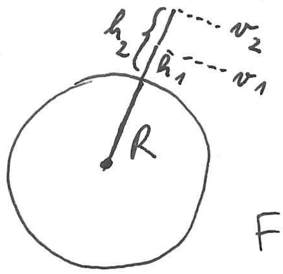
UN

2. Izstrelek izstrelimo navpično navzgor. Na nadmorski višini 2000 km ima hitrost 4 km/s v smeri navpično navzgor. Kolikšna je hitrost tega izstrelka na nadmorski višini 3000 km ? Zanemarite zračni upor.

$R = 6400\,\mathrm{km}$, $g_0 = 9.81\,\mathrm{m\,s^{-2}}$

$v_1 = 4\,\mathrm{km/s} = 4000\,\mathrm{m/s}$, $h_1 = 2\cdot10^6\,\mathrm{m}$, $h_2 = 3\cdot10^6\,\mathrm{m}$



$$W_{p,1} + \frac{m v_1^2}{2} = W_{p,2} + \frac{m v_2^2}{2}$$

$$m v_2^2 = 2(W_{p,1} - W_{p,2}) + m v_1^2$$

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$$v_2 = \sqrt{\frac{2}{m}\left(\frac{-m g_0 R^2}{(R+h_1)} + \frac{m g_0 R^2}{(R+h_2)}\right) + v_1^2}$$

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$$v_2 = \sqrt{2 g_0 R\left[\frac{R}{R+h_2} - \frac{R}{R+h_1}\right] + v_1^2}$$

$$\underline{\underline{v_2 = 2.41\,\mathrm{km/s}}}$$

$$F = -G\frac{mM}{r^2}$$

$$F = -\frac{dW_p}{dr} = -\frac{d}{dr}\left(-G\frac{mM}{r}\right)$$

$$W_p = -G\frac{mM}{r}$$

$$\boxed{W_p = -G\frac{mM}{(R+h)}}$$

ker je $g_0 = G\frac{M}{R^2}$

$$\Downarrow$$

$$GM = g_0 R^2$$

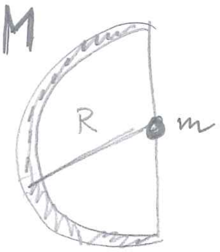
$$\Downarrow$$

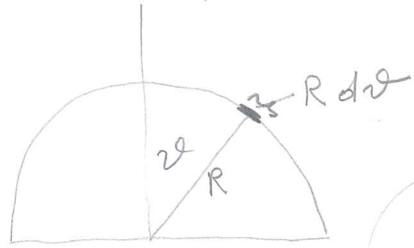
$$\boxed{W_p = -\frac{m g_0 R^2}{(R+h)}}$$

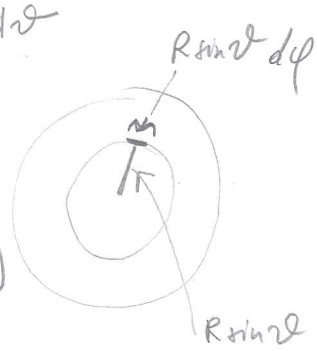
$g_0 = 9.81\,\mathrm{m\,s^{-2}}$

$W_p = mgh$

nal | gravitacija : M



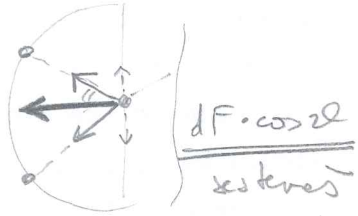




$$\boxed{\sigma = \frac{M}{S} = \frac{M}{2\pi R^2}}$$

$$dS = (R\,d\vartheta)(R\sin\vartheta\,d\varphi)$$

$$\boxed{dF = \varkappa\,\frac{m\cdot dM}{R^2}\cdot\cos\vartheta}$$



$$F = \iint \frac{\varkappa\cdot m\cdot M\;dS}{2\pi R^2\cdot R^2}\cdot\cos\vartheta =$$

$$\boxed{dM = \sigma\cdot dS}$$

$$= \iint \frac{\varkappa m M}{2\pi R^4}\cos\vartheta\,R^2\sin\vartheta\,d\vartheta\,d\varphi =$$

