1. Kolikšno celotno povprečno kinetično energijo imajo molekule kisika (O_2) pri temperaturi 20° ? Kolikšna je ustrezna hitrost teh molekul ?

1996

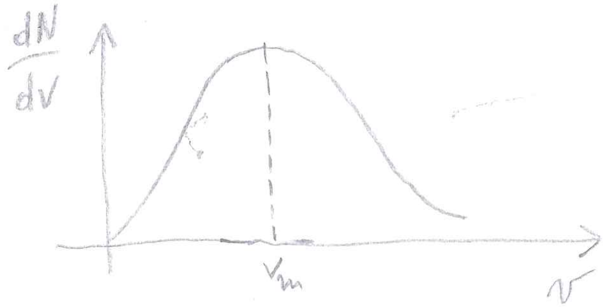
$$\overline{W}_k = \frac{5}{2} kT = 10{,}1 \cdot 10^{-21}\ J$$

$$\overline{W}_{k,trans} = \frac{3}{2} kT = 6{,}1 \cdot 10^{-21}\ J$$

$$\frac{1}{2} m_1 \overline{v^2} = \frac{3}{2} kT$$

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m_1}} = \sqrt{\frac{3kTN_A}{M}} = 476\ ms^{-1}$$

1. Kolikšna je najverjetnejša hitrost molekul kisika (O_2) pri kateri ima Maxwellova porazdelitev molekul po velikosti hitrosti v (dN/dv) maksimum ? (T = 300 K)



$$\frac{dN}{dv} = \frac{4N}{\sqrt{\pi}} \left(\frac{m_1}{2kT}\right)^{3/2} v^2 e^{-\frac{m_1 v^2}{2kT}}$$

$$\frac{dN}{dv} = 0 \Rightarrow v_m = \sqrt{\frac{2kT}{m_1}} = \sqrt{\frac{2kTN_A}{M}} = 395\ m/s$$

6. Kolikšen je volumen posode v kateri je $2\cdot10^{20}$ molekul kisika (O_2) s povprečnim kvadratom hitrosti $36\cdot10^{4}$ m^2s^{-2}, če je v posodi tlak $1{,}28\cdot10^{3}$ Pa ? (M_{O_2} = 32 kg/kmol)

1995 M–Ž

$$\frac{m_1\overline{v^2}}{2} = \frac{3}{2}kT$$

$$\Downarrow$$

$$T = \frac{m_1\overline{v^2}}{3k}, \quad m_1 = \frac{M}{N_A}$$

$$\Downarrow$$

$$T = \frac{M\cdot\overline{v^2}}{N_A\cdot 3\cdot k}$$

$$pV = \frac{m}{M}RT$$

$$pV = \frac{N\cdot m_1}{N_A\cdot m_1}\cdot R\cdot T, \quad R = N_A\cdot k$$

$$V = \frac{1}{p}N\cdot k\cdot T$$

$$V = \frac{N\cdot M\cdot\overline{v^2}}{p\cdot 3\cdot N_A} = 1\,l$$

FOR kolokvij 90/91 KTP2

3. Izračunaj celotno povprečno kinetično energijo treh molov ~~idealnega~~ enoatomnega plina pri temperaturi 227 °C. Kolikšna je povprečna ~~hitrost~~ hitrosti (ALI PA KVADRAT) ene molekule tega plina pri navedeni temperaturi ?($N_A=6\cdot10^{26}$ molekul, R = 8300 J/(kmol·K)) ~~M=40 kg~~

(A)

(B) isto za DVOATOMNI PLIN

$T = 500\,K$

$N_A = 6\cdot10^{26}$

$k = 1{,}38\cdot10^{-23}$ J/K

$\overline{W}_k = 18{,}6\cdot10^3$ J

ENOATOMNI PLIN:

a) $W_k = N\cdot\frac{3}{2}kT = 18.63$ kJ

$N = \frac{3}{1000}\cdot6\cdot10^{26} = 18\cdot10^{23}$

b) $\frac{3}{2}kT = \frac{m\overline{v^2}}{2} \Rightarrow 3kT = m\overline{v^2}$

DVOATOMNI PLIN:

a) $W_k = N\frac{5}{2}kT$

b) $\frac{3}{2}kT = \frac{mv^2}{2}$

$\overline{v^2} = \frac{3kT}{m} = \frac{3kTN_A}{M} = \frac{3\cdot1.38\cdot10^{-23}\,500\cdot6\cdot10^{26}}{40} \Rightarrow$

