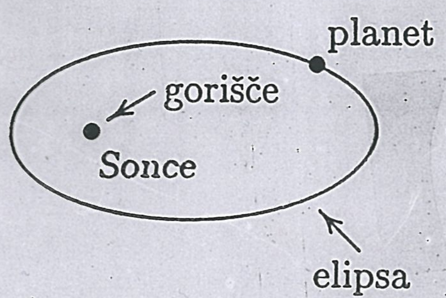
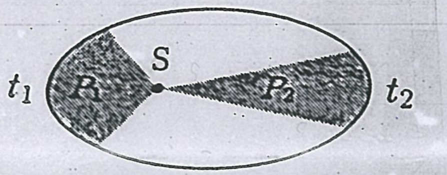
KEPLERJEVI ZAKONI

Johannes Kepler
(1571–1630)

I.



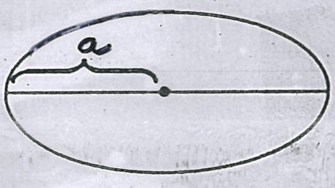
II.



$$\frac{P_1}{t_1} = \frac{P_2}{t_2}$$

$t \equiv$ čas

III.

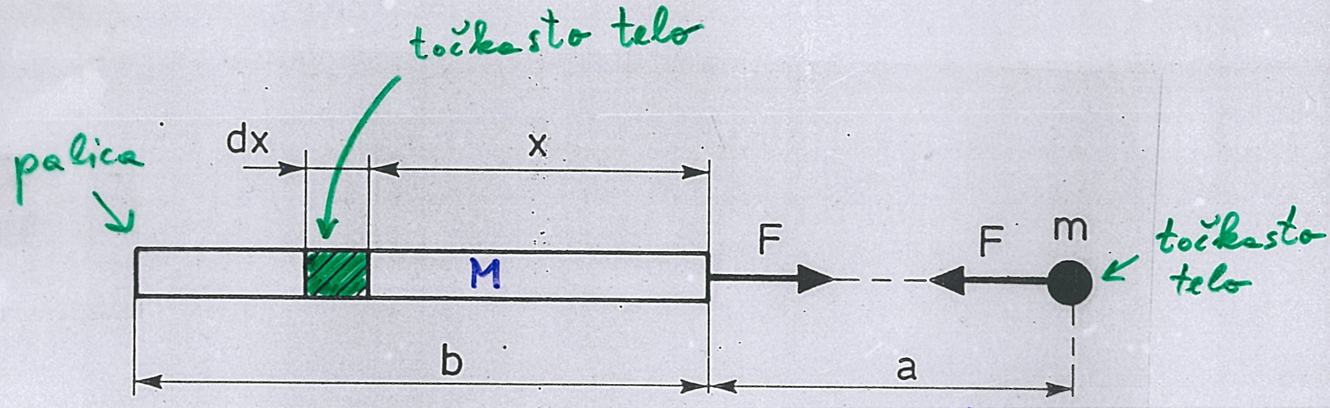


$$\frac{a^3}{t_0^2} = \kappa$$

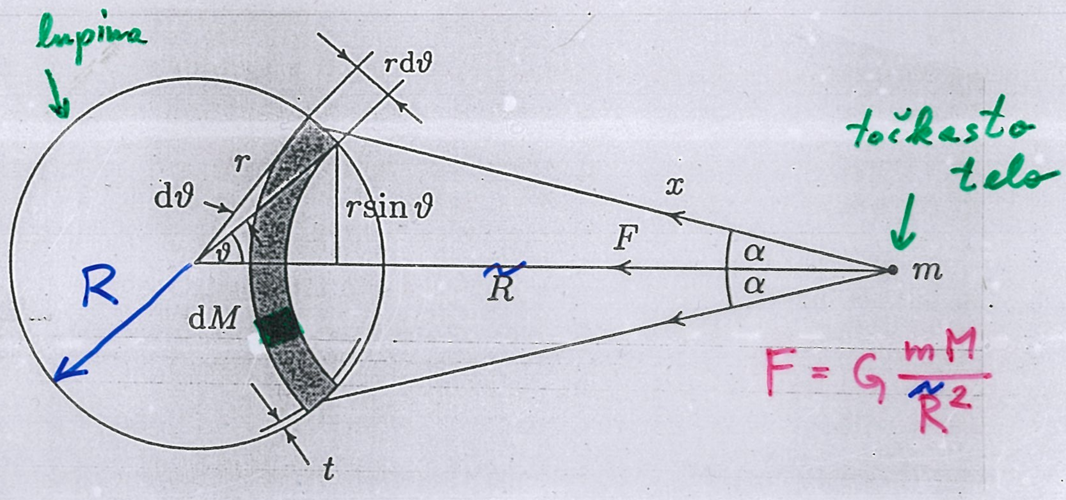
Planet	r [UA]	t_0 [leto]	$\frac{r^3}{t_0^2}$ $\left[\frac{UA^3}{leto^2}\right]$
Merkur	0,387	0,241	0,998
Venera	0,723	0,616	0,996
Zemlja	1,000	1,000	1,000
Mars	1,524	1,88	1,001
Saturn	5,20	11,86	1,000
Jupiter	9,54	29,46	1,000
Uran	19,18	84,0	1,001
Neptun	30,06	164,8	1,001
Pluton	39,5	247,7	1,001

