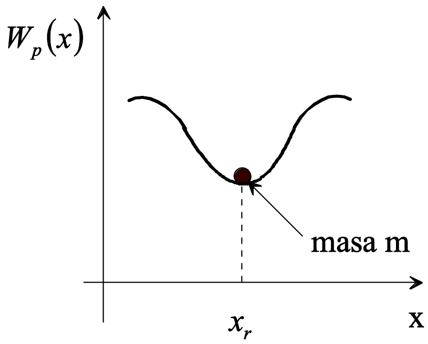
HARMONIČNO NIHANJE

Harmonično nedušeno nihanje

Splošno:

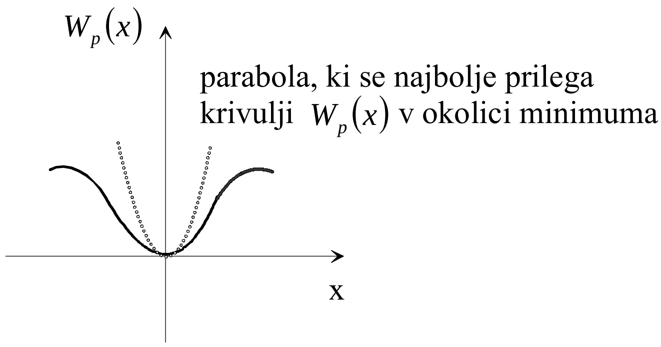
Potencialna energija (1 – D):



Pogoj za minimum potencialne energije:

$$\left.\frac{dW_p}{dx}\right|_{x=x_r} = 0, \quad \left.\frac{d^2W_p}{dx^2}\right|_{x=x_r} > 0$$

Izhodišče koordinatnega sistema premaknemo:



za x<<1 razvijemo $W_p(x)$ v Taylorjevo vrsto

$$W_p(x) = W_p(x=0) + x\left.\left(\frac{dW_p}{dx}\right)\right|_{x_r=0} + \frac{x^2}{2}\left.\left(\frac{d^2W_p}{dx^2}\right)\right|_{x_r=0} + \ldots \cong \frac{x^2}{2}\left.\left(\frac{d^2W_p}{dx^2}\right)\right|_{x_r=0}$$

$$W_p(x) \cong k\frac{x^2}{2}, \qquad k = \left.\left(\frac{d^2W_p}{dx^2}\right)\right|_{x_r=0} > 0. \qquad F = -\frac{dW_p}{dx} = -kx.$$

II. Newtonov zakon za gibanje mase m:

$$ma = -\mathrm{k}x \qquad m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\mathrm{k}x, \qquad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{\mathrm{k}}{m}x\,.$$

nastavek za rešitev :

$$x = x_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right].$$

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = x_0\left(\frac{2\pi}{t_0}\right)\cos\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right] \qquad v_0 \equiv x_0\left(\frac{2\pi}{t_0}\right)$$

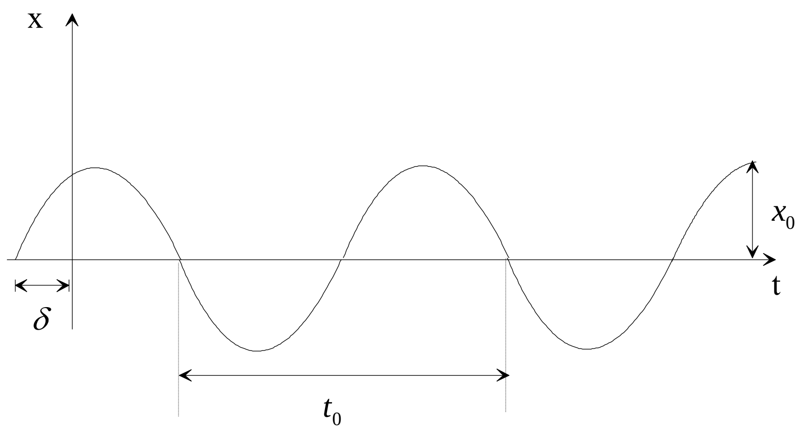
$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -x_0\left(\frac{2\pi}{t_0}\right)^2 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right].$$

preizkus:

$$\left(\frac{2\pi}{t_0}\right)^2 = \frac{k}{m}, \qquad t_0 = 2\pi\sqrt{\frac{m}{k}}\,.$$

HARMONIČNO ali SINUSNO nihanje

$$x = x_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$$



velja zveza $\omega_0 t_0 = 2\pi \quad \Rightarrow \quad \boxed{\omega_0 = \frac{2\pi}{t_0}}$

$\omega_0 =$ lastna krožna frekvenca

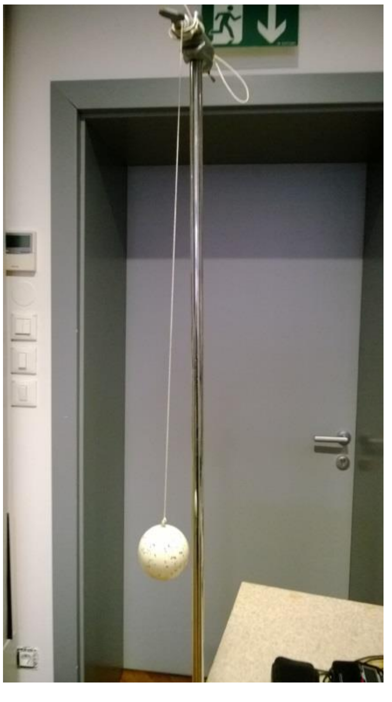
lastna frekvenca :

$$\nu = \frac{1}{t_0} \quad \Rightarrow \quad \omega_0 = 2\pi\nu$$

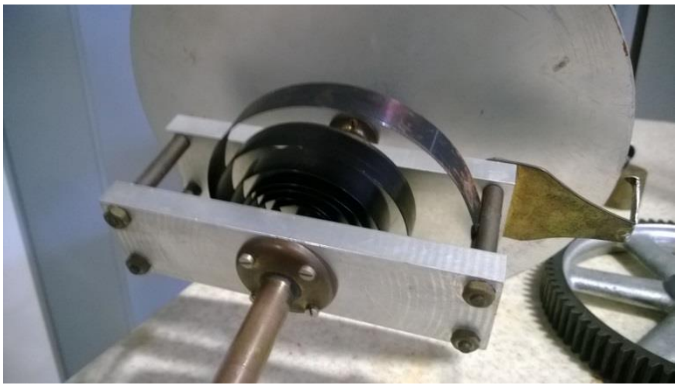
Primeri nihal, ki nihajo sinusno

nihalo na vijačno vzmet (viseče)