$\overline{v^2} = \frac{3\cdot1.38\cdot500\cdot1000\cdot6}{40} = (557.3)^2\,m^2/s^2$

$\bar{v}^2 = 310583{,}3\,\frac{m^2}{s^2}$

$v = \sqrt{v_x^2+v_y^2+v_z^2}$

$\bar{v}^2 \neq \overline{v^2}$

$\bar{v} \neq \sqrt{\overline{v^2}}$

4. Porazdelitev velikega števila molekul idealnega plina po hitrosti lahko opišemo z Maxwellovo porazdelitveno funkcijo

$$N(v) = 4\cdot\Pi\cdot N\cdot\left[\frac{m}{2\cdot\Pi\cdot k\cdot T}\right]^{3/2}\cdot v^2\cdot\exp\left[-\frac{m\cdot v^2}{2\cdot k\cdot T}\right],$$

kjer je N število molekul v vzorcu, m masa ene molekule, k Boltzmannova konstanta, T temperatura in v velikost hitrosti. Kolikšna je v tem modelu najverjetnejša velikost hitrosti molekule kisika O_2 pri temperaturi 300 K. (M_{O_2} = 32 kg/kmol)

$$\frac{dN}{dv} = 2v(\ )\exp + (\ )v^2\left(-\frac{2m_1 v}{2kT}\right)\exp = 0$$

$$1 - v^2\frac{m_1}{2kT} = 0$$

$$v = \sqrt{\frac{2kT}{m_1}} = \sqrt{\frac{2kT\cdot N_A}{M}} = 394\ m/s$$

$$\left(\frac{2\cdot 1{,}38\cdot 10^{-23}\cdot 300\cdot 6\cdot 10^{26}}{32}\right)^{1/2}$$

Popršt 1991   Str(K)/225   ~ Elektr/12 KTP 7

4. Pri kateri temperaturi je število molekul kisika (O_2), ki imajo hitrosti v intervalu 249-251 m/s, enako številu molekul, ki imajo hitrosti v intervalu 549-551 m/s?

$v = 250\ m/s \qquad \Delta v = 2\ m/s$

$\tilde{v} = 550\ m/s \qquad \Delta\tilde{v} = 2\ m/s$

$M_{O_2} = 32$

$$\left(\frac{dN}{N}\right)_v = \alpha v^2 e^{-\frac{\bar{m}v^2}{2kT}} dv$$

v^2 pride od integracije v 3-D: $dV_v = 4\pi v^2 dv$

$$dN = N\alpha v^2 e^{-\frac{\bar{m}v^2}{2kT}} dv$$

$dN = d\tilde{N}$ $\qquad$ $dv = \Delta v$, $d\tilde{v} = \Delta\tilde{v}$

$\Downarrow$

$$v^2 e^{-\frac{\bar{m}v^2}{2kT}} \Delta v = \tilde{v}^2 e^{-\frac{\bar{m}\tilde{v}^2}{2kT}} \Delta\tilde{v}$$

$\Downarrow$

$$\left(\frac{\tilde{v}}{v}\right)^2 = e^{\frac{\bar{m}}{2kT}(\tilde{v}^2 - v^2)} \Rightarrow \ln\left(\frac{\tilde{v}}{v}\right)^2 = \frac{\bar{m}}{2kT}(\tilde{v}^2 - v^2) \Rightarrow$$

$$T = \frac{\bar{m}}{2k} \cdot \frac{(\tilde{v}^2 - v^2)}{2\ln\left(\frac{\tilde{v}}{v}\right)} = \frac{M}{2R}\,\frac{(\tilde{v}^2 - v^2)}{2\cdot\ln\left(\frac{\tilde{v}}{v}\right)} = 293,4\ K$$ ✓

$$\frac{\bar{m}}{k} = \frac{\bar{m} N_A}{k N_A} = \frac{M}{R}$$

$$\frac{16\,(55^2 - 25^2)\,100}{8314 \cdot 2 \cdot \ln\left(\frac{550}{250}\right)} = 293,4\ K$$ ✓

