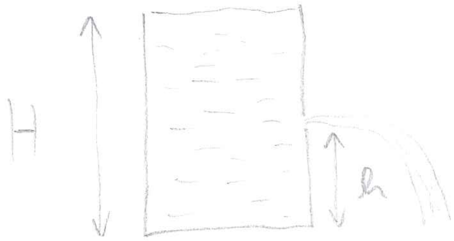
1992

4. Posoda višine 30 cm je polna vode. Na kolikšni višini od vrha posode moramo zvrtati luknjico s presekom 4 mm^2 v steno posode, da bo volumski pretok izstopajočega curka vode $6.9\cdot10^{-6}$ m^3/s ?



$H = 0{,}3$ m

$S = 4\cdot10^{-6}$ m^2

$\phi_v = 6{,}9\cdot10^{-6}$ m^3/s

$$p + \frac{1}{2}\rho v'^2 + \rho g H = p + \frac{1}{2}\rho v^2 + \rho g h$$

($v' = 0$)

$$\phi_v = S\cdot v$$

$$v = \frac{\phi_v}{S} = 1{,}725\ \frac{m}{s}$$

$$h = H - \frac{v^2}{2g} = 15\ \text{cm}$$

1. Volumski tok viskozne tekočine skozi neko kapilaro podaja Poiseuilleov zakon : $\Phi_V = \pi \Delta p r^4/8\eta l$, kjer je Δp tlačna razlika med koncema kapilare, r polmer kapilare, η viskoznost tekočine in l dolžina kapilare. Za koliko procentov se spremeni tok skozi kapilaro, če se radij kapilare zmanjša za 1% ?

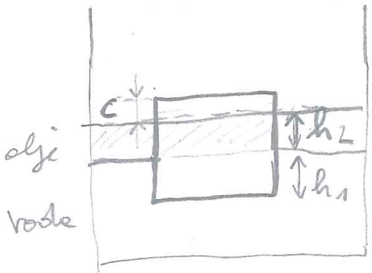
$$\phi_v = \frac{\alpha \Delta p r^4}{\eta l}$$

$$d\phi_v = \frac{\alpha \Delta p}{\eta l} 4r^3 dr \qquad d\phi_v \sim \Delta\phi_v \quad , \quad dr \sim \Delta r$$

$$\frac{\Delta\phi_v}{\phi_v} = \frac{\alpha \Delta p}{\eta l} \frac{4r^3 \Delta r \eta l}{\alpha \Delta p r^4} = \frac{4 \Delta r}{r} = -4\%$$

7. V valjasti posodi z radijem 10 cm plava na vodi lesena kocka z robom 10 cm in gostoto 0,5 g/cm^3. Koliko kubičnih decimetrov olja z gostoto 0,8 g/cm^3 moramo vliti na vodo, da bo oljna gladina 4 cm pod zgornjo ploskvijo kocke ? FOR

$g = 9.82\ ms^{-2}$



$r = 0{,}1\ m$, $c = 4\ cm$

$a = 0{,}1\ m$

$\rho_k = 500\ kg/m^3$

$\rho_o = 800\ kg/m^3$

$\rho_v = 1000\ kg/m^3$

$$a = c + h_1 + h_2$$

$$\Downarrow$$

$$h_1 = a - c - h_2$$

$$a^3 \cdot \rho_k = h_1 \cdot a^2 \cdot \rho_v + h_2 \cdot a^2 \rho_o$$

$$a^3 \rho_k = (a-c) a^2 \rho_v - h_2 a^2 \rho_v + h_2 a^2 \rho_o$$

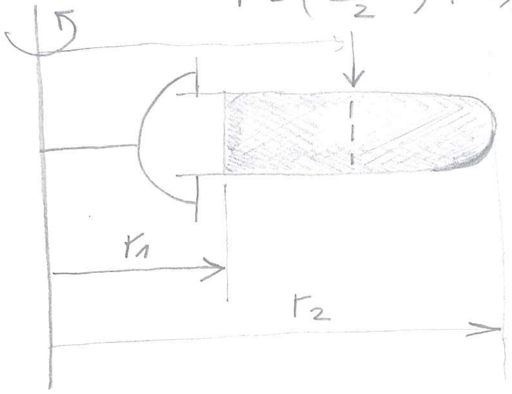
$$h_2 (\rho_v - \rho_o) = (a-c)\rho_v - a\rho_k = a(\rho_v - \rho_k) - c\rho_v$$

$$h_2 = a \cdot \frac{(\rho_v - \rho_k)}{(\rho_v - \rho_o)} - c \cdot \frac{\rho_v}{(\rho_v - \rho_o)} = 5\ cm$$

$$V_o = (\pi r^2 - a^2) h_2 = 1{,}07\ dm^3$$

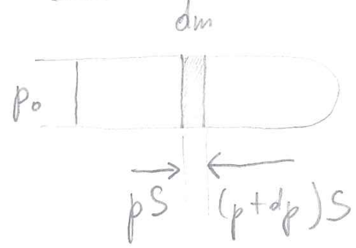
6. Z vodo napolnjena epruveta se vrti v centrifugi s frekvenco 4000 vrt./min. Gladina vode je oddaljena od osi vrtenja 8 cm, dno epruvete pa 18 cm. Kolikšen je tlak na sredini med gladino vode in dnom epruvete ? Zunanji zračni tlak je 10^5 Pa. 1592

$\tilde{r} = \left(\frac{r_2 - r_1}{2}\right) + r_1 = 0{,}13\ m$, $\nu = 4000\ \frac{vrt}{min} = 66{,}7\ s^{-1}$



$r_1 = 0{,}08\ m$

$r_2 = 0{,}18\ m$, $\rho = 10^3\ kg/m^3$



$p_0 = 10^5\ Pa$

$$dp \cdot S = dm\, r\, \omega^2 = S dr \cdot \rho \cdot r \cdot \omega^2$$

$$\int_{p_0}^{\tilde{p}} dp = \int_{r_1}^{\tilde{r}} \rho \omega^2 r\, dr$$

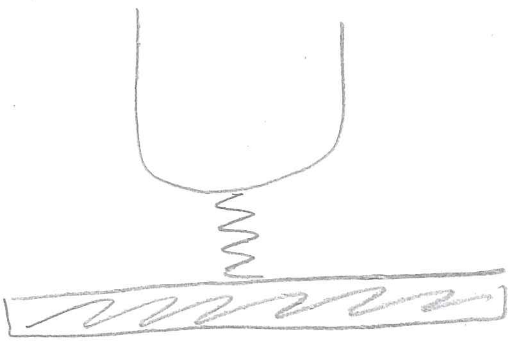
$$\tilde{p} = p_0 + \frac{1}{2} \rho \omega^2 (\tilde{r}^2 - r_1^2) = (1 + 9{,}22) \cdot 10^5\ Pa$$

5

FOR Kolokvij 80/81 T1

4. Na tehtnico postavimo lonec, ki delno napolnjen z vodo, tehta 1 kg. Ne da bi se dotaknili stene lonca ali dna, potisnemo 5 cm globoko v vodo krajišče lesenega kola. Koliko pokaže tehtnica, če ima kol pravokotni presek 10 cm^2 in gostoto $0{,}7\cdot 10^3$ kg/m^3? Koliko pa pokaže tehtnica, če isto ponovimo s popolnoma enakim železnim kolom, ki ima desetkrat večjo gostoto kot lesen kol ? (gostota vode je 10^3 kg/m^3, $g = 9{,}8$ ms^{-2})

$(F_c = 10.3\,N)$



$S = 10\cdot 10^{-4}\ m^2$

$h = 0.05\ m$

$\rho_{vode} = 10^3\ kg/m^3$

$g = 9.8\ ms^{-2}$

$F_c = F_{vode} + F_{vzg} = \underline{\underline{10.3\,N}}$

$F_{vode} \rightarrow 9.8\,N$, $F_{vzg} \rightarrow 0.5\,N$

$F_{vzg} = V\cdot\rho_v\cdot g = S\cdot h\cdot\rho_v\cdot g = 10\cdot 10^{-4}\cdot 0.05\cdot 10^3\cdot 9.8 =$

$= 10\cdot 10^{-1}\cdot 0.05\cdot 9.8 = \underline{\underline{0.5\,N}}$

$g = 10\ ms^{-2}$, $F_c = 10.5\ N$

6

Naloga

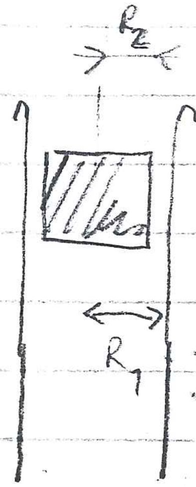
Valj polmera $R_2 = 4.8\ cm$ spustimo v z oljem napolnjeno valjasto posodo z notranjim polmerom $R_1 = 5\ cm$.

