izpit 1991

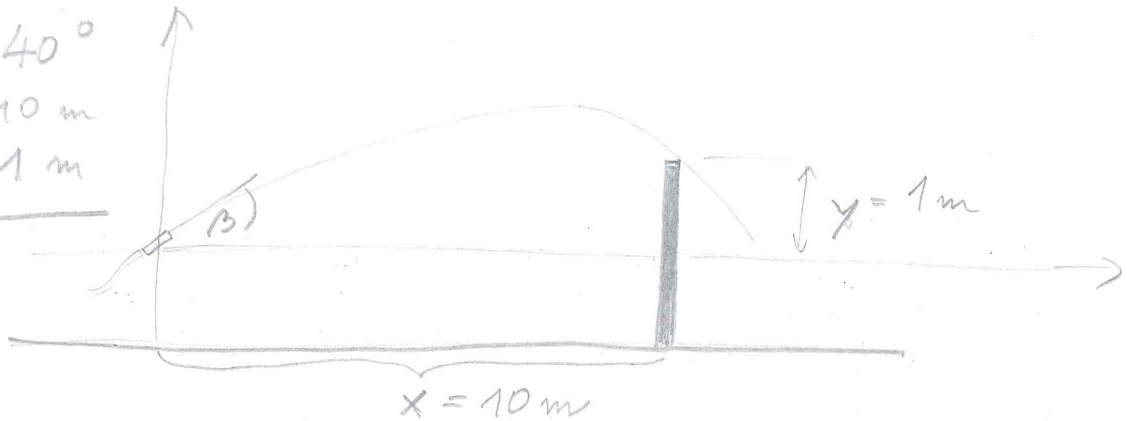
M7

2. Vrtnar usmeri curek vode iz gumijaste cevi pod kotom 40° glede na vodoravno smer tako, da curek vode ravno še pada preko 2 m visoke žive meje, katere zadnja stran je 10 m odaljena krajišča cevi iz katerega teče voda. Kolikšna je hitrost vode, ki izhaja iz cevi, če je krajišče cevi v vrtnarjevi roki na višini 1 m od tal ?

$\beta = 40^\circ$

$x = 10\ m$

$y = 1\ m$



$v_x = v_0 \cos\beta$ $\qquad x = \int v_x\, dt$

$v_y = v_0 \sin\beta - gt$ $\qquad y = \int v_y \cdot dt$

$x = v_0 \cos\beta \cdot t \Longrightarrow \boxed{t = x/(v_0 \cdot \cos\beta)}$

$y = v_0 \cdot \sin\beta \cdot t - \frac{g t^2}{2}$

$\Downarrow$

$$y = \frac{v_0 \sin\beta \cdot x}{v_0 \cos\beta} - \frac{g \cdot x^2}{2 \cdot v_0^2 \cdot \cos^2\beta}$$

$$y = x \cdot \operatorname{tg}\beta - \tfrac{1}{2} g x^2 / (v_0^2 \cdot \cos^2\beta)$$

$$\tfrac{1}{2} g x^2 / (v_0^2 \cdot \cos^2\beta) = x \operatorname{tg}\beta - y$$

$$\boxed{v_0 = \left[\frac{\frac{1}{2} g x^2}{(x \cdot \operatorname{tg}\beta - y) \cdot \cos^2\beta} \right]^{1/2}}$$

$g = 9{,}8\ m\,s^{-2}$

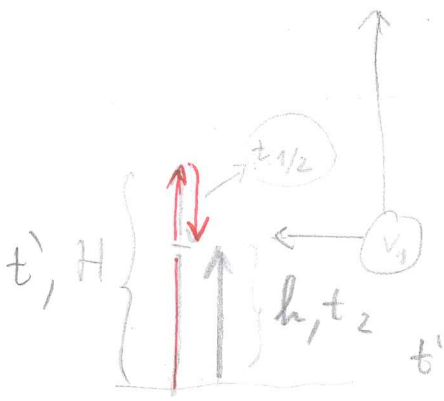
$= 10{,}63\ \frac{m}{s}$

4. Dve telesi vržemo vertikalno navzgor z iste točke in z enako začetno hitrostjo 24,5 m/s v časovnem razmiku 0,5 s. Na kateri višini se bosta telesi srečali ?