Zemlja kroži okrog Sonca po eliptičnem tiru, ki ga v prvem približku aproksimiramo s krožnico; njena povprečna odaljenost (r) od Sonca je $149{,}5 \cdot 10^6$ km (pozimi 147,0 milj. km, poleti 152,0 milj. km); ta razdalja se v astronomiji uporablja kot enota dolžine (UA):

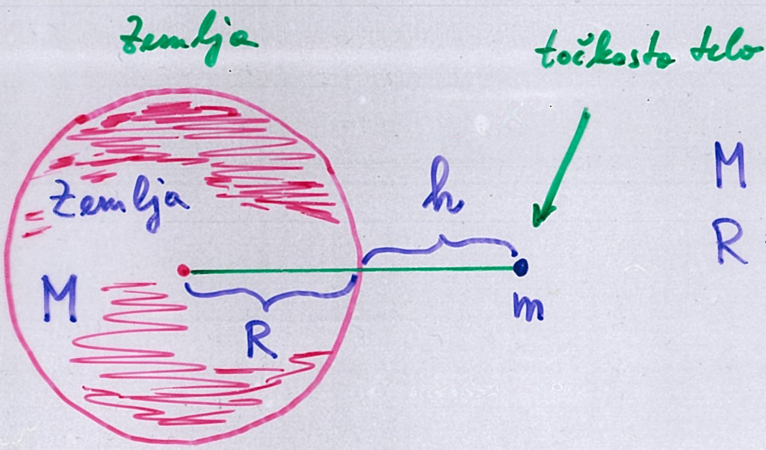
$$1 \text{ UA} = 149{,}5 \cdot 10^6 \text{ km} = 1{,}495 \cdot 10^{11} \text{ m}$$



$$F = G\frac{mM}{a(a+b)}$$



$$F = G\frac{mM}{\cancel{x}\,R^2}$$

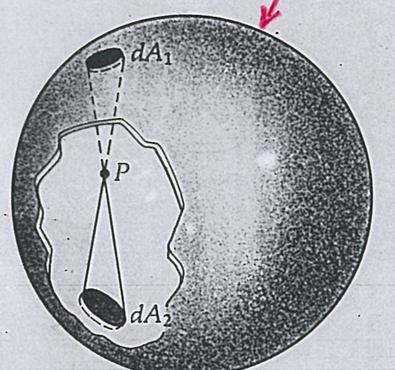


$M \equiv$ masa Zemlje

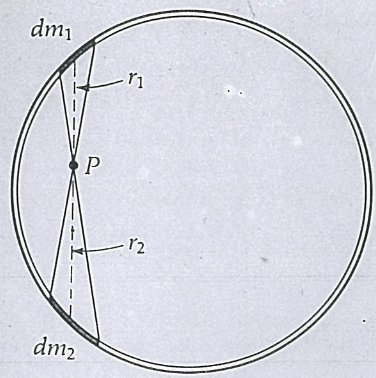
$R \equiv$ polmer Zemlje

$$\boxed{F = G\frac{Mm}{(R+h)^2}}$$

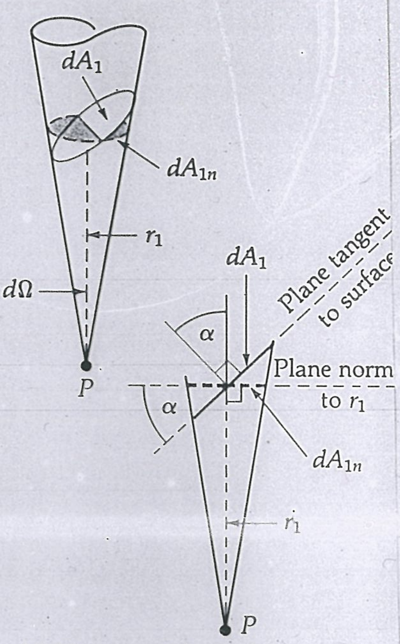
lupina



(a) The point P is arbitrarily located inside a uniform, thin, spherical shell. Equal elemental solid angles $d\Omega$ extend in opposite directions from P to intercept areas dA_1 and dA_2 on the shell.