$\rho_{olja} \equiv \rho_0 = 0.8\ g/cm^3$

$\rho_{valja} \equiv \rho = 5.8\ g/cm^3$

$\eta = 48\ Ns/m^2$ {viskoznost olja}

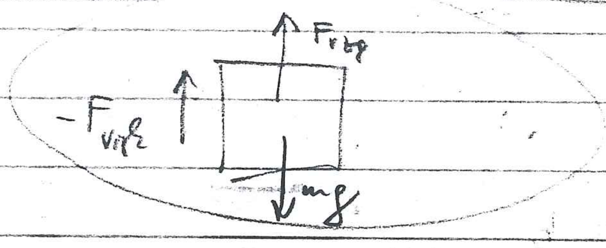
S kakšno hitrostjo se giblje valj navzdol v stacionarnem stanju?



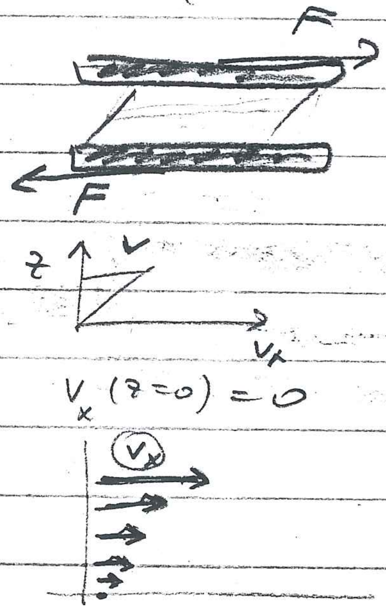
$$F_{vzgon} = \pi R_2^2 h \rho_0 \cdot g$$

$$F_{viskoz} = \eta \cdot \underbrace{2\pi R_2 \cdot h}_{S} \cdot [v/(R_1 - R_2)]$$

$$F_{teže} = mg = g(\pi R_2^2 \cdot h)\rho$$



$$\frac{F}{S} = \eta \frac{dv_x}{dz}$$



stacionarno stanje $\Rightarrow$ $a = 0$, $\Rightarrow$ $\Sigma F = 0$

$$F_{teže} = F_{vzg} + F_{visk}$$

$$g(\pi R_2^2 h)\rho = \pi R_2^2 h \cdot \rho_0 \cdot g + \eta\, 2\pi R_2 h \cdot [v/(R_1 - R_2)]$$

$$\Downarrow$$

$$v = \frac{g(\rho - \rho_0) R_2 \cdot (R_1 - R_2)}{2\eta}$$

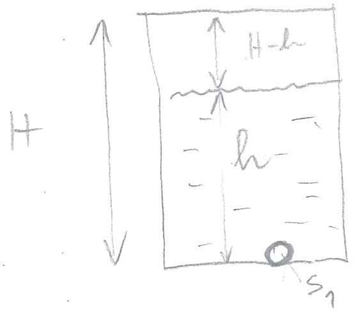
$v = 0.05\ m/s$

$v = 5\ cm/s$

kolokvij 30/31 T3

8. Cilindričen sod notranje višine 70 cm in notranje osnovne ploskve 600 cm^2 je do vrha napolnjen z vodo. Na dnu soda je odprtina s presekom 1 cm^2, ki je v začetku zamašena. S kakšno hitrostjo začne iztekati voda iz odprtine, ko jo odmašimo ? Koliko časa je potrebno, da se višina gladine vode v sodu zmanjša na polovico glede na začetno višino ? Koliko časa pa je potrebno, da se sod popolnoma izprazni ?

A-M Se 51/9-4



$H = 0.7\ m$

$S = 600 \cdot 10^{-4}\ m^2 = 6 \cdot 10^{-2}\ m^2$

$S_1 = 10^{-4}\ m^2$

$\rho g h + \frac{1}{2}\rho v^2 + p = konst$

$$\rho g h + \frac{1}{2}\rho v_g^2 = \frac{1}{2}\rho v^2 \qquad S v_g = S_1 \cdot v \Rightarrow v_g = \frac{S_1}{S} v$$

$$-S_1 v\, dt = S \cdot dh$$

h se zmanjšuje ⇓ (−)

$$g h + \frac{1}{2}\frac{S_1^2}{S^2} v^2 = \frac{1}{2} \cdot v^2$$

$$2gh = v^2\left(1 - \frac{S_1^2}{S^2}\right) \Rightarrow v^2 = 2gh / \left(1 - \frac{S_1^2}{S^2}\right)$$

$$\sqrt{2gh} \cong v = \sqrt{\frac{2g}{1 - \frac{S_1^2}{S^2}}} \cdot h^{1/2} = A \cdot h^{1/2}$$

$$\int_0^t -S_1 A\, dt = \int_H^h S \frac{dh}{h^{1/2}}$$

$$-\frac{S_1 A t}{S} = \int_H^h h^{-1/2} dh = 2 h^{1/2} \Big|_H^h$$

$$-\frac{S_1 A t}{S \cdot 2} = (h)^{1/2} - H^{1/2}$$

$$t = \frac{2 \cdot S}{S_1 A}\left[H^{1/2} - h^{1/2}\right]$$

$A = 4.4317$

začetna hitrost:

(v) $v_{zač} = A \cdot H^{1/2} = 3{,}7\ \frac{m}{s}$

prazen sod:

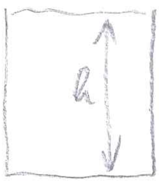
(1) $h = 0 \Rightarrow t = \frac{2 \cdot S}{S_1 A} H^{1/2} = 227\ s$

na pol prazen sod:

(1/2) $h = \frac{H}{2} \Rightarrow t = \frac{2S}{S_1 A} \cdot \left[H^{1/2} - \left(\frac{H}{2}\right)^{1/2}\right] = 66\ s$

FOR - 1996

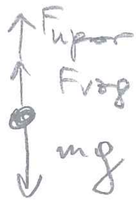
4. Na vodno gladino stresemo različno velike ~~okrogle celice~~ kroglice z gostoto $1.05\cdot 10^3\ \mathrm{kg/m^3}$. Zaradi sile teže se ~~celice~~ kroglice pričnejo gibati navzdol. Izračunajte radij tistih ~~celic~~ kroglic, ki se po 10 urah ravno usedejo na dno. Razdalja od gladine do dna je 10 cm. Koeficient viskoznosti vode je $1.5\cdot 10^{-3}$ kg/ms. Gostota vode je $1.03\cdot 10^3\ \mathrm{kg/m^3}$.