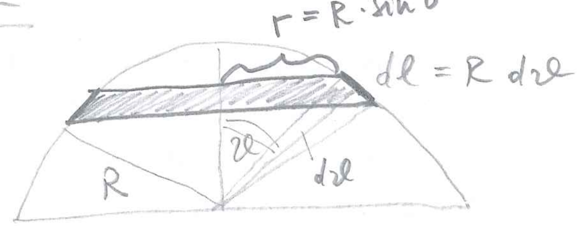
$$= \frac{\varkappa m M}{2\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^{2\pi}\sin\vartheta\cos\vartheta\,d\vartheta\,d\varphi = \frac{\varkappa m M}{2\pi R^2}\int_0^{1}\int_0^{2\pi} d\varphi\cdot\underbrace{x\cdot dx}_{\frac{x^2}{2}} = \frac{\varkappa m M}{2\pi R^2}\cdot\pi =$$

$$\sin\vartheta = x$$

$$\cos\vartheta\,d\vartheta = dx, \quad d\vartheta = \frac{dx}{\cos\vartheta}$$

$$= \frac{\varkappa m M}{2R^2}$$

drugi način



$$dS = 2\pi r\,dl =$$

$$= 2\pi R\sin\vartheta\,R\,d\vartheta$$

$$F = \int \varkappa\,\frac{m\,\sigma\,dS}{R^2}\cos\vartheta =$$

Naloga 1988

Umetni satelit izgubi zaradi trenja 1 % celotne mehanske energije. Ocenite ~~za~~ koliko odstotkov so se povečali ali zmanjšali:

a) polmer tira

b) hitrost satelita

c) obhodni čas

Privzemite da je gibanje satelita krožno.

Delaj z diferencialnimi količinami

(glej si Strnad str. 64, 93, 88) Kladnik vaje str. 154

$$\frac{\Delta W}{|W|} = -1\%$$

$$F = \underbrace{\varkappa \frac{m \cdot m_z}{r^2}} = mg \quad , \quad m_z \equiv \text{masa zemlje}$$

$$\boxed{g = \frac{\varkappa m_z}{r^2}} \quad , \text{ na površini zemlje je } \boxed{g_0 = \frac{\varkappa m_z}{r_0^2}}$$

$$\Rightarrow \boxed{g = g_0 \frac{r_0^2}{r^2}} \qquad r_0 = 6.4 \cdot 10^6\ m$$

$$-\vec{g}\,d\vec{s} = -g\,ds\cos\varphi = g\,dr, \quad dr = -ds \cdot \cos\varphi$$

$$\Delta W_p = -\int m\vec{g}\,d\vec{s} = \int mg\,dr = \int_{r'} mg_0 \frac{r_0^2}{r^2}\,dr = -mg_0\frac{r_0^2}{r} - \left(-mg_0 r_0^2/r'\right) = mg_0 r_0^2\left(\frac{1}{r'} - \frac{1}{r}\right)$$

satelit enakomerno kroži: $\underset{\text{gravitacijski pospešek}}{mg} = \underset{\text{centripetalni pospešek}}{m\frac{v^2}{r}} \Rightarrow mg_0\frac{r_0^2}{r^2} = m\frac{v^2}{r}$

$$\boxed{g_0 \frac{r_0^2}{r} = v^2}$$

$$W_p = -mg_0\frac{r_0^2}{r}$$

$$W_k = \frac{mv^2}{2} = \frac{m}{2} g_0 \frac{r_0^2}{r}$$

$$W_T = W_p + W_k = -\frac{1}{2} m g_0 \frac{r_0^2}{r}$$

$$dW_T = +\frac{1}{2} m g_0 \frac{r_0^2}{r^2}\,dr$$

$$\frac{\Delta W_T}{|W_T|} = \frac{dr}{r} = -0.01$$

$$-\frac{g_0 r_0^2}{r^2}\,dr = 2v\,dv$$

$$-v^2\frac{dr}{r} = 2v\,dv$$

$$\boxed{-\frac{dr}{r} = 2\frac{dv}{v}}$$

$$t_{0n} = \frac{2\pi(r-dr)}{v+dv}, \quad t_{0stari} = \frac{2\pi r}{v}$$

$$\frac{dt_0}{t_{0stari}} = \frac{t_{0stari} - t_{0n}}{t_{0stari}} = 1 - \frac{t_{0n}}{t_{0s}} =$$

