\pi\varepsilon_0 R}\right)^{1/2} = \underline{0{,}283 \cdot 10^6\ \frac{m}{s}}}$$ ✓

$$\frac{2 \cdot 1{,}6 \cdot 10^{-19} \cdot 10^{-9}}{1{,}67 \cdot 10^{-27} \cdot 4 \cdot \pi \cdot 8{,}85 \cdot 10^{-12} \cdot 2 \cdot 10^{-2}} =$$

$$\frac{3{,}2 \cdot 10^{-28}}{1{,}67 \cdot 4 \cdot \pi \cdot 8{,}85 \cdot 2 \cdot 10^{-41}} = 0{,}283 \cdot 10^{6}$$

1992

2. Središči dveh kovinskih kroglic polmerov 2 cm in 4 cm sta oddaljeni 100 cm. Vsaka od površin kroglic je spojena z enim izmed polov baterije napetosti 300 V. Oceni kolikšna sila deluje med kroglicama ? 24.9.

$r_1 = 2\,cm$

$r_2 = 4\,cm$

$R = 100\,cm$

$U = 300\,V$

$F = ?$

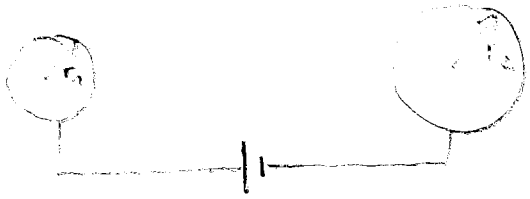
$R \gg r_1, r_2$

$\Downarrow$

naboja sferno-simetrična

$\Downarrow$

$\Downarrow$



$\varepsilon_0 E\, 4\pi r^2 = Q$

$E = \frac{Q}{4\pi\varepsilon_0 r^2} = -\frac{d\varphi}{dr}$

$\varphi = \frac{Q}{4\pi\varepsilon_0 r}$

$$\boxed{U \cong \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1} - \frac{Q_2}{r_2}\right)} \quad / \quad \boxed{Q_1 + Q_2 = 0}$$

torej:

$$\boxed{U = \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$$

$\Downarrow$

$$|Q_1| = |Q_2| = \frac{4\pi\varepsilon_0 U r_1 r_2}{r_1 + r_2}$$

$\Downarrow$

$$\boxed{F \cong \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{R^2} = \frac{4\pi\varepsilon_0}{R^2} \frac{U^2 r_1^2 r_2^2}{(r_1 + r_2^2)^2} = \underline{1{,}78 \cdot 10^{-9}\,N}}$$

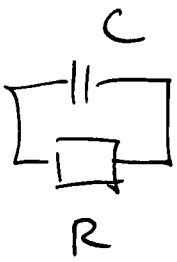
2. Kondenzator s kapaciteto 35 μF, naelektren z napetostjo 60 V, praznimo preko upora. Po 10 ms je padec napetosti na kondenzatorju 50 V. Kolikšen je celoten upor kroga ? 1997

$C = 35\ \mu F$

$U_0 = 60\ V$

$U_1(t_1 = 10\ ms) = 50\ V$

$R = ?$



$$U_1 = U_0\, e^{-\frac{t_1}{RC}}$$

$$\ln \frac{U_1}{U_0} = -\frac{t_1}{RC}$$

$$R = \frac{t_1}{C \cdot \ln \frac{U_0}{U_1}} = \underline{\underline{1567\ \Omega}}$$

$$\frac{10 \cdot 10^{-3}}{35 \cdot 10^{-6} \ln \frac{6}{5}} = \frac{10}{35 \cdot 10^{-3} \ln \frac{6}{5}} = \underline{\underline{1567\ \Omega}}$$

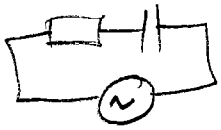
5

1995
RoR

5. Vezje z zaporedno vezanim uporom R = 50 Ω in kondenzatorjem C = 100 μF je priključeno v mestno omrežje (U_{ef} = 220 V, ν = 50 Hz). Kolikšna je amplituda toka I_0, ki teče skozi vezje ?