matematično nihalo

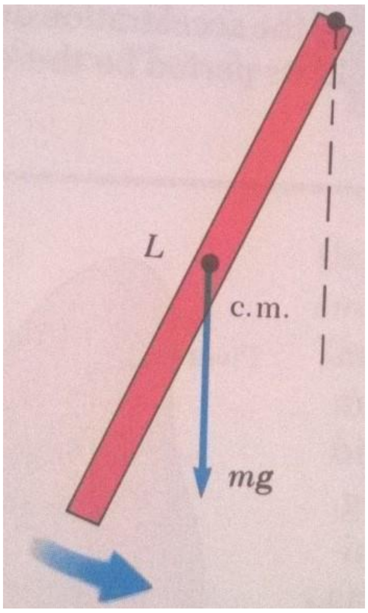


sučno nihalo na polžasto vzmet



torzijsko sučno nihalo

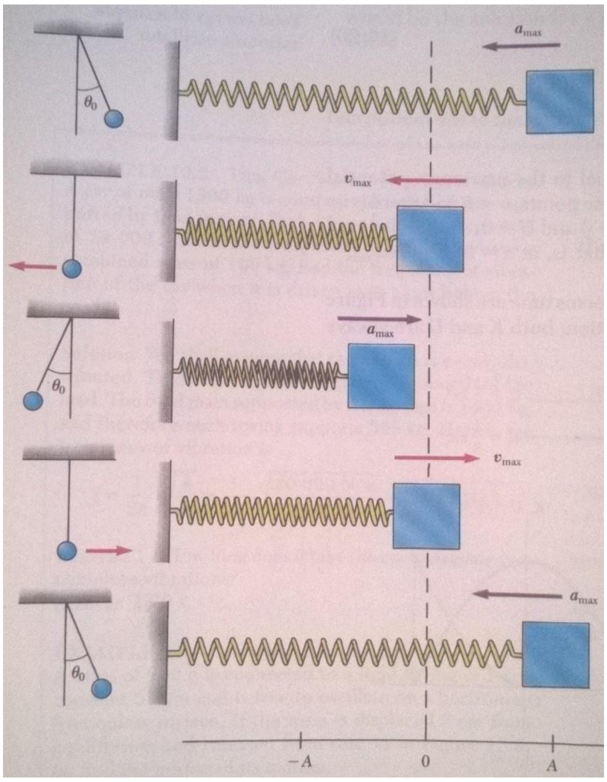
fizično nihalo



$$x = x_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right].$$

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} = x_0\left(\frac{2\pi}{t_0}\right)\cos\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right] \qquad v_0 \equiv x_0\left(\frac{2\pi}{t_0}\right)$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -x_0\left(\frac{2\pi}{t_0}\right)^2 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right].$$



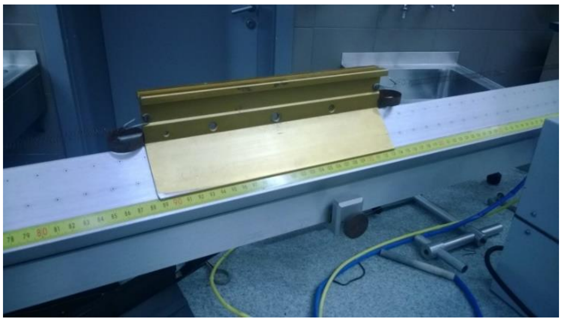
$$\varphi = \varphi_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$$

$$\omega = \frac{d\phi}{dt} = \phi_0\left(\frac{2\pi}{t_0}\right)\cos\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$$

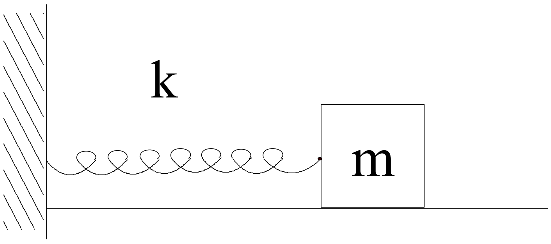
$$\alpha = \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -\varphi_0\left(\frac{2\pi}{t_0}\right)^2 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right],$$

Primeri nihal, ki nihajo sinusno – teoretični opis

Nihalo na vijačno vzmet



HORIZONTALNO gibanje mase m



$$m\frac{\mathrm{d}^2x}{\mathrm{d\,t}^2}=-kx$$

nastavek : $$x=x_0\sin\left[\left(\frac{2\pi}{t_0}\right)t+\delta\right],$$

nihajni čas $$t_0=2\pi\sqrt{\frac{m}{k}}$$

Energija harmoničnega nihanja

- **Kinetična** energija mase

$$W_k = \frac{m v^2}{2} = \frac{1}{2} m x_0^2 \left(\frac{2\pi}{t_0}\right)^2 \cos^2\left[\left(\frac{2\pi}{t_0}\right) t + \delta\right]. \qquad v_0 \equiv x_0 \left(\frac{2\pi}{t_0}\right)$$

- **Potencialna** energija

$$W_p = \frac{k x^2}{2} = \frac{k}{2} x_0^2 \sin^2\left[\left(\frac{2\pi}{t_0}\right) t + \delta\right].$$

- **Celotna energija**

$$W = W_k + W_p = \frac{1}{2} m x_0^2 \left(\frac{2\pi}{t_0}\right)^2 \cos^2\left[\left(\frac{2\pi}{t_0}\right) t + \delta\right] + \frac{k}{2} x_0^2 \sin^2\left[\left(\frac{2\pi}{t_0}\right) t + \delta\right].$$

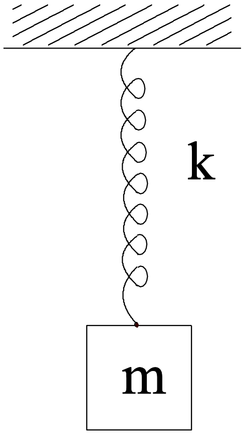
$$\left(\frac{2\pi}{t_0}\right)^2 = \frac{k}{m}$$

$$W = \frac{1}{2} k x_0^2 \left\{\cos^2\left[\left(\frac{2\pi}{t_0}\right) t + \delta\right] + \sin^2\left[\left(\frac{2\pi}{t_0}\right) t + \delta\right]\right\} = \frac{1}{2} k x_0^2 .$$

$$\frac{1}{2} k x_0^2 = \frac{1}{2} k v_0^2 \Big/ \left(\frac{2\pi}{t_0}\right)^2 = \frac{1}{2} m v_0^2 . \qquad v_0 \equiv x_0 \left(\frac{2\pi}{t_0}\right)$$

$$W = \frac{1}{2} k x_0^2 = \frac{1}{2} m v_0^2 .$$

VERTIKALNO gibanje mase m



$$\bar{x} = x_r + x \ .$$

ravnovesje : $kx_r = mg \ .$

II. Newtonov zakon : $ma = mg - k\bar{x}$

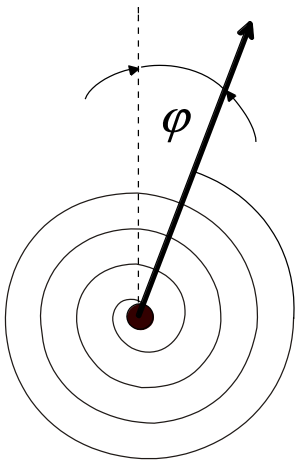
$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = mg - k(x_r + x)$$

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -kx$$

$$x = x_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right], \quad t_0 = 2\pi\sqrt{\frac{m}{k}}$$

Nihalo na polžasto vzmet (sučno sinusno nihanje)

$$M = -D\varphi$$



$$M = -D\varphi \qquad\qquad M = J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$$

$$J\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -D\varphi \qquad\qquad \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -\frac{D}{J}\varphi$$

nastavek: $\varphi = \varphi_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$

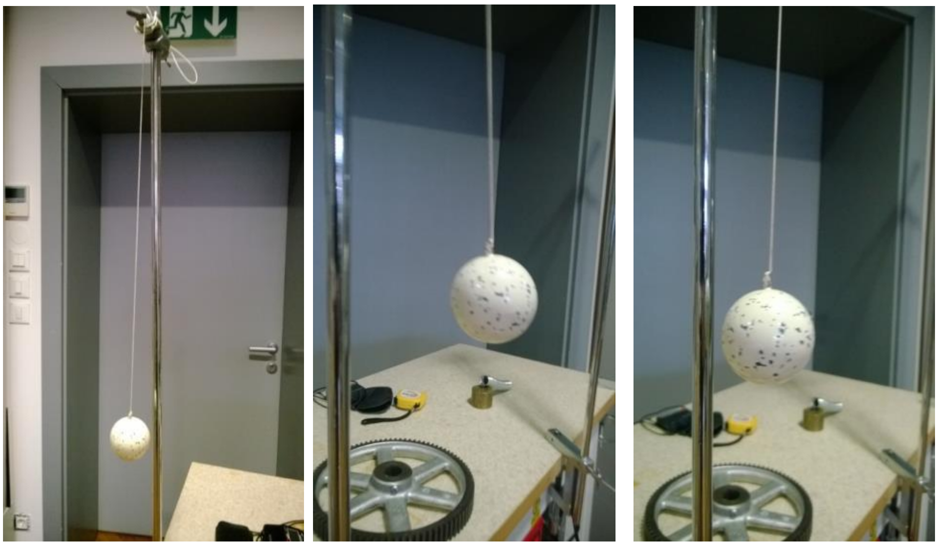
kotna hitrost: $\omega = \frac{d\phi}{dt} = \phi_0\left(\frac{2\pi}{t_0}\right)\cos\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right] \qquad \omega_0 = \varphi_0\left(\frac{2\pi}{t_0}\right),$