Elektronsko

$v_0 = 24{,}5\ m/s \quad \tau = 0{,}5\ s$

hitrost obeh teles v tej točki enaka, ker $\Delta v = 0$ za obe telesi, ki imata isto začetno hitrost



$v = v_0 - g t'$

$0 = v_0 - g t' \Rightarrow \boxed{t' = \frac{v_0}{g}}$

$t' + t_{1/2}: \ v = -v_1, \ v_1 = g \cdot t_{1/2} \Rightarrow \boxed{t_{1/2} = v_1/g}$

čas leta prvega telesa: $t_1 = t' + t_{1/2} = \boxed{\frac{v_0}{g} + \frac{v_1}{g} = t_1}$

čas leta drugega telesa: $v_1 = v_0 - g t_2 \Rightarrow \boxed{t_2 = (v_0 - v_1)/g}$

$t_1 - t_2 = \left[\tau = \frac{v_0}{g} + \frac{v_1}{g} = \frac{(v_0 - v_1)}{g}\right] \Rightarrow \boxed{\frac{2v_1}{g} = \tau} \Rightarrow \boxed{v_1 = \frac{\tau g}{2}}$

torej:

$$t_2 = \frac{v_0}{g} - \frac{v_1}{g} = \frac{v_0}{g} - \frac{\tau g}{2} = \frac{2v_0 - g\tau}{2g} = \begin{matrix} 2.2\ s \ (g=10) \\ 2.25\ s \ (g=9.8) \end{matrix}$$

$v = v_0 - gt$

$$h = \int_0^{t_2} v\,dt = v_0 t_2 - \frac{g t_2^2}{2} = \frac{2v_0^2 - g\tau v_0}{2g} - \frac{g(2v_0 - g\tau)^2}{2 \cdot 4g^2} =$$

$$= \frac{2v_0^2}{2g} - \frac{g\tau v_0}{2g} - \frac{1}{8g}\left(4v_0^2 - 4v_0 g\tau + g^2\tau^2\right) =$$

$$= \frac{2v_0^2}{2g} - \frac{\tau v_0}{2} - \frac{v_0^2}{2g} + \frac{v_0\tau}{2} - \frac{g^2\tau^2}{2g} = \frac{v_0^2}{2g} - \frac{g\tau^2}{8} = h =$$

$$= \begin{matrix} 30.26\ m \ (g=9.82) \\ 28.7\ m \ (g=10) \end{matrix}$$

✓

ALI KRAJŠE

2. Dve telesi vržemo vertikalno navzgor z iste točke in z enako začetno hitrostjo 24,5 m/s v časovnem razmiku 0,5 s. Na kateri višini se bosta telesi srečali ?