$\rho_k = 1{,}05\cdot 10^3\ \mathrm{kg/m^3}$

$\rho = 1{,}03\cdot 10^3\ \mathrm{kg/m^3}$

$\eta = 1.5\cdot 10^{-3}\ \mathrm{kg/ms}$



$t_0 = 10\ \mathrm{h}$

$h = 0.1\ \mathrm{m}$

$$v = \frac{h}{t_0}$$

$$6\pi r \eta v + \frac{4\pi r^3}{3}\rho g = \frac{4\pi r^3}{3}\rho_k g$$

$$r = \sqrt{\frac{9\eta h}{2t_0(\rho_k-\rho)g}} = 9.65\ \mu\mathrm{m}$$

3\. Kroglica s polmerom 1,5 mm in maso 6 mg pada s hitrostjo 4 $\cdot 10^{-4}$ m/s v tekočini z viskoznostjo $8\cdot10^{-2}$ kg/(m·s). Enako velika kroglica z maso 4.5 mg pa se v tej tekočini dviga. Kolikšna je velikost hitrosti te kroglice ?

1994

$g = 10\,\mathrm{m\,s^{-2}}$

padanje:

$r_1 = r = 1{,}5\cdot10^{-3}$ m

$m_1 = 6\cdot10^{-6}$ kg

$v_1 = 4\cdot10^{-4}$ m/s

$\eta = 8\cdot10^{-2}$ kg/m·s

dviganje:

$r_2 = r = 1{,}5\cdot10^{-3}$ m

$m_2 = 4{,}5\cdot10^{-6}$ kg , $v_2 < 0$

$|v_2| = ?$

padanje: $m_1 g = 6\pi r\eta v_1 + m_0 g$

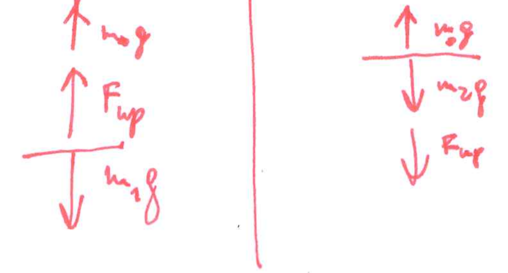
dviganje: $m_0 g = m_2 g - 6\pi r \eta v_2$

rešitev: sistem enačb $\{m_0, v_2\} = ?$

$$m_1 g = m_2 g + 6\pi r\eta v_1 - 6\pi r\eta v_2$$

$$|v_2| = -v_2 = \frac{(m_1 - m_2)g - 6\pi r\eta v_1}{6\pi r\eta} = 6{,}23\cdot10^{-3}\ \frac{\mathrm{m}}{\mathrm{s}}$$

6

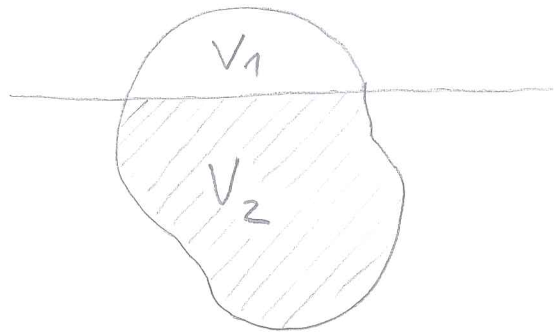


FOR-1996

2. V morju plava ledena gora z gostoto $\rho_{led} = 0.9\ g/cm^3$. Volumen ledene gore, ki štrli iz vode je $195\ m^3$. Kolikšen je celoten volumen ledene gore ? ($\rho_{vode} = 1.03\ g/cm^3$)

$\rho_l = 0.9\ g/cm^3$, $V_1 = 195\ m^3$

$\rho_{vod} = 1{,}03\ g/cm^3$



$$(V_1+V_2)\rho_l g = V_2 \rho_v g$$

$$V_1 \rho_l g = V_2 g(\rho_v - \rho_l)$$

$$V_2 = \frac{V_1 \rho_l}{(\rho_v - \rho_l)} = 1350\ m^3$$

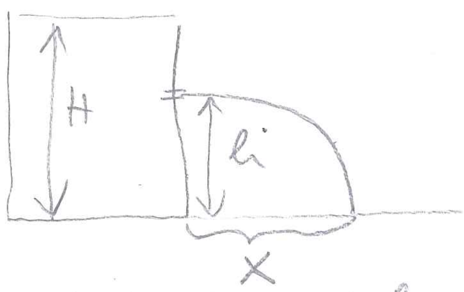
$$V = V_1 + V_2 = (1350 + 195)\ m^3$$

5. Posoda višine 40 cm je polna vode. Na višini 30 cm od dna posode izvrtamo v steno posode majhno luknjico. Ocenite kolikšen je domet izstopajočega curka vode ?

$H = 40\ cm$

$h = 30\ cm$

$x = ?$

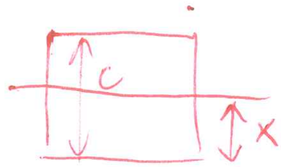


$$p_0 + \frac{\rho v^2}{2} + \rho g H = p_0 + \frac{1}{2}\rho v^2 + \rho g h$$

$$v = \sqrt{2g(H-h)} \qquad h = \frac{gt^2}{2} \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$x = v \cdot t = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}} = 0{,}35\ m$$

5. Lesen kvader s stranicama 10 cm, 20 cm in 30 cm plava v vodi tako, da sta njegovi največju ploskvi vzporedni z vodno gladino. Kako globoko pod vodo je spodnja ploskev kvadra ? (gostota lesa je 0.8 g/cm^3)



$a = 30\ cm$

$b = 20$ "

$c = 10\ cm$

$\rho_0 = 1 g/cm^3$

$\rho_1 = 0{,}8 g/cm^3$

$$\rho_1 abc = \rho_0 abx$$

$$x = c\frac{\rho_1}{\rho_0} = 0{,}8c = \underline{\underline{8\ cm}}$$

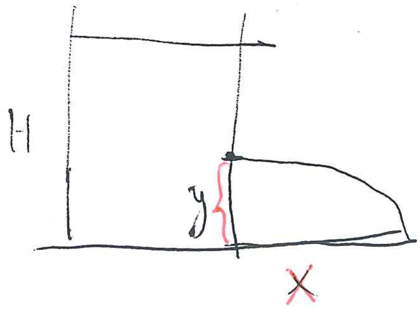
---

4. Pokončna posoda je do višine 3 m napolnjena z vodo in se nahaja na vodoravni podlagi. V stransko steno posode izvrtamo luknjico na višini 90 cm. Kolikšen je domet iztekajočega curka ?

$H = 3m$

$y = 90 cm$

$x = ?$



$$x = vt$$

$$y = \frac{gt^2}{2} \quad t = \sqrt{\frac{2y}{g}}$$

$$\rho g H = \frac{1}{2}\rho v^2 + \rho g y$$

$$v = \sqrt{2g(H-y)}$$

$$x = \sqrt{4y(H-y)} =$$

$$= \sqrt{4 \cdot 0{,}9 \cdot 2{,}1} = \underline{2{,}75 m}$$

Kocka s stranico

1992

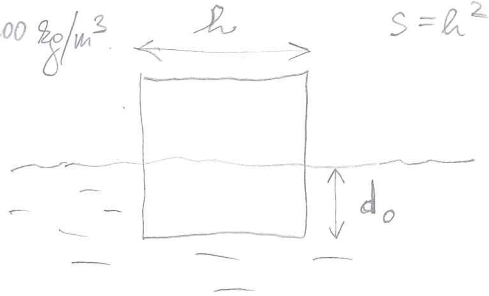
3. ~~Kvader višine~~ 10 cm in gostote 500 kg/m^3 plava v vodi. Nekoliko ~~ga~~ potisnemo v vodo in spustimo, da začne nihati. Kolikšen je njen nihajni čas ? Upor vode zanemarimo !