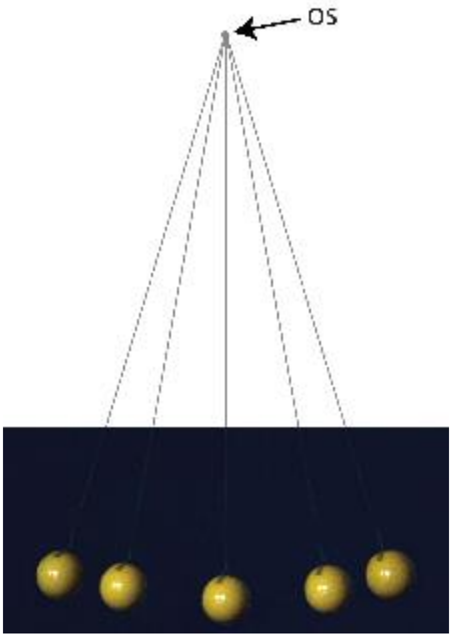
kotni pospešek : $\alpha = \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -\varphi_0\left(\frac{2\pi}{t_0}\right)^2 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right],$

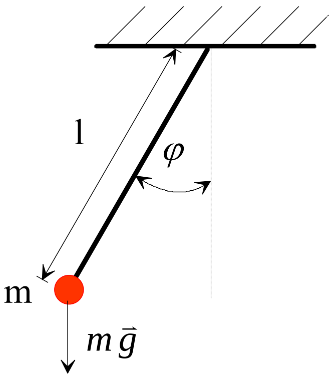
$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -\left(\frac{2\pi}{t_0}\right)^2\varphi \qquad\qquad \left(\frac{2\pi}{t_0}\right)^2 = \frac{D}{J}$$

$$t_0 = 2\pi\sqrt{\frac{J}{D}}$$

Matematično nihalo







φ = odmik

$$J = m l^2$$

predpostavka : φ << 1 $\sin\varphi \cong \varphi$

$$M \cong -mgl\varphi$$

$$M = J \frac{\mathrm{d}^2 \varphi}{\mathrm{d}t^2}$$

$$-mgl\varphi = ml^2 \frac{\mathrm{d}^2 \varphi}{\mathrm{d}t^2}\,,$$

$$\frac{\mathrm{d}^2 \varphi}{\mathrm{d}t^2} = -\frac{g}{l}\varphi$$

nastavek : $\varphi = \varphi_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$

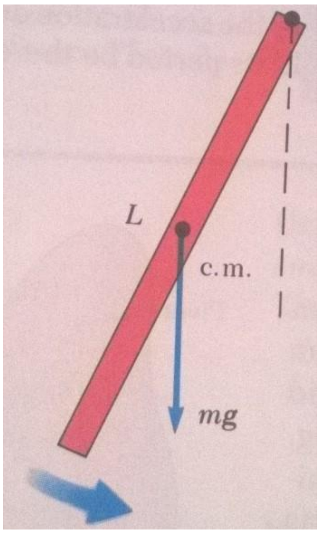
$$\frac{d^2\varphi}{dt^2} = -\frac{g}{l}\varphi$$

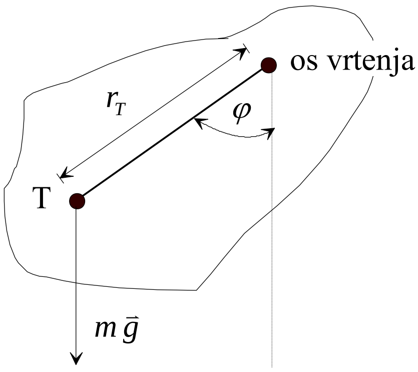
$$\varphi = \varphi_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$$

$$\frac{d^2\varphi}{dt^2} = -\varphi_0 \left(\frac{2\pi}{t_0}\right)^2 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right],$$

$$\left(\frac{2\pi}{t_0}\right)^2 = \frac{g}{l}, \qquad t_0 = 2\pi\sqrt{\frac{l}{g}}$$

Fizično nihalo





$$M = -mgr_T \sin\varphi$$

predpostavkA $(\sin\varphi \cong \varphi)$ za $(\varphi << 1)$: $M \cong -mgr_T\varphi$

$$M = J\,\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$$

$$-m\,g\,r_T\,\varphi = J\,\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$$

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -\left(\frac{mg\,r_T}{J}\right)\varphi.$$

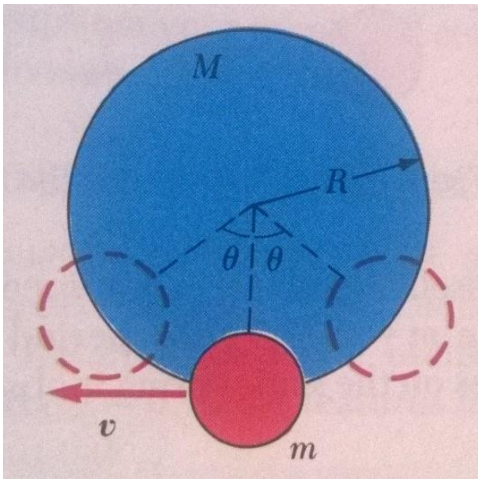
nastavek: $\varphi = \varphi_0 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right]$

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = -\varphi_0\left(\frac{2\pi}{t_0}\right)^2 \sin\left[\left(\frac{2\pi}{t_0}\right)t + \delta\right],$$

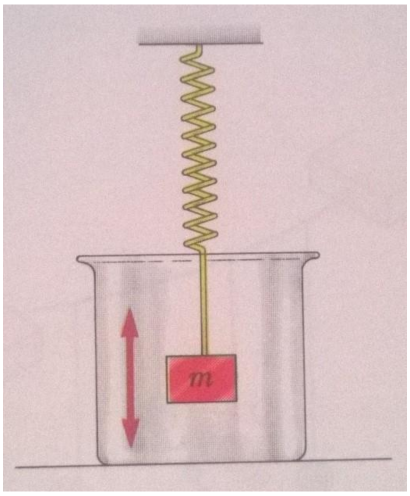
$$\left(\frac{2\pi}{t_0}\right)^2 = \frac{m\,g\,r_T}{J}, \qquad t_0 = 2\pi\sqrt{\frac{J}{m\,g\,r_T}}$$

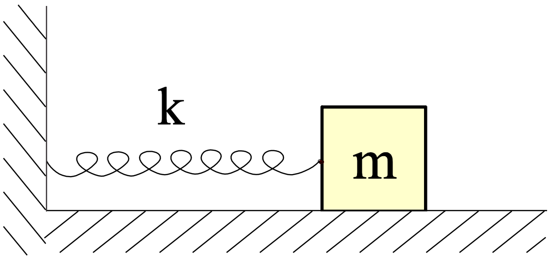
poseben primer (matematično nihalo): $J = m\,r_T^2$ in $r_T = l$

Primer :