1894

AL1

$v_0 = 24.5 \frac{m}{s}$

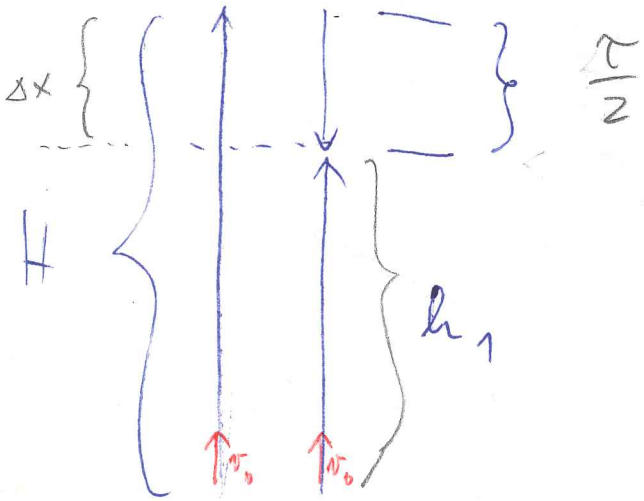
$\tau = 0.5 \frac{m}{s}$

$v^2 = v_0^2 - 2gh$

$0 = v_0^2 - 2gH$

$H = \frac{v_0^2}{2g} = 30.01\,m \ (g=10)$

$= 30.56\,m \ (g=9.81)$



$\Delta x = \frac{g t^2}{2} = 0.312\,m \ (g=10)$

$0.307\,m \ (g=9.81)$

$h_1 = H - \Delta x = 29.7\,m \ (g=10)$

$30.25\,m \ (g=9.81)$

1442

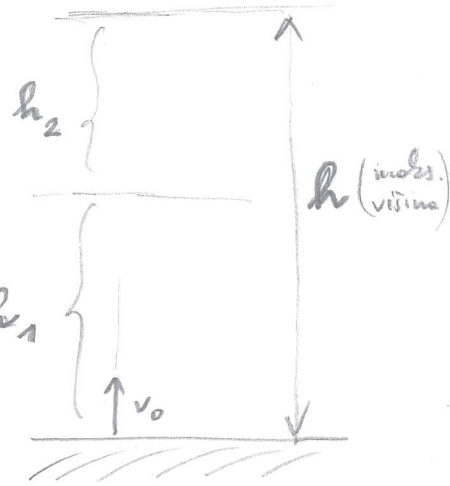
1. Telo vrežemo vertikalno navzgor z začetno hitrostjo 10 m/s. V trenutku, ko telo doseže najvišjo lego, vržemo drugo telo z isto začetno hitrostjo tudi vertikalno navzgor. Na kateri višini se bosta telesi srečali ? Upor zraka zanemarimo.

FOR

$g = 9.82\ m\,s^{-2}$

$v_0 = 10\ m/s$

$$h = \frac{v_0^2}{2g}$$



čas je isti (t):

$$h_2 = \frac{g}{2} \cdot t^2 \qquad h_1 = v_0 t - \frac{g}{2} t^2$$

$$h = h_1 + h_2 \Rightarrow h = v_0 t = v_0 \sqrt{\frac{2h_2}{g}}$$

$$h_2 = h - h_1$$

$$h = v_0 \sqrt{\frac{2(h-h_1)}{g}}$$

$$\Downarrow$$

$$g \cdot \frac{h^2}{v_0^2} = 2h - 2h_1 \Rightarrow h_1 = h - \frac{g h^2}{2 v_0^2} = \frac{v_0^2}{2g} - \frac{g v_0^4}{4g^2 \cdot 2 v_0^2} =$$

$$= \frac{3}{8} \frac{v_0^2}{g}$$

$$h_1 = \frac{3}{8} \frac{v_0^2}{g} = 3.82\ m$$ ✓

1. Z vrha stolpa vržemo istočasno dve telesi z enako začetno hitrostjo 10 m/s. Prvo telo vržemo pod kotom 30°, drugo telo pa pod kotom 60° glede na horizontalo. Na kakšno medsebojni oddaljenosti se nahajata telesi 2 s po metu ? (A-L)