$R = 50\ \Omega$, $\nu = 50\ Hz$

$C = 10^{-4}\ F$



$U_{ef} = 220\ V$, $U_0 = 220 \cdot \sqrt{2}\ V$ $\boxed{U_{ef} = \frac{U_0}{\sqrt{2}}}$

$Z_R = R$, $Z_L = i\omega L$, $Z_C = -i\frac{1}{\omega C}$

$I_0 = ?$

$U_0 = Z \cdot I_0$

$|U_0| = |Z| \cdot I_0$

$$U_0 = \left(R^2 + \frac{1}{(\omega C)^2}\right)^{1/2} I_0$$

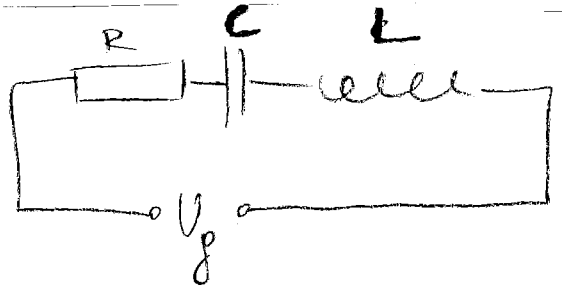
$$I_0 = \frac{U_{ef} \cdot \sqrt{2}}{\left[R^2 + \frac{1}{(\omega C)^2}\right]^{1/2}} = \underline{\underline{5.25\ A}}$$

$\omega = 2\pi\nu = 314\ s^{-1}$

$\frac{1}{(\omega C)^2} = 1014$

(ter generatorjem frekvenčne napetosti)

V vezju z zaporedno vezanim uporom, kondenzatorjem in tuljavo teče efektivni tok 300 mA in se troši povprečna moč 3 mW. Tok zaostaja za napetostjo za 30°. Kolikšna je impendanca vezja ?

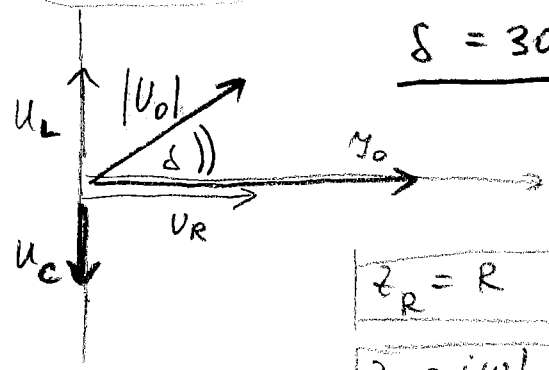


$u_g = U_0 e^{i\omega t}$

$I_{ef} = 300\ mA$

$\overline{P} = 3\ mW$

$\delta = 30°$



$$\boxed{U_0 = \left[R + i\left(\omega L - \frac{1}{\omega C}\right)\right] I_0}$$

$$\boxed{Z_R = R}$$

$$\boxed{Z_L = i\omega L}$$

$$\boxed{Z_C = -i\frac{1}{\omega C}}$$

$$\boxed{tg\,\delta = \frac{\omega L - \frac{1}{\omega C}}{R}}$$

$$\boxed{\overline{P} = \frac{1}{2} I_0 |U_0| \cos\delta = \frac{1}{2} I_0 \cdot \underbrace{U_{R0}}_{|U_0|\cos\delta} = \frac{1}{2} I_0 (R I_0) = \frac{1}{2} R I_0^2 = R (I_{ef})^2}$$

$$\boxed{I_{ef} = I_0/\sqrt{2}}$$

$$\boxed{U_{ef} = U_0/\sqrt{2}}$$

$$\Downarrow$$

(1) $R = \overline{P}/I_{ef}^2 = 0{,}0333\ \Omega = \frac{1}{30}\ \Omega$

(2) $\omega L - \frac{1}{\omega C} = R \cdot tg\,\delta = 0{,}01924\ \Omega$

$$|U_0| = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2} \cdot I_0$$

(3) $\boxed{|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 0{,}0385\ \Omega}$

ALI: $\overline{P} = \frac{1}{2} I_0 |U_0| \cos\delta = I_{ef} U_{ef} \cdot \cos\delta$

$= \frac{1}{2} I_0 |Z| I_0 \cos\delta =$

$= \frac{1}{2} I_0^2 |Z| \cos\delta \Rightarrow |Z| = \frac{\sqrt{2}\, U_{ef}}{\sqrt{2}\, I_{ef}} = \frac{\overline{P}}{I_{ef}^2 \cos\delta} = 0.0385\ \Omega$

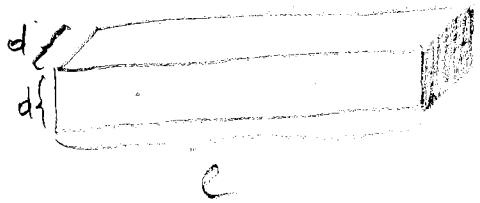
$Z = 0{,}033 + i\,0{,}019$

$|U_0| = |Z|\, I_0$

$\nu = 50\ s^{-1}$

(4)