$$= 1 - \frac{(r-dr)\,v}{r\,(v+dv)} = 1 - \frac{(1-\frac{dr}{r})}{(1+\frac{dv}{v})}$$

(3)

3) Izstrelek izstrelimo v smeri navpično navzgor z začetno hitrostjo 7200 m/s. Oceni, kolikšno višino bo dosegel! Zanemari zračni upor! Radij Zemlje je 6400 km. 1994

$$F = -\varkappa \frac{Mm}{r^2} = -\frac{d}{dr}\left(-\frac{\varkappa Mm}{r}\right)$$

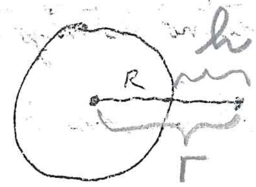
$$F = mg = m\left(\frac{\varkappa M}{r^2}\cdot\frac{R^2}{R^2}\right) = m\underbrace{\left(\frac{\varkappa M}{R^2}\right)}_{g_0}\frac{R^2}{r^2}$$

$R = 6400\,km$

$g_0 = 10\,m/s^2$

$v = 7200\,m/s$

$$\frac{1}{2}mv^2 - \frac{\varkappa Mm}{R} = -\frac{\varkappa Mm}{r}$$



$$g_0 = \frac{\varkappa M}{R^2}$$

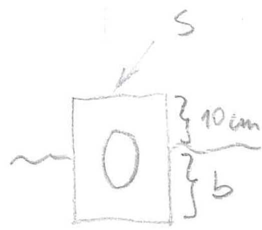
$$\frac{1}{2}v^2 - g_0R = -g_0\frac{R^2}{r}$$

$$r = \frac{-g_0R^2}{\frac{v^2}{2} - g_0R} = 10756{,}3\,km$$

$$h = r - R = \underline{4356{,}3\,km} \quad (za\ g_0 = 10\,m/s^2)$$

(4)

4) Valj iz aluminija z gostoto 2.7 g/cm^3 ima osnovno plskev s ploščino 100 cm^2 in višino 50 cm. V valju je votlina. Valj položimo v vodo tako, da stoji pokonci. Osnovni ploskvi sta vzporedni z gladino vode. Zgornja osnovna ploskev je 10 cm nad vodno gladino. Kolikšen je volumen votline v valju? Gostota vode je 1 g/cm^3, gostoto zraka v votlini zanemari. 1994



$\rho_{Al} = 2{,}7\,g/cm^2$, $\rho_{H_2O} = 1\,g/cm^3$ $\quad V_0$ = volumen votline

$S = 100\,cm^2$

$h = 50\,cm$

$$F_g = m_{Al}\cdot g = (Sh - V_0)\rho_{Al}\cdot g$$

$$F_{vzg} = S\cdot b\cdot \rho_{H_2O}\, g$$

$$F_g = F_{vzg}$$

$$(Sh - V_0)\rho_{Al} = Sb\cdot\rho_{H_2O}$$

$$\boxed{V_0 = S\left(h - b\frac{\rho_{H_2O}}{\rho_{Al}}\right) = \underline{3518{,}5\,cm^3}}$$

(5)

5. Na kateri višini glede na površino Zemlje kroži satelit, če se giblje s hitrostjo 6 km/s ? Polmer Zemlje je 6400 km ?

$v = 6000\,m/s$

$R = 6400\,km$

$h = ?$

$$F = m \cdot g = \varkappa \frac{M \cdot m}{r^2} = \varkappa \frac{M \cdot m \; R^2}{(R+h)^2 \, R^2} = m \, g_0 \frac{R^2}{(R+h)^2}$$

$$m \frac{v^2}{(R+h)} = m g_0 \cdot \frac{R^2}{(R+h)^2}$$

$$(R+h) = \frac{g_0 \cdot R^2}{v^2}$$

$$h = \frac{g_0 \cdot R^2}{v^2} - R = R\left(\frac{g_0 R}{v^2} - 1\right) \cong 5000\,km$$

4750 km ?