$h = 0{,}1\ m$, $\rho_0 = 1000\ kg/m^3$

$\rho = 500\ kg/m^3$

$t_0 = ?$

$S = h^2$



$h S \rho g = d_0 S \rho_0 g$

$d_0 = h \cdot \rho / \rho_0 = \underline{0{,}05\ m}$

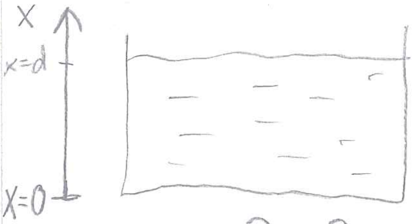
$\underbrace{h S \rho}_{m} \cdot \ddot{x} = - x S \rho_0 g$

$\ddot{x} = - \overbrace{g \cdot \frac{\rho_0}{\rho} \cdot \frac{1}{h}}^{\omega_0^2} \cdot x$

$$\boxed{t_0 = 2\pi \cdot \left[h \cdot \rho / (\rho_0 \cdot g)\right]^{1/2}} = \underline{0{,}444\ s}$$

2. Gostota tekočine v posodi je linearna funkcija razdalje od dna posode, gostota na dnu je ρ_o = 1,19 g/cm^3, na gladini pa je 10 % manjša (razdalja od dna posode do gladine tekočine je 1m). Ocenite kolikšno delo moramo opraviti, da predmet mase 0,01 kg in volumna 5 cm^3 telo počasi vzdignemo z dna posode na gladino ! (delo zaradi upora tekočine zanemarimo)

1996



$\rho_0 = 1{,}19\ g/cm^3\ (x=0)$, $d = 1m$

$m = 0.01\ kg$

$\rho_1 = 1{,}07\ g/cm^3\ (x=d)$, $V = 5\ cm^3$

$$\rho(x) = \frac{\rho_1 - \rho_0}{d}\cdot x + \rho_0$$

$$F(x) = mg - Vg\cdot\rho(x)$$

$$A = \int_0^d F(x)\,dx = \int_0^d \left[mg - Vg\left(\frac{\rho_1-\rho_0}{d}x + \rho_0\right)\right]dx =$$

$$= V\frac{gd}{2}\left(2\cdot\frac{m}{V} - \rho_1 - \rho_0\right) = \underline{\underline{4{,}27\cdot 10^{-2}\ J}}$$

6. Žoga gostote 750 kg/m^3 lebdi na meji dveh tekočin, ki se ne mešajo. Gostota spodnje tekočine je 820 kg/m^3, gostota zgornje pa 658 kg/m^3. Določite kolikšen del žoge je potopljen v zgornji tekočini !

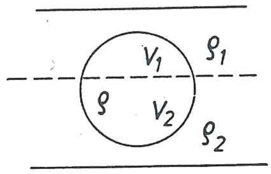
Neka je u gornjoj tečnosti gustoće ρ_1 potopljen dio lopte zapremine V_1, a u donjoj tečnosti gustoće ρ_2 dio lopte zapremine V_2.

Ako lopta lebdi, mora biti ispunjen uslov

$$\rho V g = \rho_1 V_1 g + \rho_2 V_2 g, \qquad (1)$$

gdje je V ukupna zapremina lopte, a ρ njena gustoća. Iz uslova da je

$$V = V_1 + V_2$$



i relacije (1) dobija se

$$V_1 = \frac{\rho_2 - \rho}{\rho_2 - \rho_1} V$$

$$V_2 = \frac{\rho - 1}{\rho_2 - 1} V,$$

tj.

$$V_1 = 0{,}432\ V$$

$$V_2 = 0{,}568\ V.$$

1994

1. V toplotno izolirani posodi se nahaja voda v kateri plava košček ledu mase 0,1 kg v katerem se nahaja svinčena kroglica mase 5 g. Koliko toplote moramo dovesti, da bi kroglica začela toniti ? (ρ_{Pb} = 11,3 g/cm^3, ρ_{led} = 0,9 g/cm^3, q_{tal} = 3,3·10^5 J/kg, T_{vode} = 0^{o}C)

$m_0 = 100\ g$ $\qquad \rho_v$ ≡ gostota vode

$m_2 = 5\ g$ $\qquad m_1$ ≡ nova masa ledu

$$m_1 + m_2 = V \cdot \rho_v$$ ← zahteva $F_{vzg} = F_g$

$$V = \frac{m_1}{\rho_e} + \frac{m_2}{\rho_{Pb}}$$

$$m_1 + m_2 = \rho_v \left(\frac{m_1}{\rho_e} + \frac{m_2}{\rho_{Pb}} \right)$$

$$m_1 = m_2 \frac{(\rho_{Pb} - \rho_v)\rho_e}{(\rho_v - \rho_L)\rho_{Pb}} = 8{,}2\ m_2$$

$\Delta m = m_0 - m_1 = 59\ g$, $\quad Q = \Delta m\, q_{tal} = 19{,}5 \cdot 10^3\ J$

---

2. 20 cm dolga cev je sestavljena iz treh delov. Dolžina prvega dela je 9 cm, njegov polmer pa 0,5 mm, dolžina drugega dela je 5 cm in polmer 0,8 mm. Polmer tretjega dela je 1 mm. Tlak na začetku cevi je 2·10^3 Pa, tlak na koncu cevi pa je 10^3 Pa. Viskoznost tekočine, ki teče skozi cev je 10^{-3} kg/(m·s). Ocenite volumski pretok skozi cev ?

$$\Phi_V = \frac{\pi r^4 \Delta p}{8 l \eta}$$

$$\Phi_V = \frac{\Delta p}{R}$$

$$R = \frac{8 l \eta}{\pi R^4}$$

$l_1 = 0{,}09\ m$, $R_1 = 0{,}5 \cdot 10^{-3}\ m$ $\qquad \Delta p = 10^3\ Pa$

$l_2 = 0{,}05\ m$, $R_2 = 0{,}8 \cdot 10^{-3}\ m$ $\qquad \eta = 10^{-3}\ kg/(m \cdot s)$

$l_3 = 0{,}06\ m$, $R_3 = 10^{-3}\ m$

$$\Phi_V = \frac{\Delta p}{\frac{8 l_1 \eta}{\pi R_1^4} + \frac{8 l_2 \eta}{\pi R_2^4} + \frac{8 l_3 \eta}{\pi R_3^4}} = \frac{\pi \cdot \Delta p}{8\eta \left[\frac{l_1}{R_1^4} + \frac{l_2}{R_2^4} + \frac{l_3}{R_3^4} \right]} =$$

$$\frac{\pi \cdot 10^3}{8 \cdot 10^{-3} \cdot \frac{10^{-2}}{10^{-12}} [162{,}2]} = \frac{\pi \cdot 10^3 \cdot 10^{-7}}{8 \cdot 162{,}2} = 2{,}42 \cdot 10^{-7}\ m^3/s$$

2. Železna votla krogla tehta na zraku 270 N, v vodi pa 230 N. Kolikšen je volumen votline ? Gostota vode je 1000 kg/m^3, gostota železa pa je 7800 kg/m^3.