Skozi dolg vodnik s kvadratnim presekom $S = 4.10^{-6}\ m^2$ teče tok 10 A. Koliko prostih nosilcev naboja je v vsakem metru tega vodnika, če nastane, ko ga vtaknemo v magnetno polje 0,2 T, med z magnetnim poljem vzporednima stranicama napetostna razlika 1 μV? (Hallov pojav) 1996



$S = 4.10^{-6}\ m^2$
$I = 10\ A$
$B = 0.2\ T$
$U = 1\ \mu V$

$$n = \frac{N}{V} = \frac{N}{S\cdot l} = \left(\frac{N}{l}\right)/S$$

$$\boxed{S = d^2} \Rightarrow \boxed{d = \sqrt{S} = 2\cdot 10^{-3}\ m}$$

$$j = \frac{I}{S} = \frac{dq/dt}{S} = \frac{d(x\cdot S\cdot n)e_0}{dt\cdot S} = n e_0 v \Rightarrow \boxed{v = \frac{I}{S\cdot n\cdot e_0}}$$

$$\boxed{e_0 E = e_0 v B}$$

$$\Downarrow$$

$$\left(\frac{U}{d}\right) = \frac{I}{S\cdot n\cdot e_0} B$$

$$\boxed{n = \frac{I\cdot B\cdot d}{U\cdot S\cdot e_0}}$$

$$\left(\frac{N}{l}\right)/S$$

$$\vec{F} = I\int d\vec{l}\times B$$

$$= \frac{dq}{dt} d\vec{l}\times B$$

$$\boxed{\vec{F} = e\vec{v}\times\vec{B}}$$

$$\Rightarrow \left(\frac{N}{l}\right)\cdot\frac{1}{S} = \frac{I\cdot B\cdot d}{U\cdot S\cdot e_0}$$

$$\Downarrow$$

$$\boxed{\left(\frac{N}{l}\right) = \frac{I\cdot B\cdot d}{U\cdot e_0}}$$

$$\Downarrow$$

$$\frac{N}{l} = 2.5\cdot 10^{22}\ m^{-1}$$

$$B = \frac{U\cdot e_0\cdot d}{I}$$

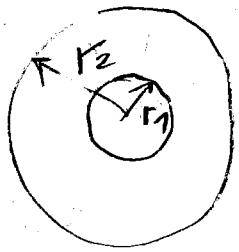
$$\boxed{\begin{array}{l} I = 5A \\ B = 0.4T \end{array}}$$

$$B = \frac{\left(\frac{N}{l}\right)U\cdot e_0}{I\cdot d}$$

obrni

1585

4. Koaksialni vodnik ima premer žile 2 mm in notranji premer plašča 6 mm. Med obema je izolator z dielektričnostjo 5 in s prebojno jakostjo električnega polja 35 kV/cm. Kolikšna je največja napetost, ki jo smemo priključiti na vodnik.



$r_1 = 10^{-3}\,m$

$r_2 = 3\cdot 10^{-3}\,m$

$\varepsilon = 5$

$E_m = 35\ kV/cm$

$\boxed{\oint D\,dS = e}$ $\boxed{D = \varepsilon_0 \varepsilon E}$

$D\,2\pi r l = e \Rightarrow E = \left(\frac{e}{2\pi\varepsilon\varepsilon_0 l}\right)\frac{1}{r}$

$\boxed{E_m = \frac{e}{2\pi\varepsilon\varepsilon_0 l}\frac{1}{r_1}} \Rightarrow \frac{e}{2\pi\varepsilon\varepsilon_0 l} = E_m r_1$

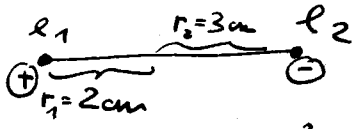
$E = -\operatorname{grad}\varphi$

$\Delta\varphi = -\int E\,dx$

$\Delta\varphi = -\int_{r_2}^{r_1}\left(\frac{e}{2\pi\varepsilon\varepsilon_0 l}\right)\frac{1}{r} = \left(\frac{e}{2\pi\varepsilon\varepsilon_0 l}\right)\ln\frac{r_2}{r_1} = E_m\cdot r_1 \ln\frac{r_2}{r_1} =$

$= \frac{35\cdot 10^3}{10^{-2}}\cdot 10^{-3}\ln 3 = 3845\ V$

5. Izračunajte jakost električnega polja na zveznici med dvema točkastima nabojema $e_1 = 2\cdot 10^{-6}$ As in $e_2 = -1\cdot 10^{-6}$ As na razdalji 2 cm od prvega naboja (e_1). Naboja sta oddaljena 5 cm.