test 1991

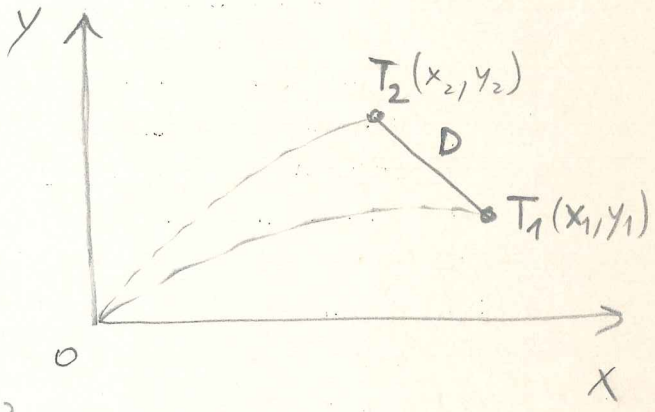
$\alpha_1 = 30^\circ$

$\alpha_2 = 60^\circ$

$v_0 = 10\ m/s$

$t = 2\ s$

$D = ?$



$$y_1 = v_0 \cdot t \sin\alpha_1 - \frac{1}{2} g t^2 = -9.62\ m$$

$$x_1 = v_0 t \cos\alpha_1 = 17,32\ m$$

$$y_2 = v_0 t \sin\alpha_2 - \frac{1}{2} g t^2 = -2.3\ m$$

$$x_2 = v_0 t \cos\alpha_2 = 10,00\ m$$

$$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = 10.35\ m$$ ✓

1) Z istega mesta vržemo hkrati dva kamna. Prvega z začetno hitrostjo 10 m/s pod kotom 30° poševno navzgor proti vodoravnici, drugega pa z začetno hitrostjo 15 m/s. Pod kolikšnim kotom glede na vodoravnico moramo vreči drugi kamen, da bo 1 sekundo po metu razdalja med kamnoma 7 m?

1φ2/03

$v_{10} = 10\ m/s \qquad v_{20} = 15\ m/s \qquad t = 1\ s$

$\varphi_1 = 30\ m/s \qquad \varphi_2 = ? \qquad d = 7\ m$

$y_1 = v_{10} \sin\varphi_1 t - \frac{gt^2}{2}$, $y_2 = v_{20} \sin\varphi_2 t - \frac{gt^2}{2}$

$x_1 = v_{10} \cos\varphi_1 t$, $x_2 = v_{20} \cos\varphi_2 t$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d^2/t^2 = (v_{20}\cos\varphi_2 - v_{10}\cos\varphi_1)^2 + (v_{20}\sin\varphi_2 - v_{10}\sin\varphi_1)^2$$

$$\frac{d^2}{t^2} = v_{20}^2\cos^2\varphi_2 - 2v_{20}v_{10}\cos\varphi_2\cos\varphi_1 + v_{10}^2\cos^2\varphi_1 + v_{20}^2\sin^2\varphi_2 - 2v_{20}v_{10}\sin\varphi_1\sin\varphi_2 + v_{10}^2\sin^2\varphi_1$$

$$\frac{d^2}{t^2} = -2v_{20}v_{10}\cos(\varphi_2 - \varphi_1) + v_{20}^2 + v_{10}^2$$

$$\boxed{\cos(\varphi_2 - \varphi_1) = \frac{v_{20}^2 + v_{10}^2 - \frac{d^2}{t^2}}{2v_{20}v_{10}}} = 23.1$$

$$\Downarrow$$

$$\varphi_2 = \varphi_1 + 23^\circ = \underline{53^\circ}$$

$\cos(\varphi_1 - \varphi_2) = 23.1$

$$\Downarrow$$

$$\varphi_2 = \varphi_1 - 23.1 \approx \underline{7^\circ}$$

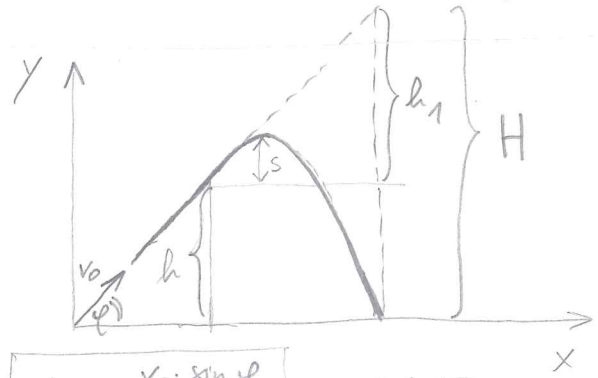
3. Letalo leti poševno navzgor s hitrostjo 250 m/s s kotom 60° proti vodoravnici. V višini 400 m spusti svinčeno kroglo in nadaljuje let v nespremenjeni smeri. Kolikšna je razdalja med krajem, ki ga zadene krogla in letalom v trenutku, ko pade krogla na tla? Pod kakšnim kotom proti vodoravnici udari krogla na tla ? Upor zraka zanemarimo.