Dušeno nihanje





$$F_d = -2m\beta v$$

β = koeficient dušenja

$$m\ddot{x} = -kx - 2\beta m\dot{x}$$

$\dot{x} \equiv \frac{\mathrm{d}x}{\mathrm{d}t}$ $\ddot{x} \equiv \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$$

$$\omega_0^2 = \left(\frac{2\pi}{t_0}\right)^2 = \frac{k}{m}$$

nastavek: $x(t) = y(t)e^{-\beta t}$ $\dot{x} = \dot{y}e^{-\beta t} - \beta y e^{-\beta t}$ $\ddot{x} = \ddot{y}e^{-\beta t} - 2\beta\dot{y}e^{-\beta t} + \beta^2 y e^{-\beta t}$

$$\ddot{y}e^{-\beta t} - 2\beta\dot{y}e^{-\beta t} + \beta^2 y e^{-\beta t} + 2\beta(\dot{y}e^{-\beta t} - \beta y e^{-\beta t}) + \omega_0^2 y(t)e^{-\beta t} = 0$$

$$e^{-\beta t}\left[\ddot{y} + \left(\omega_0^2 - \beta^2\right)y\right] = 0$$

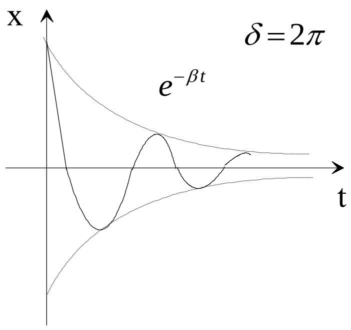
$$e^{-\beta t}\left[\ddot{y}+\left(\omega_0^2-\beta^2\right)y\right]=0$$

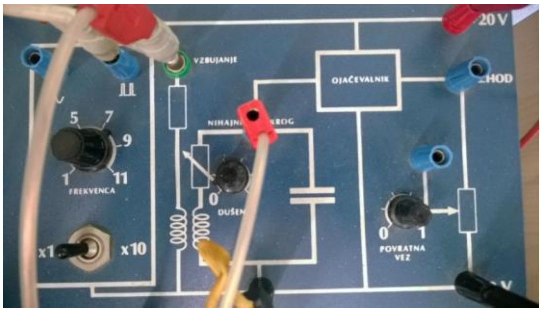
$$\ddot{y}+\left(\omega_0^2-\beta^2\right)y=0$$

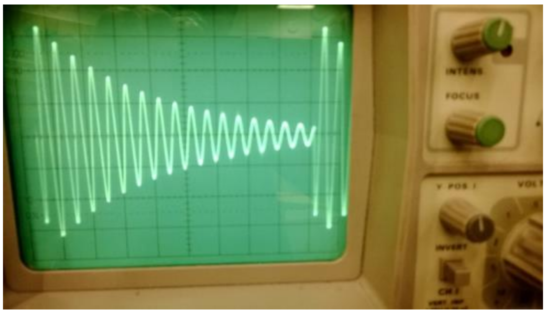
rešitev :

$$y=x_0\cos\left(\tilde{\omega}_0 t+\delta\right) \qquad \tilde{\omega}_0=\left(\omega_0^2-\beta^2\right)^{\frac{1}{2}}$$

$$\beta<\omega_0 \qquad x=y(t)e^{-\beta t}=x_0\, e^{-\beta t}\cos\left(\tilde{\omega}_0 t+\delta\right)$$







$\beta < \omega_0$: $\qquad \ddot{y} + \left(\omega_0^2 - \beta^2\right) y = 0 \qquad \ddot{y} = -\left(\omega_0^2 - \beta^2\right) y$

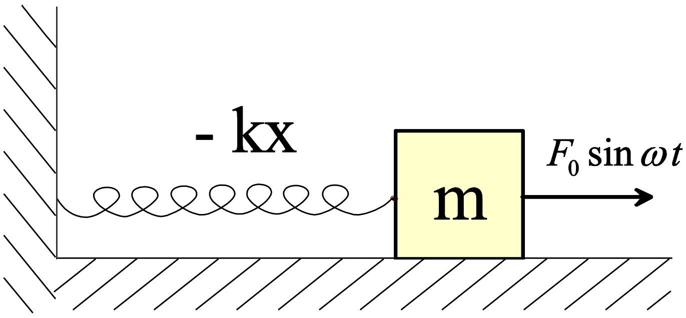
$$y = x_0 \cos\left(\tilde{\omega}_0 t + \delta\right)$$

$$x = y(t) e^{-\beta t} = x_0 \, e^{-\beta t} \cos\left(\tilde{\omega}_0 t + \delta\right) \qquad \tilde{\omega}_0 = \left(\omega_0^2 - \beta^2\right)^{\frac{1}{2}}$$

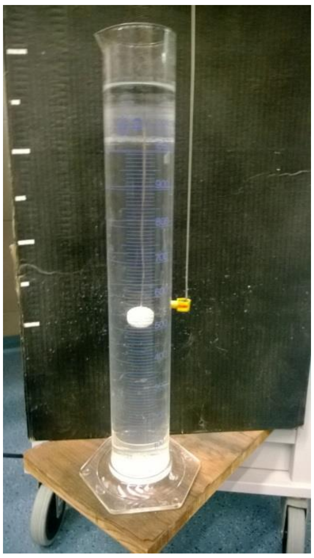
$\beta > \omega_0$: $\qquad \ddot{y} = \left(\beta^2 - \omega_0^2\right) y \qquad y = x_0 \, e^{-\alpha t} \; , \;\; \alpha > 0$

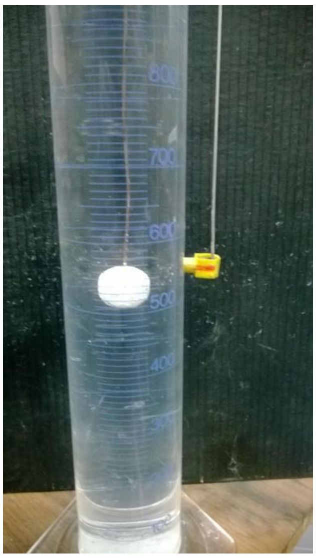
$$x = y(t) e^{-\beta t} = x_0 \, e^{-\alpha t} e^{-\beta t} = x_0 \, e^{-(\alpha + \beta) t}$$

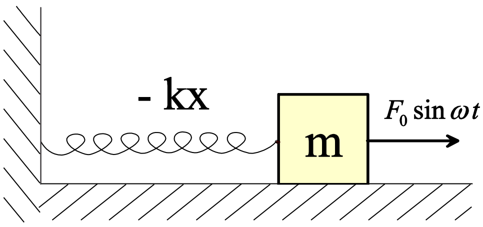
Vsiljeno nihanje



$$F(t) = F_0 \sin \omega t$$







II. Newtonov zakon:

$$m\ddot{x} = -kx - 2\beta m\dot{x} + F_0 \sin\omega t$$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \frac{F_0}{m}\sin\omega t \qquad \omega_0^2 = k/m$$

Za $\omega << \omega_0$: $\ddot{x} \to 0,\ \dot{x} \to 0$:

$$\omega_0^2 x \cong \frac{F_0}{m}\sin\omega t \qquad x = \frac{F_0}{m\omega_0^2}\sin\omega t$$

Splošno rešitev : $x(t) = x_h(t) + x_p(t)$

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0 \qquad x_h = x_0 e^{-\beta t}\sin\tilde{\omega}_0 t \qquad \tilde{\omega}_0 = \sqrt{\omega_0^2 - \beta^2}$$

$$x_h(t \to \infty) \to 0$$

nastavek : $x_p(t) = x_0 \sin(\omega t + \delta)$

ω = krožna frekvenca vsiljene sile

PREIZKUS ter x_0 in $\delta = ?$

$$x_p(t) = x_0 \sin(\omega t + \delta) = x_0 \sin\omega t\cos\delta + x_0\cos\omega t\sin\delta$$

$$x_p(t) = A_1 \sin\omega t + A_2\cos\omega t \qquad x_0\cos\delta = A_1 \qquad x_0\sin\delta = A_2$$

$$x_p(t) = A_1 \sin \omega t + A_2 \cos \omega t$$
$$\dot{x}_p = A_1 \omega \cdot \cos \omega t - A_2 \omega \cdot \sin \omega t$$
$$\ddot{x}_p = -A_1 \omega^2 \cdot \sin \omega t - A_2 \omega^2 \cdot \cos \omega t$$