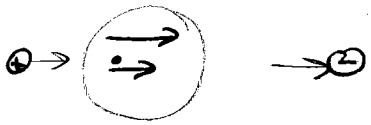


$r_1 = 2\cdot 10^{-2}\,m$

$r_2 = 3\cdot 10^{-2}\,m$

$E = \frac{e_1}{4\pi\varepsilon_0 r_1^2} - \frac{|e_2|}{4\pi\varepsilon_0 r_2^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{e_1}{r_1^2} + \frac{|e_2|}{r_2^2}\right) =$

$= 5{,}5\cdot 10^7\ \frac{V}{m}$



(5)

E 38

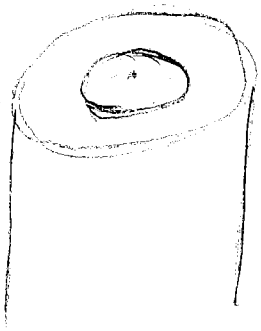
2. Po valjasti žici z radijem 1 cm teče po vsem preseku enakomerno razdeljen el. tok 100 A. Kolikšna je ~~gostota~~ jakost magnetnega polja v vodniku v razdalji 0,5 cm od osi. (rezultat: $B = 10^{-2}\ \frac{Vs}{m^2}$)

$$B = \mu_0 H$$

$$\boxed{\oint \vec{H}\, d\vec{s} = I}$$

$$\oint B\, ds = \mu_0 I$$

$$I = \frac{de}{dt} = j_0 S_0$$



$$\oint B\, ds = j_0 \pi r^2 \mu_0$$

$$B \cdot 2\pi r = j_0 \pi r^2 \mu_0$$

$$\boxed{B = \frac{I}{\pi r_0^2} \cdot \frac{\pi r^2 \mu_0}{2\pi r} = \frac{I r \mu_0}{2\pi r_0^2}}$$

$$= \frac{200 \cdot 0.005 \cdot 4\pi \cdot 10^{-7}}{(0.01)^2 \cdot 2\pi} \frac{A\, m\, Vs}{m^2\, A\, m} \quad \left[\frac{Vs}{m^2}\right]$$

$$= 10^{-3}\ \frac{Vs}{m^2} \checkmark$$

$$H = 795\ \frac{A}{m}$$

Izračunaj še energijo mag. polja v žici na dolžinskem metru?

$$\eta = \frac{\int w\, dV}{l} = \frac{\int \frac{1}{2}\frac{B^2}{\mu_0}\, dV}{l}, \quad dV = 2\pi r\, l\, dr$$

3 Ovoj iz tanke bakrene žice s specifičnim uporom $1{,}7\cdot10^{-8}\ \Omega\cdot m$ in radijem 12 cm leži v magnetnem polju z gostoto 2,0 T, ki je pravokotna na ravnino ovoja. V kolikšnem času se sme magnetna poljska gostota LINEARNO zmanjšati na nič, ne da bi se ovoj pretrgal ? Gostota magnetnega pojema linearno s časom. Napetost pri kateri se bakrena žica pretrga je 200 N/mm^2.