$N_1 = 270\ N$

$N_2 = 230\ N$

$\rho_0 = 1000\ kg/m^3$

$\rho_{Fe} = 7800\ kg/m^3$

$\rho_{Fe} V_e g = N_1 \longrightarrow V_e = N_1 / \rho_{Fe} \cdot g$

$\rho_{Fe} V_e g - \rho_0 (V_e + V_v) g = N_2$

$$N_1 - \frac{\rho_0 N_1}{\rho_{Fe}} - \rho_0 V_v g = N_2$$

$$V_v = \frac{1}{\rho_0 g}(N_1 - N_2) - \frac{N_1}{\rho_{Fe} g} = 5{,}38 \cdot 10^{-4}\ m^3 = 0{,}54\ l$$

0.004

$V_e + V_v = 4\ l$

Popr. t. 1991 $g = 9.8\ m\,s^{-2}$

FI

3. Aluminijasta kroglica polmera 0,5 cm pade z višine 0,05 m v viskozno tekočino (η = 0,05 Pa·s). Kakšna bo hitrost kroglice čez 2 sekundi ? Gostota aluminija je 2,7 g/cm^3, gostota tekočine pa 1,2 g/cm^3. (čas štejš od padca v vodo)

podatki:

$r = 5\cdot 10^{-3}\ m$

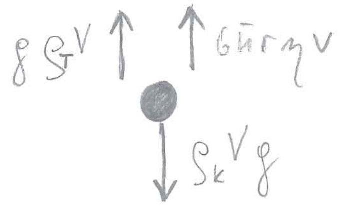
$h = 0{,}05\ m$

$\eta = 0{,}05\ Pa\cdot s$

$t = 2\ s$

$\rho_k = 2700\ kg/m^3$

$\rho_T = 1200\ kg/m^3$



$v_0 = \sqrt{2gh} = 0.99\ \frac{m}{s}$

$h = \frac{g t_0^2}{2} \Rightarrow t_0 = \sqrt{\frac{2h}{g}} = 0.1\ s$

$$-6\pi r\eta v - \rho_T V_k g + \rho_k V_k g = \rho_k V_k \frac{dv}{dt}$$

$\alpha = \frac{6\pi r\eta}{\rho_k V_k}$

$\beta = \frac{(\rho_k - \rho_T)\cdot V_k \cdot g}{\rho_k \cdot V_k}$

$\beta = 1 - \frac{\rho_T}{\rho_k}$

$$-\alpha v + \beta = \frac{dv}{dt}$$

$$\int_{v_0}^{v} \frac{dv}{\beta - \alpha v} = \int_{t_0}^{t} dt$$

$$\beta - \alpha v = (\beta - \alpha v_0)\, e^{-\alpha(t-t_0)}$$

$$v = \frac{\beta}{\alpha} - \left(\frac{\beta}{\alpha} - v_0\right) e^{-\alpha(t-t_0)}$$

$t_0 = 0$: $v = 0.168\ m/s \simeq \frac{\beta}{\alpha} = v_{rav}$

$t_0 = 0.1\,s$: $v = 0.168\ m/s \simeq \frac{\beta}{\alpha} = v_{rav}$

LIMITA:

če $\frac{dv}{dt} \simeq 0$

$$v_r = \frac{\rho_k V_k g - \rho_T V_k g}{6\pi r\eta} = \frac{\beta}{\alpha}$$

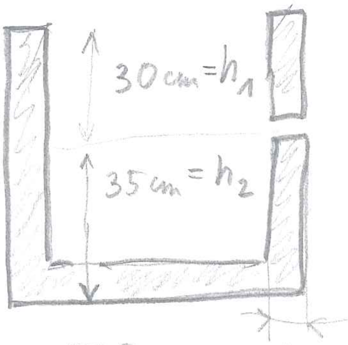
$v_r = 0.17\ m/s$

kolokvij 90/91

(T6)

5. 65 cm [illegible] visoka jeklena posoda z debelino sten 5 cm je do vrha napolnjena s ~~tekočino~~ VODO, ki ima viskoznost 10^{-3} kg/ms. 30 cm od vrha kotla je zamašena luknjica s premerom 1 mm. S kakšno hitrostjo izstopi voda iz luknjice, če jo odmašimo? Kako daleč od zunanje stene kotla pade curek tekočine na podlago, na kateri stoji kotel ?
(za volumski tok skozi ravno enakomerno debelo cev velja: $\Phi_V = \pi r^4 \Delta p / 8\eta l$, r = polmer cevi, l = dolžina cevi, η = viskoznost tekočine, Δp = razlika tlakov med koncema cevi)

(A-M)



$S = \pi r^2$ $\quad l = 5\,cm$

$l = 0.05\,m$

$2r = 1 \cdot 10^{-3}\,m$

$\Delta p = \rho g h_1$

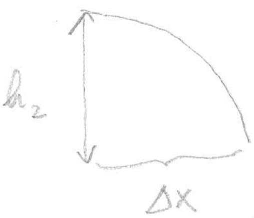
$\eta = 10^{-3}\,kg/ms$

$$\Phi_V = \frac{dV}{dt} = \frac{S\,dx}{dt} = S \cdot v$$

$\rho g h_1 = \Delta p$

$$v = \frac{\Phi_V}{S} = \frac{\pi r^4 \Delta p}{S 8 \eta l} = \frac{\pi r^4 \Delta p}{\pi r^2 8 l \eta} = \frac{r^2 \Delta p}{8 l \eta} =$$

$$= \frac{r^2 \rho g h_1}{8 l \eta} = \frac{(0.5)^2 \cdot 10^{-6} \cdot 1000 \cdot 10 \cdot 0.3}{8 \cdot 0.05 \cdot 0.001} = 1.875\,\frac{m}{s}$$



$v_x = -gt$

$s = \frac{gt^2}{2} \Rightarrow t = \sqrt{\frac{2h_2}{g}}$

$$\Delta x = v \cdot \sqrt{\frac{2h_2}{g}} = 1.875 \sqrt{\frac{2 \cdot 0.35}{10}} = 0.496\,m$$

... premerom 5 mm in gostoto 2,6 g/cm³, ki pada v viskozni tekočini, ima v nekem trenutku hitrost 1 m/s. Ocenite kolikšen je pospešek kroglice v tem trenutku ? Ocenite tudi kolikšna bo stacionarna hitrost kroglice v tej tekočini ? Gostota tekočine je 1,2 g/cm³, viskoznost pa $6\cdot 10^{-2}$ Pa·s.

1

$g = 10\ ms^{-2}$

$r = 5\cdot 10^{-3}\ m$

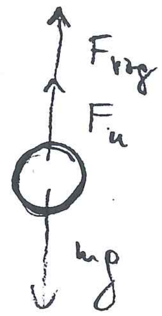
$v(t) = 1\ m/s$

$\rho = 2{,}6\cdot 10^{3}\ kg/m^3$

$\rho_t = 1{,}2\cdot 10^{3}\ kg/m^3$

$\eta = 6\cdot 10^{-2}\ Pa\cdot s$

$v = ?$



$$\boxed{mg = 6\pi r \eta v_s + m_t g} \quad \text{ravnovesje}$$

$$v_s = \frac{(m - m_t) g}{6\pi r \eta} = \frac{(\rho - \rho_t) 4\pi r^3 \cdot g}{3\cdot 6\pi r \eta} \Rightarrow$$

B) $$\boxed{v_s = \frac{(\rho - \rho_t) 2 r^2 g}{9\eta} \cong} \quad \underline{1.3\ \frac{m}{s}} \ \text{(stacionarna hitrost)}$$

A) $$ma = mg - 6\pi r \eta v - m_t g$$

$$a = g - \frac{6\pi r \eta v}{m} - \frac{m_t}{m} g = g\left(1 - \frac{m_t}{m}\right) - \frac{6\pi r \eta v}{m}$$

$$= g\left(1 - \frac{\rho_t}{\rho}\right) - \frac{3\cdot 6\pi r \eta v}{4\pi r^3 \rho} = g\left(1 - \frac{\rho_t}{\rho}\right) - \frac{9\eta v}{2 r^2 \rho} =$$

$$= \underline{1.13\ \frac{m}{s^2}}$$

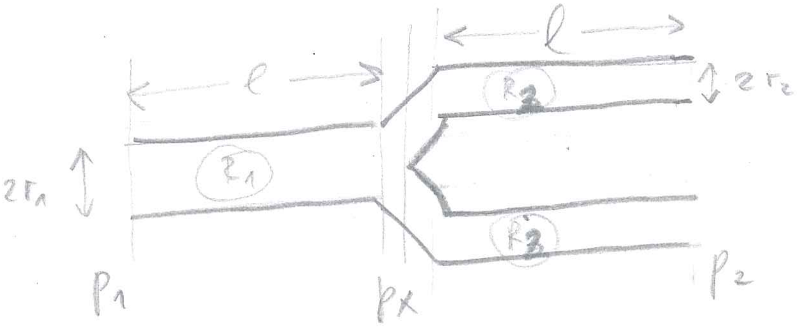
Zdravij 90/91

(T4)

1. Po 10 cm dolgi cevi s premerom 0,4 cm, ki se razcepi v dve enaki 10 cm dolgi cevi s premeroma 0,2 cm, se pretaka tekočina z viskoznostjo $2 \cdot 10^{-3}$ kg/ms. Tlak na začetku široke cevi je $2 \cdot 10^{3}$ N/m^2, na koncu obeh tanjših cevi pa je $0{,}64 \cdot 10^{3}$ N/m^2. Izračunaj celotni volumski tok skozi opisani sistem cevi!