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = \frac{F_0}{m} \sin \omega t$$

$$-A_1 \omega^2 \sin \omega t - A_2 \omega^2 \cos \omega t + 2\beta A_1 \omega \cos \omega t - 2\beta A_2 \omega \sin \omega t +$$
$$+ \omega_0^2 A_1 \sin \omega t + \omega_0^2 A_2 \cos \omega t = \frac{F_0}{m} \sin \omega t$$

$$-A_1 \omega^2 - 2\beta A_2 \omega + \omega_0^2 A_1 = \frac{F_0}{m}$$
$$-A_2 \omega^2 + 2\beta A_1 \omega + \omega_0^2 A_2 = 0$$

$$\left(\omega_0^2 - \omega^2\right) A_1 - 2\beta \omega A_2 = \frac{F_0}{m} \qquad \left(\omega_0^2 - \omega^2\right) A_2 + 2\beta \omega A_1 = 0$$

$$x_0 \cos \delta = A_1 \qquad x_0 \sin \delta = A_2$$

$$\frac{A_2}{A_1} = \frac{x_0 \sin \delta}{x_0 \cos \delta} = tg\, \delta = \frac{-2\beta\omega}{\omega_0^2 - \omega^2}$$

$$\left(\omega_0^2 - \omega^2\right) A_1 - 2\beta \omega A_2 = \frac{F_0}{m} \quad /(\ )^2$$

$$\left(\omega_0^2 - \omega^2\right) A_2 + 2\beta \omega A_1 = 0 \quad /(\ )^2$$

$$\left(\omega_0^2 - \omega^2\right)^2 \left(A_1^2 + A_2^2\right) + 4\beta^2 \omega^2 \left(A_1^2 + A_2^2\right) = \left(\frac{F_0}{m}\right)^2$$

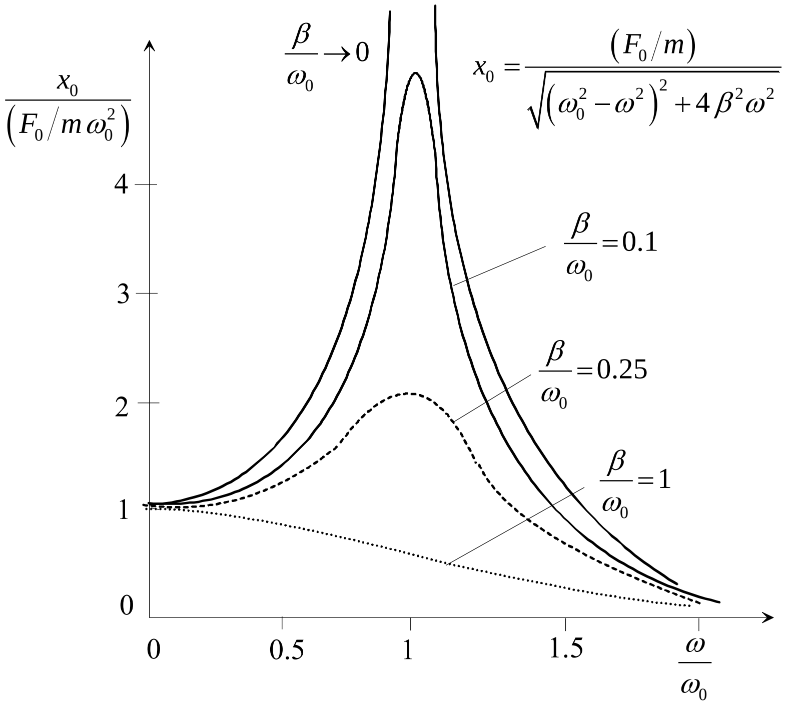
$$x_0 \cos \delta = A_1 \qquad x_0 \sin \delta = A_2 \qquad A_1^2 + A_2^2 = x_0^2 \cos^2 \delta + x_0^2 \sin^2 \delta = x_0^2$$

$$\left(\omega_0^2 - \omega^2\right)^2 x_0^2 + 4\beta^2 \omega^2 x_0^2 = \left(\frac{F_0}{m}\right)^2$$

$$\left(\omega_0^2-\omega^2\right)^2 x_0^2+4\beta^2\omega^2 x_0^2=\left(\frac{F_0}{m}\right)^2$$

$$x_0=\frac{\left(\frac{F_0}{m}\right)}{\sqrt{\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2}}$$