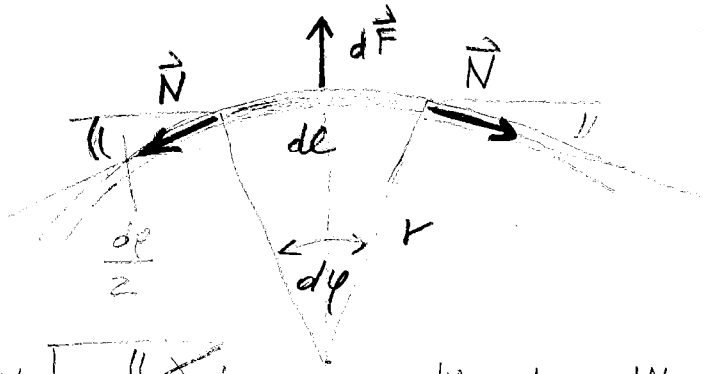
$r = 0{,}12\ m$

$B_0 = 2\ T$

$\sigma_m = 200\ N/mm^2 = 2\cdot10^8\ N/m^2$

$\xi = 1{,}7\cdot10^{-8}\ \Omega\cdot m$

$d\vec{F} = I\, d\vec{l} \times \vec{B}$, $dl = r\, d\varphi$



$\Delta t = ?$

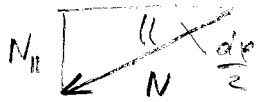
$\Delta B = B_0$, $\boxed{S = \pi r^2}$

$$\boxed{U_i = \frac{d(B\cdot S)}{dt} = S\cdot\frac{B_0}{\Delta t}}$$

$$\boxed{R = \frac{\xi\cdot 2\pi r}{S_0}}$$

$$\boxed{I = \frac{U_i}{R} = \frac{S\cdot B_0}{R\cdot\Delta t}}$$

$$\boxed{dF = I\, dl\, B}$$



$\sin\frac{d\varphi}{2} \approx \frac{d\varphi}{2} = \frac{dN_{\parallel}}{N}$

$dN_{\parallel} = N\cdot\frac{d\varphi}{2}$

ravnovesje: $\boxed{2\cdot dN_{\parallel} = dF}$

$\Downarrow$

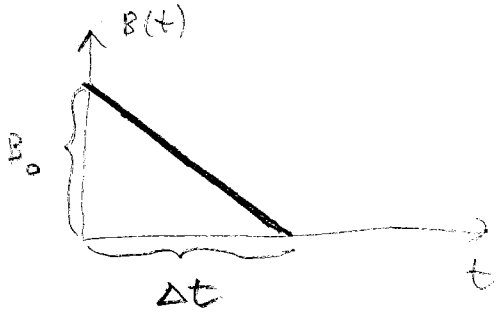
$$\boxed{N\cdot d\varphi = dF}$$

$$\boxed{\sigma = \frac{N}{S_0}}$$

$N\, d\varphi = I\, dl\, B$

$\sigma S_0\, d\varphi = \frac{S\cdot B_0}{R\,\Delta t}\cdot \overbrace{r\, d\varphi}\cdot B$

$\sigma S_0 = \frac{\pi r^2\cdot B_0\cdot S_0\cdot r\cdot B}{\xi\, 2\pi r\cdot\Delta t}$



$$\boxed{\sigma = \frac{r^2\cdot B_0\cdot B}{2\cdot\xi\cdot\Delta t}} \Rightarrow \sigma = \sigma_m,\ \check{c}e\ B = B_0 \Rightarrow \boxed{\sigma_m = \frac{r^2 B_0^2}{2\cdot\xi\cdot\Delta t}} \Rightarrow$$

$$\boxed{\Delta t = \frac{r^2\cdot B_0^2}{2\cdot\xi\cdot\sigma_m} = \underline{8{,}47\ ms}}$$

2. Točkovni izvor EM valovanja z močjo P = 100 W izotropno oddaja EM valovanje z valovno dolžino λ= 0,1 m. Kolikšna je amplituda jakosti eletričnega polja na razdalji 5m od izvora ?

$P = 100\,W$
$\lambda = 0.1\,m$
$r = 5\,m$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ As/Vm$

$$w = w_e + w_m = 2\,\frac{1}{2}\,\varepsilon_0 E^2 = \varepsilon_0 E^2$$

$$E = E_0 \cos(\,) \ : \quad \overline{w} = \varepsilon_0 E_0^2\,\overline{\cos^2(\,)} = \frac{1}{2}\,\varepsilon_0 E_0^2 \quad , \quad j = \overline{w}\,c$$

$$j = \frac{1}{2}\,\varepsilon_0 E_0^2(r)\cdot c = \frac{P_1}{4\pi r^2} \Rightarrow E_0(r) = \sqrt{\frac{P_1}{2\pi\varepsilon_0 c r^2}} = \underline{\underline{15{,}5\ \frac{V}{m}}}$$

$$\sqrt{\frac{100}{2\cdot\pi\cdot 8{,}85\cdot 3\cdot 10^{-4}\cdot\ \cdot 25}} = 1{,}548\cdot 10^{1}\ \frac{V}{m}$$

4. Prevodno kroglico s polmerom 2 cm, ki nosi naboj na površini $2\cdot10^{-6}$ As, z zelo dolgo prevodno nitko, povežemo z drugo kroglico, na kateri je 10^{-6} As naboja in meri v polmeru 5 cm. Kolikšna sta končna ~~gostota~~ naboja na kroglah ?