(za volumski tok skozi ravno enakomerno debelo cev velja: $\Phi_V = \pi r^4 \Delta p / 8 \eta l$, r = polmer cevi, l = dolžina cevi, η = viskoznost tekočine, Δp = razlika tlakov med koncema cevi)

N-7



$r_1 = 2 \cdot 10^{-3}$ m, $l = 0.1$ m

$r_2 = 10^{-3}$ m,

$\eta = 2 \cdot 10^{-3}$ kg/ms

$p_1 = 2 \cdot 10^{3}$ N/m^2

$p_2 = 0.64 \cdot 10^{3}$ N/m^2

$\Delta p = p_1 - p_2 = 1.36 \cdot 10^{3} \frac{N}{m^2}$

$$\Phi_V = \frac{\pi r^4}{8 \eta l} \cdot \Delta p = \frac{\Delta p}{R}$$

$$R = \frac{8 \eta l}{\pi r^4}$$

$$\frac{1}{\tilde{R}_2} = \frac{1}{R_2} + \frac{1}{R_2} = \frac{2}{R_2} \Rightarrow \tilde{R}_2 = \frac{R_2}{2}$$

$$R = R_1 + \frac{R_2}{2}$$

$$R = \frac{8 \eta l}{\pi r_1^4} + \frac{1}{2} \frac{8 \eta l}{\pi r_2^4} = \frac{8 \eta l}{\pi} \left(\frac{1}{r_1^4} + \frac{1}{2} \frac{1}{r_2^4} \right) = \frac{8 \eta l}{\pi} \frac{2 r_2^4 + r_1^4}{2 r_2^4 r_1^4}$$

$$\Phi_V = \frac{\Delta p}{R} = 4{,}75 \cdot 10^{-6} \ m^3 s^{-1}$$

$$= \frac{\pi}{4 \eta l} \frac{r_2^4 r_1^4 \Delta p}{2 r_2^4 + r_1^4}$$

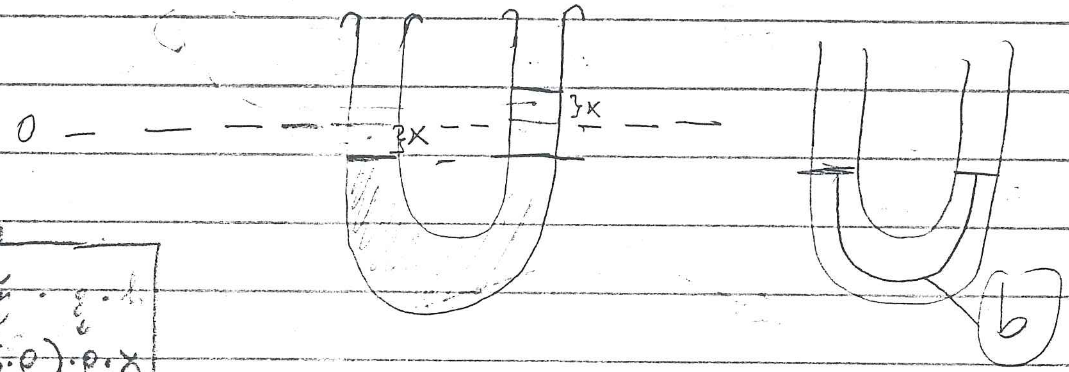
Naloga 1590

$m = 1.2\ kg$

$\rho_{Hg} = 13.6\ g/cm^3$

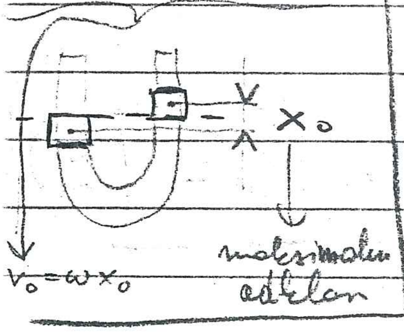
$2R = 1.2\ cm$

V cevi v obliki črke U je 1.2 kg živega srebra. Notranji premer cevi je 1.2 cm. S kolikšnim nihajnim časom ~~frekvenco~~ lahko zaniha živosrebrni steber, če ga spravimo iz ravnovesne lege? Gostota Hg je $13.6\ g/cm^3$.



DRUGI NAČIN

$$(bS\rho)\frac{v_0^2}{2} = (x_0 \cdot S \cdot \rho)\cdot g \cdot x_0$$



$$ma = -\underbrace{2xS}_{\Delta V}\cdot\rho g$$

$$a = -\frac{2S\rho g}{m}\cdot x$$

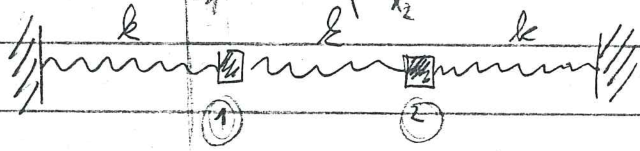
$$bS\rho = m \Rightarrow \frac{S\rho}{m} = \frac{1}{b}$$

$$a = -\frac{2g}{b}\cdot x$$

$$\omega^2 = \frac{2g}{b}$$

$$\omega^2 = \frac{2S\rho g}{m}$$

Naloga 1580



(glej Strnad str. 140 / 142)

$k' = \frac{k}{8}$

Dve enaki vzmetni nihali z lastno frekvenco $\nu_1 = 2\ Hz$ sta med seboj povezani z vzmetjo, ki je osemkrat šibkejša od vzmeti vsakega od nihal. S kolikšno ponavljalno dobo utripata nihali če eno od njih zanihamo z

OBRNI!

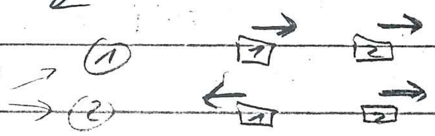
$e^{i\omega t} = \cos\omega t - i\sin\omega t$

$$m a_1 = -k x_1 + k'(x_2 - x_1)$$
$$m a_2 = -k x_2 - k'(x_2 - x_1)$$

(glej Kuščer, Mat. fizika, str. 128)

nastavek: $x_1 = A_1 e^{i\omega t}$, $x_2 = A_2 e^{i\omega t}$

OBA NIHATA NA ISTI NAČIN ⟹ LASTNO NIHANJE



(se da rešiti z $A_1 \sin(\omega t)$, $A_2 \sin(\omega t)$)

$$-A_1 m\omega^2 e^{i\omega t} = -k A_1 e^{i\omega t} + k' e^{i\omega t}(A_2 - A_1)$$
$$-A_2 m\omega^2 e^{i\omega t} = -k A_2 e^{i\omega t} - k' e^{i\omega t}(A_2 - A_1)$$

$$A_1(m\omega^2 - k - k') + k' A_2 = 0$$
$$A_1 \cdot k' + (m\omega^2 - k - k')A_2 = 0$$

det = 0 če sistem rešiter ⟹ dokaz:

množiš

$a_{11} A_1 + a_{12} A_2 = 0 \quad / A_1 a_{22}$

$a_{21} A_1 + a_{22} A_2 = 0 \quad / A_1 a_{12}$

odštej ② od ①

$a_{11} a_{22} A_1^2 - a_{21} a_{12} A_1^2 = 0$

$\underbrace{(a_{11} a_{22} - a_{21} a_{12})}_{} A_1^2 = 0$

det = 0

⇓

$$(m\omega^2 - k - k')^2 - k'^2 = 0$$
$$(m\omega^2 - k)^2 - 2k'(m\omega^2 - k) = 0$$
$$(m\omega^2 - k)\left[m\omega^2 - k - 2k'\right] = 0$$