Odvisnost amplitude x_0 od vsiljene krožne frekvence ω

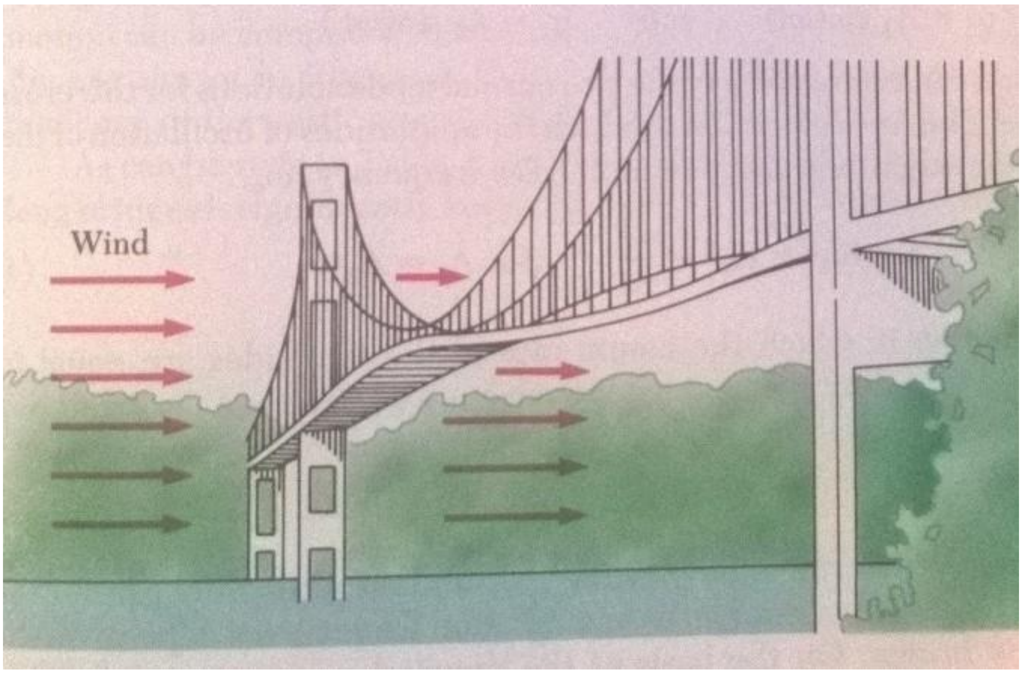


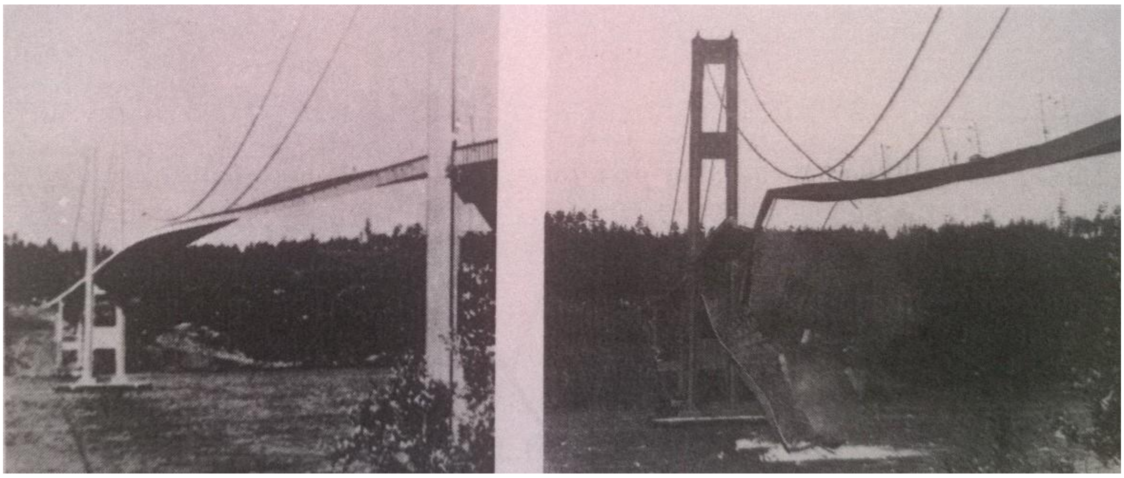
Torej:

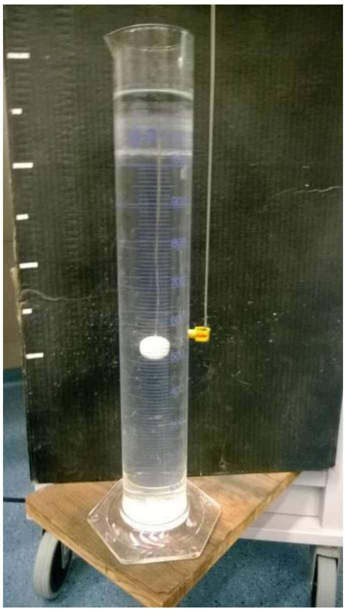
- splošna rešitev : $x = x_0\, e^{-\beta t}\sin\left(\tilde{\omega}_0 t\right)+x_0\sin\left(\omega t+\delta\right)$
- za zadosti velike čase : $x = x_0\sin\left(\omega t+\delta\right)$ s frekvenco vsiljene sile
- lastno nihanje $x_h = x_0\, e^{-\beta t}\sin\left(\tilde{\omega}_0 t\right)$ se zaduši $\left(\tilde{\omega}_0=\sqrt{\omega_0^2-\beta^2}\right)$
- $\omega \ll \omega_0 :\; x_0 \to \frac{F_0}{m\omega_0^2}$
- $\omega \cong \omega_0 :\; \frac{\mathrm{d}x_0}{\mathrm{d}\omega}=0$ je pri $\omega=\omega_{res}=\sqrt{\omega_0^2-2\beta^2}$

  za $\beta\to 0:\; \omega_{res}\to\omega_0$ in $x_0\to\infty$
- $\omega \gg \omega_0 :\; x_0 \to 0$

Zgled: resonance – nihanje visečega mostu (film na spletni učilnici)





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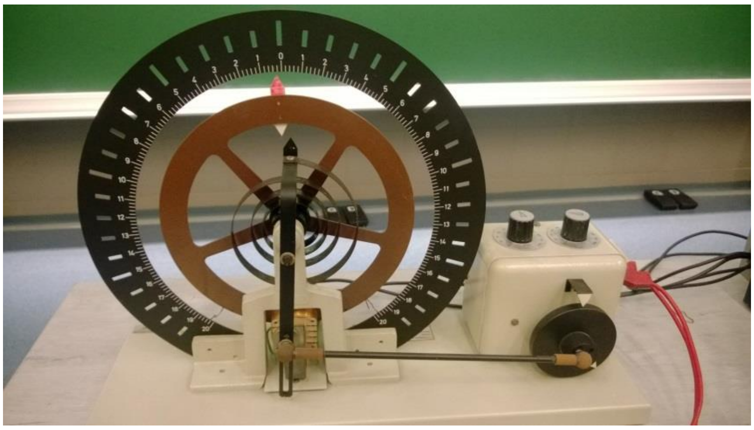

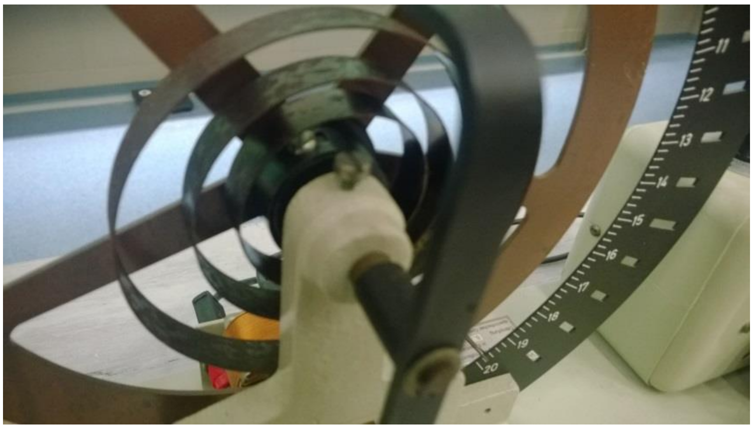
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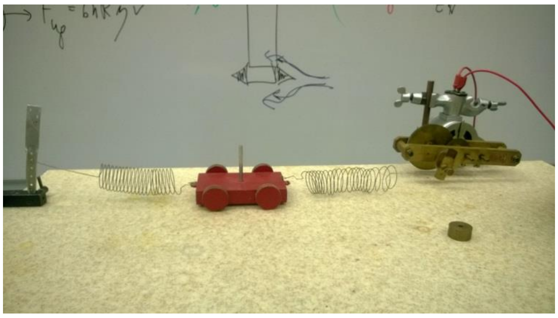
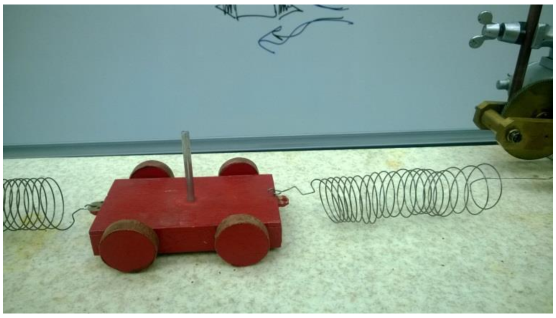
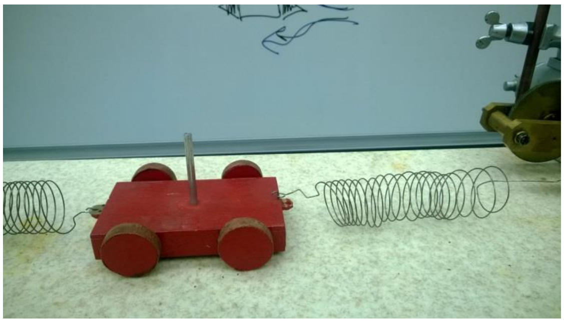






nedušeno: $\omega_0^2 = k/m \qquad t_0 = 2\pi\sqrt{\frac{m}{k}}$

dušeno: $\tilde{\omega}_0 = \left(\omega_0^2 - \beta^2\right)^{\frac{1}{2}} \qquad \omega_0^2 = k/m$

$$\tilde{\omega}_0 = \left(\omega_0^2 - \beta^2\right)^{\frac{1}{2}} = \frac{2\pi}{\tilde{t}_0} \qquad \tilde{t}_0 = \frac{2\pi}{\left(\frac{k}{m} - \beta^2\right)^{\frac{1}{2}}}$$