$1{,}38 \cdot 10^{-23}$

$1{,}01 \cdot 10^{-20}$

$\frac{5}{2}kT$

$1{,}01 \cdot 10^{3}$

10^{-23}

3. Porazdelitev molekul CO_2 po hitrosti v podaja Maxwell - Boltzmannova funkcija $\omega(v) = A\cdot v^2\cdot \exp(-m_1\cdot v^2/(2\cdot k\cdot T))$ kjer je A konstanta, m_1 masa ene molekule CO_2, k Boltzmannova konstanta in T temperatura plina. Kakšna je najverjetnejša hitrost molekul tega plina, če je koren povprečnega kvadrata hitrosti $\langle v^2\rangle$ enak 500 m/s ?

$$w(v) = A v^2 e^{-\frac{m_1 v^2}{2kT}}$$

$$\frac{dw(v)}{dv} = 2Av\, e^{-\frac{m_1 v^2}{2kT}} + Av^2 e^{-\frac{m_1 v^2}{2kT}}\left(-\frac{m_1 \cdot 2v}{2kT}\right) = 0$$

$$2 - v^{*2}\frac{m_1}{kT} = 0 \qquad v^* = \sqrt{\frac{2kT}{m_1}}$$

$$\frac{1}{2} m_1 \langle v^2\rangle = \frac{3}{2}kT \Longrightarrow \frac{kT}{m_1} = \frac{\langle v^2\rangle}{3}$$

$$v^* = \sqrt{\frac{2\langle v^2\rangle}{3}} = \sqrt{\frac{2}{3}}\cdot 500\,\frac{m}{s} = 408\,\frac{m}{s}$$

3. Porazdelitev velikega števila molekul dvoatomnega (M = 16 kg/kmol) idealnega plina po hitrosti opišemo z Maxwellovo porazdelitveno funkcijo : $\rho_v = \alpha \cdot v^2 \cdot \exp(-m_1 \cdot v^2/(2 \cdot k \cdot T))$, kjer je m_1 masa ene molekule, v pa velikost hitrosti molekule plina. Kolikšna je povprečna kinetična energija ene molekule tega plina, če je število molekul plina, ki imajo hitrosti v intervalu 249-250 m/s enako številu molekul, ki imajo hitrosti v intervalu 549-550 m/s ?

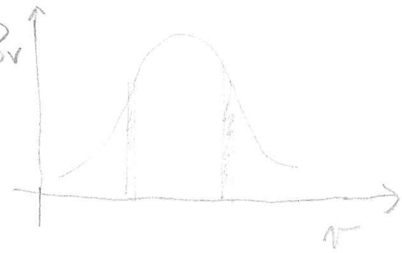
1992 1992

$M = 16$ kg/kmol

$\Delta v = 1$ m/s

$v_1 = 249{,}5$ m/s

$v_2 = 549{,}5$ m/s



$$dN_1 = dN_2$$

$$v_1^2 e^{-\frac{m_1 v_1^2}{2kT}} dv = v_2^2 e^{-\frac{m_1 v_2^2}{2kT}} dv$$

$$\left(\frac{v_2}{v_1}\right)^2 = \exp\left(\frac{m_1}{2kT}(v_2^2 - v_1^2)\right), \quad \ln\left(\frac{v_2}{v_1}\right) = \frac{m_1}{4kT}(v_2^2 - v_1^2)$$

$$kT = \frac{m_1}{4}(v_2^2 - v_1^2) / \ln\left(\frac{v_2}{v_1}\right)$$

$M = N_A \cdot m_1$

$R = N_A \cdot k$

$$\langle W \rangle = \frac{5}{2} kT = \frac{5}{2} \cdot \frac{M}{4 N_A} \cdot \frac{(v_2^2 - v_1^2)}{\ln\left(\frac{v_2}{v_1}\right)} = 5 \cdot 10^{-21} \text{ J}$$

~~ITD 18~~ KTP 13

12.) Volumen dvoatomnega idealnega plina naraste pri [redacted] segrevanju in nespremenjenem tlaku 10^5 N m^{-2} od 5 litrov na 7 litrov. Za koliko se plinu spremeni notranja energija ?