(4978)

VIS

1995/96

3. Določite nadmorsko višino (h) na kateri se v ravnini ekvatorja giblje geostacionarni satelit, t.j. satelit, ki se vedno nahaja iznad iste točke na površini Zemlje.

$R = 6400\ km$

$g_0 = 9{,}81\ ms^{-2}, \quad T = 24\ h$

$$F = m \cdot a_r$$

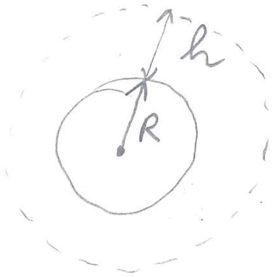
$$\boxed{m\, g_0 \frac{R^2}{(R+h)^2} = m \cdot \frac{v^2}{(R+h)}}$$

$$g_0 \frac{R^2}{(R+h)^2} = \frac{4\pi^2 (R+h)^2}{T^2 (R+h)}$$

$$\frac{g_0 T^2 \cdot R^2}{4\pi^2} = (R+h)^3$$

$$\boxed{h = \left[\frac{T^2 \cdot R^2 \cdot g_0}{4\pi^2}\right]^{1/3} - R}$$

$$h = 35.85 \cdot 10^6\ m$$



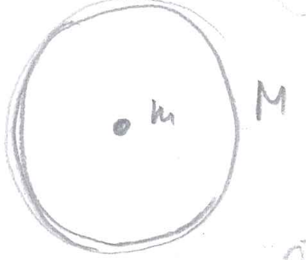
$$\boxed{v = \frac{2\pi (R+h)}{T}}$$

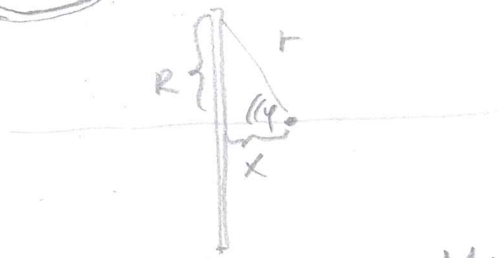
$$mg = G\frac{mM}{(R+h)^2} = m \overbrace{\underbrace{\frac{GM}{R^2}}_{g_0} \frac{R^2}{(R+h)^2}}^{g}$$

$$\boxed{g = g_0 \frac{R^2}{(R+h)^2}}$$

Gravitacija!

prel 1 obroč mase M, kroglica mase m





$$r^2 = R^2 + x^2$$

$$\cos\varphi = \frac{x}{\sqrt{R^2+x^2}}$$

$$F = \kappa \frac{Mm}{r^2} \cos\varphi = \kappa \frac{Mm\,x}{(R^2+x^2)^{3/2}}$$

$$\lim_{x \to 0} F \to \kappa \frac{Mm}{R^3} \cdot x$$

$$x \ll R$$

80./ Na kateri nadmorski višini (h) se pospešek prostega pada zmanjša na četrtino? R = 6400km

$$g = g_o R^2/(R+h)^2 = g_o/4 \quad , \quad 2R = R + h \quad , \quad h = R$$

(R = polmer Zmelje, g_o = pospešek prostega pada na površju Zemlje).

$$F = mg = G\cdot\frac{mM}{(R+h)^2} = G\frac{Mm\;R^2}{R^2\cdot(R+h)^2}$$

$$g_o$$

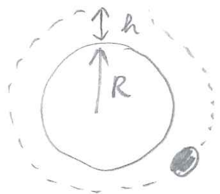
$$\boxed{g = g_o\frac{R^2}{(R+h)^2}}$$

82./ Koliko časa (T) potrebuje Lunin ~~modul~~ satelit, da obkroži Luno na višini h ~~(2000m)~~ 300 km? Pospešek prostega pada na površini Luni je g_o (~~2~~ 1.7 m/s²), polmer Lune je R (1600km). Vpliv gravitacijske privlačnosti Zemlje in Sonca zanemarimo.