vsiljeno: niha s vsiljeno krožno frekvenco ω

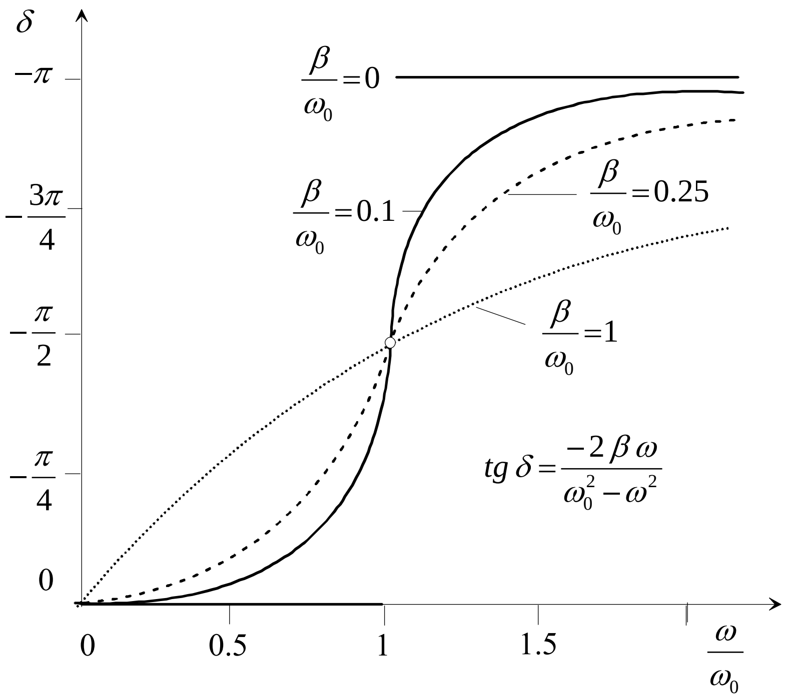
$$x = x_0\, e^{-\beta t}\sin\left(\tilde{\omega}_0\, t\right) + x_0 \sin\left(\omega t + \delta\right) \rightarrow x_0 \sin\left(\omega t + \delta\right)$$

$$x_0 = \frac{\left(\frac{F_0}{m}\right)}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}} \qquad \omega_{res} = \sqrt{\omega_0^2 - 2\beta^2} = \sqrt{\frac{k}{m} - 2\beta^2}$$

$$\omega << \omega_0 : \quad x_0 \rightarrow \frac{F_0}{m\,\omega_0^2} = \frac{F_0}{k}$$

Odvisnost faznega zamika δ od vsiljene krožne frekvence ω

$$tg\,\delta = \frac{-2\beta\omega}{\omega_0^2-\omega^2}$$



Torej:

$$\omega \ll \omega_0 : \delta \to 0 \quad \left(x_0 \to \frac{F_0}{m\,\omega_0^2}\right)$$

$$\omega \cong \omega_0 : \delta \to -\frac{\pi}{2} \quad \left(x_0 \to \infty\right)$$

$$\omega \gg \omega_0 : \delta \to -\pi \quad \left(x_0 \to 0\right)$$

Moč vsiljene sile (poglejte si v skripti ali v učbenikih - katerih PDF-ji so na Spletni učilnici ali na domači strani Laboratorija za fiziko FE)

$$\boxed{P(t)=F(t)\cdot v(t)=F_0\sin\omega t\cdot x_0\,\omega\cos(\omega t+\delta)}\ .$$

Upoštevali smo: $x=x_0\sin(\omega t+\delta)\ \Rightarrow\ v=\dot{x}=x_0\,\omega\cos(\omega t+\delta)$.

Ker $\cos(\omega t+\delta)=\cos(\omega t)\cdot\cos\delta-\sin(\omega t)\cdot\sin\delta$:

$$P(t)=F_0x_0\omega\left[\sin(\omega t)\cdot\cos(\omega t)\cdot\cos\delta-\sin^2(\omega t)\cdot\sin\delta\right]$$

Povprečna moč $\overline{P}=\frac{1}{t_0}\int_0^{t_0}P(t)\,dt$ je:

$$\overline{P}=F_0x_0\,\omega\left[\frac{\cos\delta}{t_0}\underbrace{\int_0^{t_0}\frac{1}{2}\sin(2\omega t)\,dt}_{=0}-\sin\delta\,\underbrace{\frac{1}{t_0}\int_0^{t_0}\sin^2(\omega t)\,dt}_{=\frac{1}{2}}\right]\ ,$$

torej:

$$\boxed{\overline{P}=-\frac{F_0}{2}\,\omega\,x_0\sin\delta}\ .$$

v enačbo vstavimo: $x_0=\frac{(F_0/m)}{\sqrt{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}}$ in $\sin\delta=\frac{tg\,\delta}{\sqrt{1+tg^2\delta}}$,

kjer je $tg\,\delta=\frac{-2\beta\,\omega}{\omega_0^2-\omega^2}$

in dobimo:

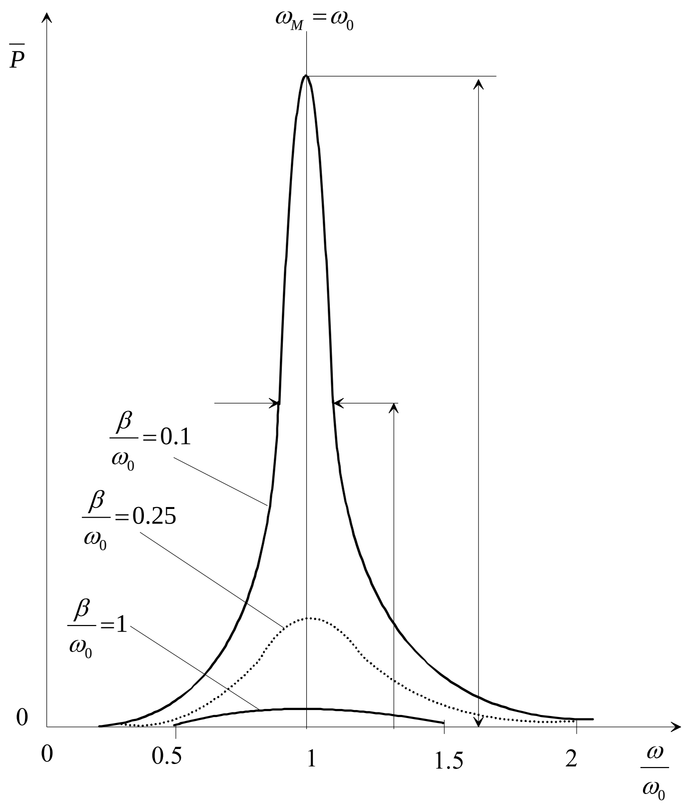
$$\overline{P}=\frac{-\frac{F_0}{2}\,\omega\,\frac{F_0}{m}}{\sqrt{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}}\cdot\frac{-\frac{2\beta\,\omega}{\omega_0^2-\omega^2}}{\sqrt{1+\frac{4\beta^2\omega^2}{(\omega_0^2-\omega^2)^2}}}\ ,$$

torej:

$$\boxed{\overline{P}=\frac{\frac{F_0^2}{m^2}\,\beta\,\omega^2}{(\omega_0^2-\omega^2)^2+4\beta^2\omega^2}}\ .$$