$m\omega^2 - k = 0 \Rightarrow \boxed{\omega_1 = \sqrt{\frac{k}{m}}}$ ①

$m\omega^2 - k - 2k' = 0 \Rightarrow \boxed{\omega_2 = \sqrt{\frac{k + 2k'}{m}}}$ ②

→ lastni nihanji

v splošnem velja: $x_1 = x_0 \cos\omega_1 t + x_0 \cos\omega_2 t =$
($\delta = 0$)

$$= x_0(\cos\omega_1 t + \cos\omega_2 t) = 2x_0 \cos\left[\tfrac{1}{2}(\omega_2 - \omega_1)t\right]\cos\omega t$$

$\omega = \frac{1}{2}(\omega_1 + \omega_2)$

$$x_0\left[\cos\left(t\frac{\omega_1 + \omega_2}{2} - t\frac{\omega_2 - \omega_1}{2}\right) + \cos\left(t\frac{\omega_1 + \omega_2}{2} - t\frac{\omega_1 - \omega_2}{2}\right)\right]$$

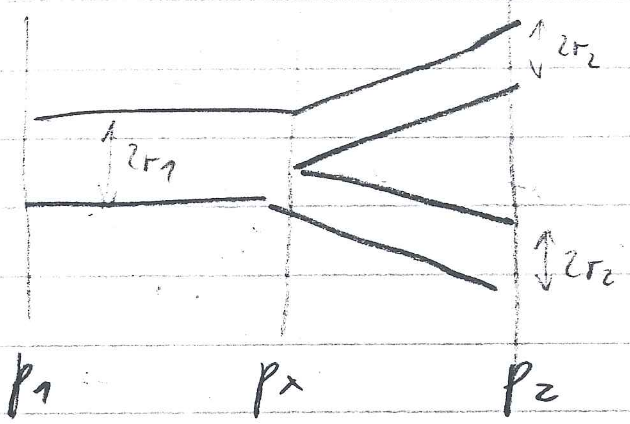
$x_0\left[\cos(\ )\cos(\ )\right]$

manjša frekvenca ↓ spreminjanje amplitude ↓ frekvenca utripanja

M - Naloga ⊗

$$\phi_V = \frac{\pi r^4}{8\eta l}$$

Po cevi ki se razcepi v dve enaki cevi se pretaka tekočina z viskoznostjo $\eta = 2\cdot10^{-3}\ kg\,m^{-1}s^{-1}$. Tlak na širokem koncu cevi $p_1 = 2\cdot10^{3}\ Nm^{-2}$, na koncu obeh tanjših cevi pa je $p_2 = 0.64\cdot10^{3}\ Nm^{-2}$. Izračunaj celotni volumski tok, če je $2r_1 = 0.4cm$, $2r_2 = 0.2cm$ $\partial\rho$



$l_1 = l_2 = l_0 = 10cm$

$\eta = 2\cdot10^{-3}\ kg/ms$

$p_1 = 2\cdot10^{3}\ N/m^2$

$p_2 = 0.64\cdot10^{3}\ N/m^2$

$\phi_V = ?$

$$\phi_{V_1} = \phi_{V_2}^{(1)} + \phi_{V_2}^{(2)}$$

1. način:

$$\boxed{\phi_{V_1} = 2\phi_{V_2}}$$

$$\frac{\pi r_1^4 (p_1 - p_x)}{8\eta l_0} = \frac{2\pi r_2^4 (p_x - p_2)}{8\eta l_0}$$

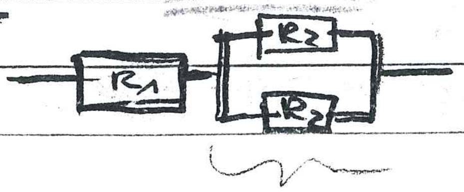
$\Downarrow$

$$p_x = \frac{p_1 + \frac{2r_2^4}{r_1^4}\cdot p_2}{1 + \frac{2r_2^4}{r_1^4}} = 1.85\cdot10^{3}\ \frac{N}{m^2}$$

$$\phi_V = 4.75\cdot10^{-6}\ m^3s^{-1}$$

2. način:

$$\phi_V = \frac{\Delta p}{R}$$



$\frac{1}{\tilde{R}} = \frac{2}{R_2} \Rightarrow \tilde{R} = \frac{R_2}{2}$

$$\Rightarrow \boxed{R = R_1 + \frac{R_2}{2}}$$

$$\phi_V = \frac{\Delta p}{R_1 + \frac{R_2}{2}}$$

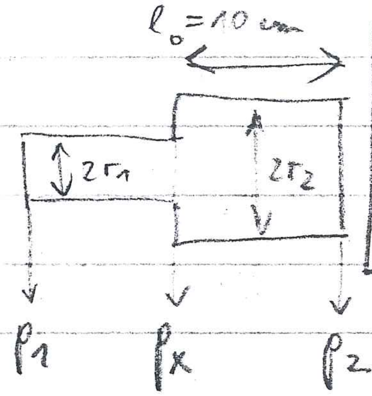
Naloga, M

Kapilara dolga 20 cm je sestavljena iz dveh 10 cm dolgih delov. Polmer prvega je 0.5 mm, drugega pa 1mm (glej sliko). Razliko tlakov med koncema kapilare je $10^3 N/m^2$. Viskoznost tekočine, ki teče skozi kapilaro je 10^{-3} kg/ms. Izračunaj volumski tok? Za volumski tok skozi ravno enakomerno debelo cev velja:

$$\phi_V = \pi r^4 \Delta p / 8 l \eta$$

$r_1 = 0.5$ mm

$r_2 = 1$ mm



$$\Delta p = p_2 - p_1 = 10^3 \ N/m^2$$

$$\phi_{v,1} = \frac{\pi r_1^4 (p_x - p_1)}{8 l_0 \eta}, \quad \phi_{v,2} = \frac{\pi r_2^4 (p_2 - p_x)}{8 l_0 \eta}$$

$$\phi_{v,1} = \phi_{v,2}$$

$$\frac{\pi r_1^4 (p_x - p_1)}{8 l_0 \eta} = \frac{\pi r_2^4 (p_2 - p_x)}{8 l_0 \eta}$$

$$r_1^4 (p_x - p_1) = r_2^4 (p_2 - p_x)$$

$$r_1^4 p_x - r_1^4 p_1 = r_2^4 p_2 - r_2^4 p_x$$

$$p_x (r_1^4 + r_2^4) = r_2^4 p_2 + r_1^4 p_1$$

$$p_x = \frac{r_2^4 p_2 + r_1^4 p_1}{(r_1^4 + r_2^4)}$$

$$\Delta p_1 = p_x - p_1$$

$$\Delta p_1 = \frac{r_2^4 p_2 + r_1^4 p_1 - p_1 r_1^4 - p_1 r_2^4}{(r_1^4 + r_2^4)} = \frac{r_2^4 (p_2 - p_1)}{(r_1^4 + r_2^4)} = \frac{r_2^4 \cdot \Delta p}{(r_1^4 + r_2^4)}$$

$$\Phi_V = \frac{\pi r_1^4}{8 l_0 \eta} \cdot \Delta p_1 = \frac{\pi r_1^4}{8 l_0 \eta} \cdot \frac{r_2^4 \cdot \Delta p}{(r_1^4 + r_2^4)} \Rightarrow$$

$$\boxed{\Phi_V = \frac{\pi \Delta p \, r_1^4 \cdot r_2^4}{8 l_0 \eta (r_1^4 + r_2^4)}}$$