$$a_r = r\omega^2 = r(2\pi/T)^2 = g_o R^2/r^2 \quad , \quad r = R + h$$

$$T = (2\pi/R)(R+h)^{3/2} g_o^{-1/2} \approx 2\pi(R/g_o)^{1/2}$$ [redacted]

$$\boxed{h = 300\ \text{km}}$$



$$F_{grav} = m\,a_r$$

$$\cancel{m}\cdot\left[g_o\cdot\frac{R^2}{(R+h)^2}\right] = \cancel{m}\cdot(R+h)\,\omega^2$$

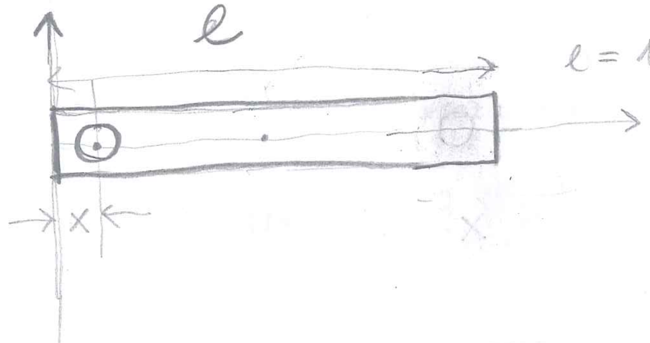
85./ Luna ima n (81) krat manjšo maso (m_L) ter m (3,7) krat manjši polmer (R_L) kot Zemlja. ~~Kolikokrat~~ Kolikšno je razmerje med težni pospešek ob (g_L) na površini ~~Lune~~ Zemlje in LUNE. ~~manjši kot na Zemlji?~~

$$g_L = Gm_L/R_L^2 = G(m_Z/R_Z^2)(m_L/m_Z)(R_Z/R_L)^2 = g_Z m^2/n$$

$$g_Z/g_L = 6,1$$

$$F = mg = G\cdot\frac{Mm}{R^2} \Rightarrow \boxed{g = G\frac{m}{R^2}}$$

4. Izračunaj težišče valjastega lesenega droga dolžine 1 m in konstantnega preseka S = 1 dm^2, ki ima na enem koncu kroglasto votlino polmera 4 cm ? Središče votline leži na geometrijski osi droga in je oddaljeno 10 cm roba droga.



$l = 1\,m$, $S = 0.01\,m^2$, $r = 0.04\,m$

$x = 0.1\,m$

$$X_T = \frac{m_v \cdot \frac{l}{2} - m_k \cdot x}{m_v - m_k} = \frac{S \cdot \frac{l^2}{2} - \frac{4\pi r^3}{3} \cdot x}{S \cdot l - \frac{4\pi r^3}{3}} = \underline{51{,}1\ cm}$$

UN

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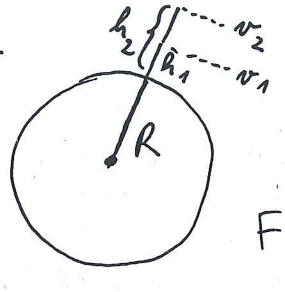

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$$v_2 = \sqrt{2 g_0 R^2\left(\frac{1}{R+h_2} - \frac{1}{R+h_1}\right) + v_1^2}$$

$$v_2 = \sqrt{2 g_0 R\left[\frac{R}{R+h_2} - \frac{R}{R+h_1}\right] + v_1^2}$$

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ker je $g_0 = G\frac{M}{R^2}$

$\Downarrow$

$GM = g_0 R^2$

$\Downarrow$

$$\boxed{W_p = -\frac{m g_0 R^2}{(R+h)}}$$

$g_0 = 9.81\,ms^{-2}$