W_n odvisna le od temperature — idealen plin — od W_k molekul

$$pV = mRT \qquad pdV = mRdT$$

$$T = \frac{pV}{mR} \qquad dT = \frac{pdV}{mR}$$

$$dW_n = m c_v dT$$

$$dW_n = m \frac{5}{2}\frac{R}{M} dT$$

$$dW_n = \frac{5}{2} mR\, dT = \frac{5}{2} \frac{mR\, pdV}{mR} = \boxed{\frac{5}{2} pdV}$$

$$\frac{5}{2} \cdot \frac{10^5 N}{m^2} \frac{2 \cdot 10^{-3} m^3}{} = \underline{\underline{5 \cdot 10^2 J}}$$

ali

$$dW_n = m c_p dT$$

$$dW_n = \frac{p(V_2 - V_1)}{\varkappa - 1} = \frac{10^5 N \cdot 2 \cdot 10^{-3} m^3}{m^2 \quad 1{,}4 - 1} = \underline{\underline{5 \cdot 10^2 J}}$$

$$\frac{c_p}{c_v} = \varkappa = 1{,}4 \text{ (je dvoatomen plin)}$$

c_p	c_v	
$\frac{5}{2}\frac{R}{M}$ (1 atom)	$\frac{3}{2}\frac{R}{M}$ (1 atom)	
$\frac{7}{2}\frac{R}{M}$ (2 atom)	$\frac{5}{2}\frac{R}{M}$ (2 atom)	3 translacije + 2 rotaciji

KTP10

Toplota 1980/81

11.) Kolikšno poprečno kinetično energijo imajo molekule kisika pri temperaturi 20°C ? Kolikšna je ustrezna (povp. kvadrat) hitrost ?

$R = 8{,}3 \cdot 10^{3}\, J\, kmol^{-1}\, st^{-1}$; $K = 1{,}38 \cdot 10^{-23}\, J\, st^{-1}$

$$\langle W_k \rangle = \frac{1}{2} m \overline{v^2} = \frac{5}{2} kT = \frac{5 \cdot 1{,}38 \cdot 10^{-23} J \cdot 293\, st}{2\, st} = \underline{\underline{1{,}01 \cdot 10^{-20} J}}$$

$$\frac{1}{2} m v^2 = \frac{3}{2} kT \quad \text{— ni potacij!}$$

(ni potacij)

$$v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kTN_a}{M}} = \sqrt{\frac{3RTN_a}{N_a M}}$$

$m = \frac{M}{N_a}$; $K = \frac{R}{N_a}$

$$v = \sqrt{\frac{3RT}{M}} =$$

$$\left[\frac{3kT}{m} = \frac{3RT}{M} \right]$$

$$v = \sqrt{\frac{3 \cdot 8{,}3 \cdot 10^{3} J \cdot 293\, st\, kmol}{kmol\, st\; 32\, kg}}$$

$$v = \sqrt{22{,}7 \cdot 10^{4} \frac{kg\, m^2}{s^2\, kg}} = \underline{4{,}76 \cdot 10^{2}\, m\, s^{-1}}$$

5) V posodi je 5 gramov dušika in 4 grami kisika. Skupni tlak je $10^7 N/m^2$. Kolikšna sta parcialna tlaka obeh plinov? Molekulska masa kisika je 32 kg/kmol, dušika pa 28 kg/kmol.

$m_1 = 5\,g$

$M_1 = 28\,kg/kmol$

$m_2 = 4\,g$

$M_2 = 32\,kg/kmol$

$p = 10^7 \frac{N}{m^2}$

$p_1 \quad p_2$

$$p = p_1 + p_2 \qquad p_1 V = \frac{m_1}{M_1} RT$$

$$\frac{p_1}{p_2} = \frac{m_1 M_2}{M_1 m_2} \qquad p_2 V = \frac{m_2}{M_2} RT$$

$$p_1 = p_2 \frac{m_1 M_2}{M_1 m_2}$$

$$p = p_2 \left(1 + \frac{m_1 M_2}{M_1 m_2}\right)$$

kisik $$p_2 = \frac{p M_1 m_2}{m_1 M_2 + m_2 M_1} = 10^7 \cdot \frac{28 \cdot 4}{5 \cdot 32 + 28 \cdot 4} = 4{,}1 \cdot 10^6 \frac{N}{m^2}$$

dušik $$p_1 = \frac{p M_2 m_1}{m_1 M_2 + m_2 M_1} = 10^7 \frac{5 \cdot 32}{5 \cdot 32 + 28 \cdot 4} = 5{,}9 \cdot 10^6 \frac{N}{m^2}$$