DRUGI NAČIN:

$$I = \frac{U}{R}, \quad \Phi_V = \frac{\Delta p}{R}, \quad R = \frac{8 l_0 \eta}{R^4 \pi}$$

$$\Phi_V = \frac{\Delta p}{R_1 + R_2} = \frac{\Delta p}{\frac{8 l_0 \eta}{\pi}\left(\frac{1}{r_1^4} + \frac{1}{r_2^4}\right)} = \frac{\pi \Delta p}{8 l_0 \eta} \frac{r_1^4 r_2^4}{(r_1^4 + r_2^4)}$$

Naloga

6. Na gladino raztopine v 10 cm visoki posodi stresemo različno velike kroglice z gostoto 1,05.10^3 kg/m^3. Zaradi teže se kroglice pričnejo gibati navzdol. Izračunaj premer tistih kroglic, ki se po enem dnevu ravno usedejo na dno posode? Raztopina je enako gosta kot voda in ima viskoznost 1,5.10^{-3} kg/ms.

T5

1995

$t_0 = 1^h$

$h = 0.1\,m$

$\rho_k = 1.05 \cdot 10^3\ kg/m^3$

$\rho = 1 \cdot 10^3\ kg/m^3$

$\eta = 1.5 \cdot 10^{-3}\ kg/ms$

$h = v \cdot t_0$

$v = \frac{h}{t_0}$

$$6\pi r \eta v + \frac{4\pi r^3}{3} \rho g = \frac{4\pi r^3}{3} \cdot \rho_k g$$

$\Downarrow$

$$r^2 = 9 \eta h / [2 t_0 (\rho_k - \rho) \cdot g]$$

$g = 10$: $2r = 7.9\ \mu m$ $\quad r = 3.95\ \mu m$

$g = 9.82$: $2r = 7.98\ \mu m$

Datum *mol. 10 (B)* *(2)*

$$\left(\frac{\beta}{\alpha} - (\beta - \alpha v_0)\cdot\frac{1}{\alpha} e^{-\alpha t}\right) dt = dy$$

$$\frac{\beta}{\alpha} t - \left(\frac{\beta}{\alpha} - v_0\right)\left(\frac{-1}{\alpha}\right)\left(e^{-\alpha t} - 1\right) = y$$

$$\frac{\beta}{\alpha} t + \left(\frac{\beta}{\alpha} - v_0\right)\frac{1}{\alpha}\left(e^{-\alpha t} - 1\right) = y$$

$$mgh = m\frac{v_0^2}{2} \qquad v_0^2 = 2gh \qquad v_0 = \sqrt{2gh} = 1\,\frac{m}{s}$$

$$v_0 = 1\,\frac{m}{s}$$

$$\beta = \left(1 - \frac{1200}{2700}\right)\cdot 10 = 5{,}56$$

$$\alpha = \frac{6\pi\ 0{,}5\cdot 10^{-2}\ 0{,}05\ 3}{4\pi\ 0{,}5^3\cdot 10^{-6}\ 2700} = 3{,}33\cdot 10^{-4-2+6}$$

$$y = \frac{5{,}56}{3{,}33}\cdot 2 + \left(\frac{5{,}56}{3{,}33} - 1\right)\frac{1}{3{,}33}\left(1 - e^{-3{,}33\cdot 2}\right) = 3{,}14\,m$$

0,201

d)

10. Aluminijasta kuglica radija r = 0,5 cm padne sa visine h = 0,05 m u viskoznu tečnost sa koeficientom η = 0,05 Pa.s. Do koje dubine će doći kroz t = 2 s. Gustina Al je ρ=2,7 g/cm^3, gustina tečnosti je ρ_o=1,2 g/cm^3.

a) 3,14 m
b) 2,76 m
c) 0,3 m
d) 3,84 m
e) ni jedan od gornjih rezultata nije tačan

3. Na gladino tekočine v 10 cm visoki posodi stresemo različno velike kroglice z gostoto $1{,}05\cdot 10^3$ kg/m^3. Zaradi teže se kroglice pričnejo gibati navzdol. Izračunaj premer tistih kroglic, ki se po enem dnevu ravno usedejo na dno posode ? Tekočina je enako gosta kot voda in ima viskoznost $1{,}5\cdot 10^{-3}$ kg/(m·s). Kroglice dosežejo ravnovesno hitrost v zanemarljivo kratkem času. 1993

$h = 10$ cm

$t_0 = 24^h$

$\rho_k = 1.05\cdot 10^3$ kg/m^3

$\rho = 10^3$ kg/m^3

$\eta = 1.5\cdot 10^{-3}$ kg/ms

$$6\pi r\eta v + \frac{4\pi r^3}{3}\rho g = \frac{4\pi r^3}{3}\rho_k g$$

$$18\eta v + 4r^2\rho = 4r^2\rho_k$$

$$9\eta v + 2r^2\rho = 2r^2\rho_k$$

$$9\eta v = 2r^2(\rho_k - \rho)g$$

$h = v\cdot t_0$

$\Downarrow$

$$v = \frac{h}{t_0}$$

$$9\eta\frac{h}{t_0} = 2r^2(\rho_k - \rho)g$$

$$r^2 = \frac{9\eta h}{t_0\cdot 2\cdot(\rho_k-\rho)g} \qquad r = \sqrt{\frac{9\eta h}{2t_0(\rho_k-\rho)g}}$$

$$\frac{9\cdot 1{,}5\cdot 10^{-3}\cdot 0.1}{9.82\; 2\cdot 24\cdot 3600\,(0.05)\cdot 10^3} = \frac{9\cdot 1.5\cdot 10^{-4}}{48\cdot 36\cdot 5\cdot 10^3\cdot 9.8} \Rightarrow$$

$$r = \sqrt{\frac{9\cdot 1.5\cdot 10^{-7}}{48\cdot 36\cdot 5\cdot 9.82}} = 3{,}989\cdot 10^{-6}\ \text{m}$$

$g = 10$: $2r = 3{,}953$ μm, $r = 7{,}90$ μm

$2r = 7.98$ μm, $r = 3.99$ μm $\rightarrow g = 9{,}82$

3. Kroglica s polmerom 5 mm in gostoto 2,6 g/cm^3 pade v viskozno tekočino s hitrostjo 1 m/s pravokotno na gladino. Gostota tekočine je 1,2 g/cm^3, viskoznost pa $6\cdot 10^{-2}$ Pa·s. Oceni kolikšna bo hitrost kroglice čez 35 sekund ?

$g = 10\ \mathrm{m s^{-2}}$

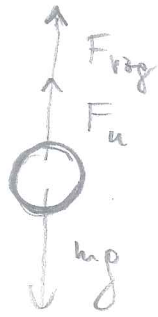
$r = 5\cdot 10^{-3}$ m

$\rho = 2{,}6\cdot 10^{3}$ kg/m^3

$\rho_t = 1{,}2\cdot 10^{3}$ kg/m^3

$\eta = 6\cdot 10^{-2}$ Pa·s

$v = ?$



$$mg = 6\pi r \eta v + m_t \cdot g$$ ravnovesje

$$v = \frac{(m - m_t)g}{6\pi r \eta} = \frac{(\rho - \rho_t)4\pi r^3 \cdot g}{3\cdot 6\pi r \eta} \Rightarrow$$

$$v = \frac{(\rho - \rho_t)2r^2 g}{9\eta} = 1.3\ \frac{\mathrm{m}}{\mathrm{s}}$$