(b) The segments of the shell within the solid angles have masses dm_1 and dm_2.



(c) The area dA_{1n} is the projection of the area dA_1 on the plane perpendicular to r_1. It equals $dA_{1n} = dA_1 \cos\alpha$.

FIGURE 16-6
A particle of mass m located arbitrarily at point P inside a uniform, thin, spherical shell.

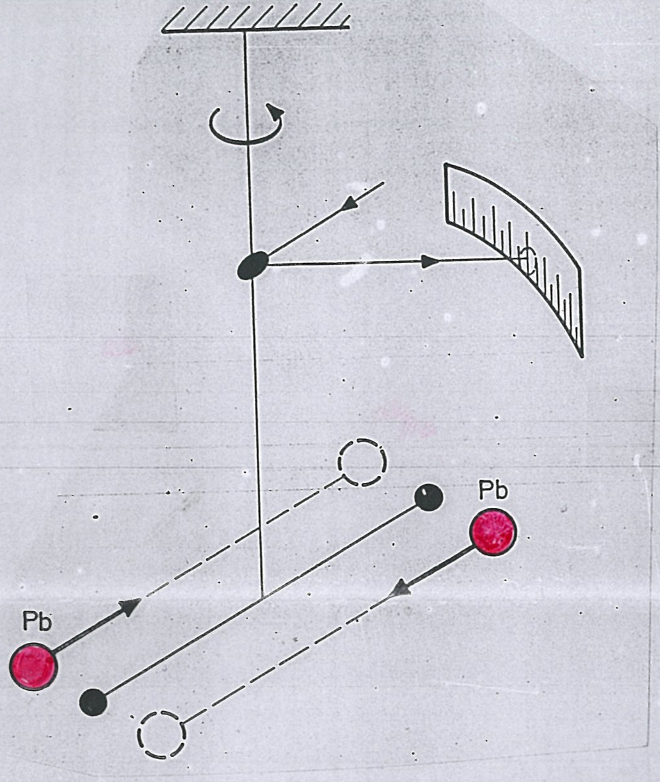
We may also obtain this last result by an interesting chain of reasoning (first pointed out by Newton) that does not involve complicated integrals. Consider the particle of mass m to be located at an arbitrary point P inside the shell. Imagine that a narrow cone is constructed with its apex at the point, extending out in an arbitrary fashion. The cone will intercept, on the shell, an element of area dA_1 (Figure 16-6a). If we project a similar cone with an equal solid angle[6] in the opposite direction, it will intercept an area dA_2. An elemental solid angle $d\Omega$ (measured in *steradians*) is defined as the ratio dA_n/r^2 where dA_n is an area *normal* to the distance r from the apex of the solid angle. For the sphere, how are dA_1 and dA_2 related to their projections dA_{1n} and dA_{2n} on planes normal to r_1 and r_2? Any straight line such as r_1 and r_2 together will intersect the sphere at two points, making the same angles α with the normal to the surface of the sphere. That is, $d\Omega = dA_{1n}/r_1^2 = (dA_1 \cos\alpha)/r_1^2 = (dA_2 \cos\alpha)/r_2^2$. With σ = the mass per unit area, the two mass elements are

$$dm_1 = \sigma\, dA_1 = \frac{\sigma\, d\Omega\, r_1^2}{\cos\alpha} \qquad \text{and} \qquad dm_2 = \sigma\, dA_2 = \frac{d\Omega\, r_2^2}{\cos\alpha}$$

These two elements of mass exert gravitational forces on the particle m. The two forces are in opposite directions, and the ratio of their magnitudes is

$$\frac{dF_1}{dF_2} = \frac{\left(\dfrac{Gm\, dm_1}{r_1^2}\right)}{\left(\dfrac{Gm\, dm_2}{r_2^2}\right)} = \frac{\left(\dfrac{\sigma\, d\Omega\, r_1^2}{\cos\alpha r_1^2}\right)}{\left(\dfrac{\sigma\, d\Omega\, r_2^2}{\cos\alpha r_2^2}\right)} = 1$$

Cavendisheva tehtnica



Lord Cavendish
l. 1798 izmeril
gravitacijsko konstanto G

$$\boxed{G = 6.67 \cdot 10^{-11}\ Nm^2/kg^2}$$

Težni pospešek na površju nebesnih teles

	g (m/s^2)	$g/g_{\oplus}$
Sonce	275	28
Merkur	2,5	0,26
Venera	8,8	0,90
Zemlja	9,8	1,00
Luna	1,6	0,16
Mars	2,0	0,20
Jupiter	26	2,65
Saturn	11	1,12
Uran	9,4	0,96
Neptun	9,8	1,00

$g_0 \equiv$ težni pospešek na površini Zemlje