$$\boxed{D=\varepsilon_0 E}$$

$e_1 = 2\cdot10^{-6}\,As$, $e_2 = 10^{-6}\,As$, $r_1 = 2\,cm$, $r_2 = 5\,cm$

$\sigma_1 = ?$ $\sigma_2 = ?$

napetosti:

v ravnovesju je napetost med kroglicama nič

$$\boxed{\underset{\text{začetek}}{e_1+e_2} = \underset{\text{konec}}{e_1'+e_2'}}\,,\quad \underset{\text{začetek}}{\frac{e_1}{4\pi\varepsilon_0 r}}\,,\quad \underset{\text{konec}}{\frac{e_1'}{4\pi\varepsilon_0 r}}\,,\quad \boxed{\frac{e_1'}{4\pi\varepsilon_0 r_1} = \frac{e_2'}{4\pi\varepsilon_0 r_2}}$$

$$\Downarrow \qquad \boxed{e_1' = e_2'\frac{r_1}{r_2}}$$

$$e_1+e_2 = e_2'\left(1+\frac{r_1}{r_2}\right) \Rightarrow$$

$$\boxed{e_2' = \frac{e_1+e_2}{1+\frac{r_1}{r_2}}} = \frac{2\cdot10^{-6}\,As + 1\cdot10^{-6}\,As}{1+0{,}4} = \frac{3\cdot10^{-6}\,As}{1{,}4} = \underline{\underline{2{,}14\cdot10^{-6}\,As}}$$

$$\boxed{e_1' = \frac{e_2' r_1}{r_2}} = \frac{2{,}14\cdot10^{-6}\,As\cdot 2\,cm}{5\,cm} = \underline{\underline{0{,}856\cdot10^{-6}\,As}}$$

$$\sigma_1 = \frac{e_1'}{4\pi r_1^2} = \frac{0{,}856\cdot10^{-6}\,As}{4\cdot3{,}14\cdot4\cdot10^{-4}\,m^2} = 1{,}7\cdot10^{-4}\,As\,m^{-2}$$

$$\sigma_2 = \frac{e_2'}{4\pi r_2^2} = \frac{2{,}14\cdot10^{-6}\,As}{4\cdot3{,}14\cdot25\cdot10^{-4}\,m^2} = 0{,}68\cdot10^{-4}\,As\cdot m^{-2}$$

$\int r^{-2} = \frac{r^{-1}}{-1}$

$$\int D\,ds = e$$

$$D\,4\pi r^2 = e,\quad E = \frac{e}{4\pi\varepsilon_0 r^2}$$

$$E = -\operatorname{grad}\varphi$$

$$U = \varphi_\infty - \varphi_{p\,ovr} = -\int_p^\infty E\,ds = \frac{e}{4\pi\varepsilon_0 r}\Big|_r^\infty$$

postavimo $\varphi_\infty = 0$

$$\boxed{U = \frac{e}{4\pi\varepsilon_0 r}}$$

Galvanometer ima vrtljivo tuljavico z 200 ovoji in s presekom 2,5 cm^2. Tuljavica se giblje v magnetnem polju z gostoto 1 T, ki ima radialno smer glede na vrtilno os. Za koliko stopinj se odkloni kazalec pri toku 10 μA, če je koeficient obeh polžastih vzmeti, ki sta povezani s tuljavico skupaj enak $2,4 \cdot 10^{-6}$ Nm ?

$N = 200$

$S = 2,5\ cm^2$

$B = 1\ T$

$I = 10\ \mu A$

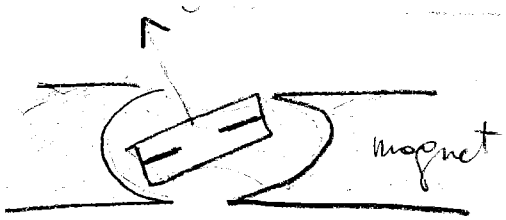
$D = 2,4 \cdot 10^{-6}\ Nm$

3)

$\int I\,dt = 5\ \mu As$

$J = 1,2 \cdot 10^{-6}\ kgm^2$

$\boxed{\sin\varphi \equiv 1}$



$p_m = NIS$

(A)

$$M = p_m B \underset{\overset{\shortparallel}{1}}{\sin\varphi} = NISB$$

$$M = NISB = D\varphi \xrightarrow{} \boxed{\varphi = NISB / D} = 0.208$$

$$= 11.9^\circ$$

$$\frac{200 \cdot 10 \cdot 10^{-6} \cdot 2,5 \cdot 10^{-4}}{2,4 \cdot 10^{-6}} = 0.208$$