2. Kolikšna mora biti temperatura zraka v balonu s prostornino 500 m^3, da lahko le ta lebdi v zraku in nosi tovor 300 kg ?
(M_{zraka} = 29 kg/kmol, p = 10^5 Pa , T = 293 K)

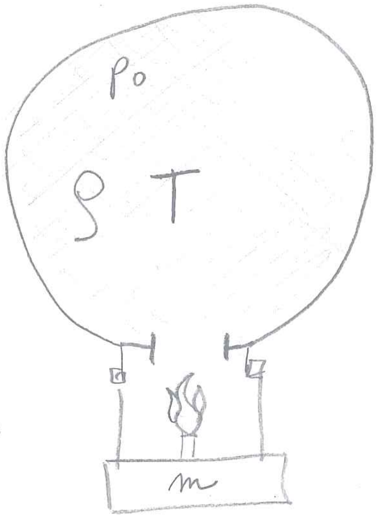
A–L 1995

T_0

p_0

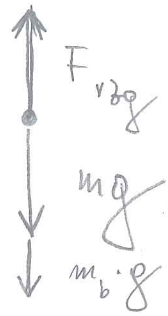
ρ_0

$\rho < \rho_0$



$m = 300\,kg$
$p_0 = 10^5\,Pa$
$M = 29\,kg/kmol$
$T_0 = 293\,K$
$V = 500\,m^3$

$T = ?$



$$pV = \frac{m}{M}RT \Rightarrow \frac{p}{\rho} = \frac{RT}{M} \Rightarrow \boxed{\rho = \frac{pM}{RT}}$$

$$\rho_0 V g = \rho V g + mg$$

$$\rho_0 V = \rho V + m$$

$$\frac{p_0 M V}{R T_0} = \frac{p_0 M V}{R T} + m$$

$$\frac{1}{T_0} = \frac{1}{T} + \frac{mR}{M p_0 V}$$

$$\boxed{\frac{1}{T} = \frac{1}{T_0} - \frac{mR}{M p_0 V}} \Rightarrow T = 590\,K$$

PE8

Naloga : V litrsko jeklenko izpraznimo vsebino dveh drugih jeklenk. V prvi jeklenki je bilo prvotno 28 g kisika (O_2), v drugi jeklenki pa 50 g CO_2 (O = 16, C = 12). Temperatura prej in potem je (bila) 300 K. Kakšen je tlak plina v končnem stanju po izpraznitvi prveh dveh jeklenk? (R = 8300 J/K kmol) Kakšna je masa povprečnega kilomola nove mešanice ?

$m_1 = 28\,g$, $M_1 = 32\,kg$

$m_2 = 50\,g$, $M_2 = 44\,kg$ $T = 300K$

$$p_1 V = \frac{m_1}{M_1} RT$$

$$p_2 V = \frac{m_2}{M_2} RT$$

$$p = (p_1 + p_2) = \left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right)\frac{RT}{V}$$

$$\boxed{p = \frac{1}{V}\left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right)RT}$$

$$\frac{m_1}{M_1} + \frac{m_2}{M_2} = \frac{m_1 + m_2}{M} \Rightarrow$$

$$\Rightarrow \boxed{M = \frac{m_1 + m_2}{m_1/M_1 + m_2/M_2}}$$

5. Kolikšen je tlak v posodi s prostornino 1 litra, če je v njej $2\cdot 10^{20}$ molekul kisika (O_2) s povprečnim kvadratom hitrosti $36\cdot 10^4\ m^2s^{-2}$. (M_{O_2} = 32 kg/kmol)

1995
A-L

$\overline{v^2} = (600)^2\ \frac{m^2}{s^2}$

$M = 32\ kg$

$N = 2\cdot 10^{20}$

$m_1 = \frac{M}{N_A}$

$\frac{m_1\overline{v^2}}{2} = \frac{3}{2}kT \Rightarrow T = \frac{m_1\overline{v^2}}{3k} = \frac{M\overline{v^2}}{3\cdot N_A\cdot k} = 462\ K$

$pV = \frac{m}{M}RT = \frac{N\cdot m_1}{N_A\cdot m_1}\cdot R\cdot T = \frac{N}{N_A}RT \Rightarrow$

$\Rightarrow p = \frac{1}{V}N\cdot k\cdot T = \frac{N\cdot M\cdot \overline{v^2}}{N_A\cdot V\cdot 3} =$

$= 1.28\cdot 10^3\ \frac{N}{m^2}$