$g = 10\,m\,s^{-1}$ 1992

$v_0 = 250\ m/s$

$\varphi = 60^\circ$

$h = 400\ m$

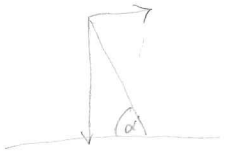


$$0 = v_0 \sin\varphi - g t_1 \Rightarrow \boxed{t_1 = \frac{v_0 \cdot \sin\varphi}{g}} = 21{,}65\ s$$

$$0 = (v_0 \sin\varphi)^2 - 2gs \Rightarrow \boxed{s = \frac{(v_0 \cdot \sin\varphi)^2}{2g}} = 2343{,}7\ m$$

$$s + h = \frac{g t_2^2}{2} \Rightarrow \boxed{t_2 = \sqrt{\frac{2(s+h)}{g}}} = 23.42\ s$$

$$H = h + (v_0 \sin\varphi)(t_1 + t_2) = 10160\ m \quad \checkmark$$



$$\vec{v}_{krogle} = (v_0 \cos\varphi,\ v_0 \sin\varphi - g(t_1 + t_2))$$

$$tg\,\alpha = \frac{v_y}{v_x}$$

$$tg\,\alpha = \frac{|v_0 \sin\varphi - g(t_1 + t_2)|}{v_0 \cos\varphi} \Rightarrow \boxed{\alpha = 62^\circ} \quad \checkmark$$

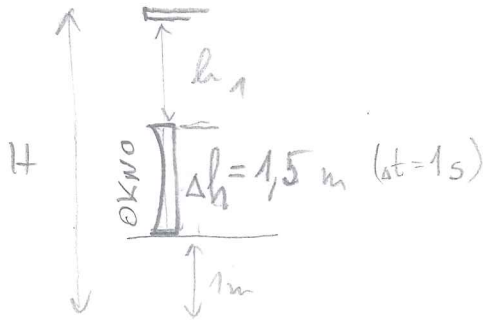
$t_1 + t_2 = 45\ s$

2

1995

F1

2. Roža v loncu pade z balkonske ograje v prvem nadstropju. Opazovalec, ki stoji v sobi v pritličju, opazi, da pada mimo okna 0,1 s. Spodnji rob okna je 1 m nad tlemi, okno pa je visoko 1,5 m. Kako visoko od tal je balkonska ograja ?



$$h_1 + \Delta h = \frac{g}{2}(t_1 + \Delta t)^2 \qquad h_1 = \frac{g t_1^2}{2}$$

$$\frac{g t_1^2}{2} + \Delta h = \frac{g}{2}(t_1 + \Delta t)^2 = \frac{g}{2} t_1^2 + g t_1 \Delta t + \frac{g}{2}\Delta t^2$$

$$\Delta h = g t_1 \Delta t + \frac{g}{2}\Delta t^2$$

$$t_1 = \frac{1}{g\Delta t}\left(\Delta h - \frac{g}{2}\Delta t^2\right)$$

$$h_1 = \frac{g}{2} t_1^2 = \frac{g}{2}\,\frac{1}{g^2 \Delta t^2}\left(\Delta h - \frac{g}{2}\Delta t^2\right)^2 = \frac{1}{2g\Delta t^2}\left(\Delta h - \frac{g}{2}\Delta t^2\right)^2$$

$$H = h_1 + \Delta h + 1\,m$$

$\Delta t = 0.1\,s \Rightarrow h_1 = 10.51$ m , $H = 13$ m

$\Delta t = 0.3\,s \Rightarrow h_1 = 0.61$, $H = 3.11$ m