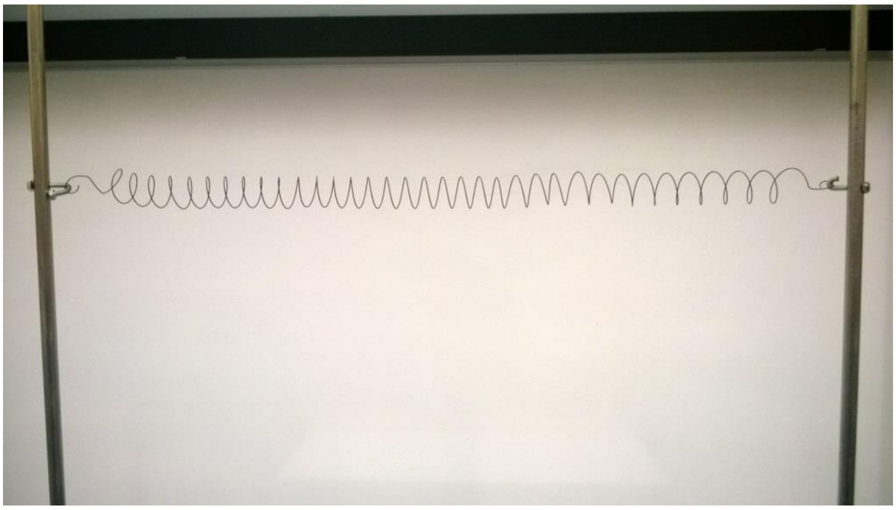
Maksimalna vrednost $\overline{P}$ je pri $\dfrac{d\overline{P}}{d\omega}=0 \;\Rightarrow\; \omega_M=\omega_0$

Sklep: za vse β je $\overline{P}$ največja pri ω_0

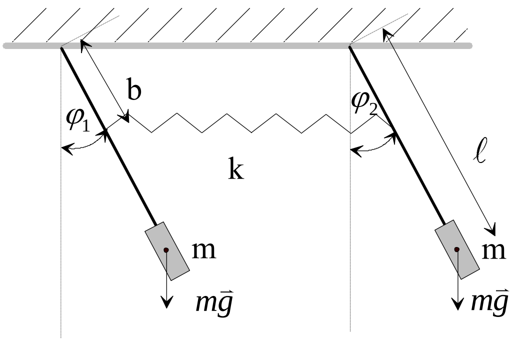


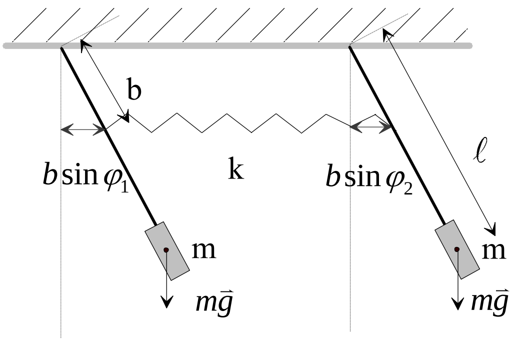
$$\overline{P}=\frac{\dfrac{F_0^2}{m^2}\beta\,\omega^2}{\left(\omega_0^2-\omega^2\right)^2+4\beta^2\omega^2}\,.$$

Sklopljeno nihanje

Dve enaki fizični nihali sta sklopljeni z vzmetjo s konstanto k





$$J\,\ddot{\varphi}_1 = -\,mg\,\ell\sin\varphi_1 + k\,\Delta x\, b\cos\varphi_1$$

$$J\,\ddot{\varphi}_2 = -\,mg\,\ell\sin\varphi_2 - k\,\Delta x\, b\cos\varphi_2$$

upoštevali $\sin\left(90^0 \pm \varphi\right) = \cos\varphi$ in $\Delta x = b\,sin\varphi_2 - b\,sin\varphi_1$.

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - ....$$

$$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - ....$$

$\varphi_1 << 1$ in $\varphi_2 << 1$

$$\sin\varphi_1 \approx \varphi_1,\quad \sin\varphi_2 \approx \varphi_2,\quad \cos\varphi_1 \approx 1,\quad \cos\varphi_2 \approx 1$$

$$\Delta x = b\sin\varphi_2 - b\sin\varphi_1 \cong b\left(\varphi_2 - \varphi_1\right)$$

$$J\ddot{\varphi}_1 = -mg\ell\sin\varphi_1 + k\Delta x b\cos\varphi_1 \qquad J\ddot{\varphi}_2 = -mg\ell\sin\varphi_2 - k\Delta x b\cos\varphi_2$$

$$\sin\varphi_1 \approx \varphi_1, \quad \sin\varphi_2 \approx \varphi_2, \quad \cos\varphi_1 \approx 1, \quad \cos\varphi_2 \approx 1$$

$$\Delta x = \quad b\sin\varphi_2 - b\sin\varphi_1 \quad \cong b(\varphi_2 - \varphi_1)$$

$$J\ddot{\varphi}_1 \cong -mg\ell\varphi_1 + k\,b^2(\varphi_2 - \varphi_1) \qquad J\ddot{\varphi}_2 \cong -mg\ell\varphi_2 - k\,b^2(\varphi_2 - \varphi_1)$$

$\omega_0^2 = \dfrac{mg\ell}{J}$ (lastna frekvenca fizičnega nihala) $\qquad D = \dfrac{kb^2}{J}$

$$\ddot{\varphi}_1 = -\omega_0^2\varphi_1 + D(\varphi_2 - \varphi_1) \qquad \ddot{\varphi}_2 = -\omega_0^2\varphi_2 - D(\varphi_2 - \varphi_1)$$

+

$$\ddot{\varphi}_1 = -\omega_0^2\varphi_1 + D(\varphi_2 - \varphi_1)$$
$$\ddot{\varphi}_2 = -\omega_0^2\varphi_2 - D(\varphi_2 - \varphi_1)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\varphi_2 + \varphi_1) = -\omega_0^2(\varphi_2 + \varphi_1)$$

rešitev: $\varphi_2 + \varphi_1 = \varphi_0\cos(\omega_1 t)$ $\qquad \omega_1 = \omega_0 = \sqrt{\dfrac{mg\ell}{J}}$

-

$$\ddot{\varphi}_1 = -\omega_0^2\varphi_1 + D(\varphi_2 - \varphi_1)$$
$$\ddot{\varphi}_2 = -\omega_0^2\varphi_2 - D(\varphi_2 - \varphi_1)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}(\varphi_2 - \varphi_1) = -(\omega_0^2 + 2D)(\varphi_2 - \varphi_1)$$

rešitev: $\varphi_2 - \varphi_1 = \varphi_0\cos(\omega_2 t)$ $\qquad \omega_2 = \sqrt{\omega_0^2 + 2D}$ $\qquad D = \dfrac{kb^2}{J}$

upoštevali : $\varphi_1 = 0, \quad \varphi_2 = \varphi_0$ ob $t = 0$

komentar : $\omega_1 = \omega_0 = \sqrt{\dfrac{m g \ell}{J}}$ $\omega_2 = \sqrt{\omega_0^2 + 2D}$ $D = \dfrac{k b^2}{J}$

ob $t = 0$: $\varphi_1 = 0 \quad \varphi_2 = \varphi_0$

$$\varphi_2 + \varphi_1 = \varphi_0$$

$$\varphi_2 - \varphi_1 = \varphi_0$$

+ $\varphi_2 + \varphi_1 = \varphi_0 \cos(\omega_1 t)$ - $\varphi_2 + \varphi_1 = \varphi_0 \cos(\omega_1 t)$

$\varphi_2 - \varphi_1 = \varphi_0 \cos(\omega_2 t)$ $\varphi_2 - \varphi_1 = \varphi_0 \cos(\omega_2 t)$

----------------------------------- -----------------------------------

$\varphi_2 = \dfrac{\varphi_0}{2}\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$ $\varphi_1 = \dfrac{\varphi_0}{2}\left[\cos(\omega_1 t) - \cos(\omega_2 t)\right]$

Upoštevamo:

$$\cos\alpha + \cos\beta = 2 \cdot \cos\left(\frac{\alpha - \beta}{2}\right)\cos\left(\frac{\alpha + \beta}{2}\right)$$

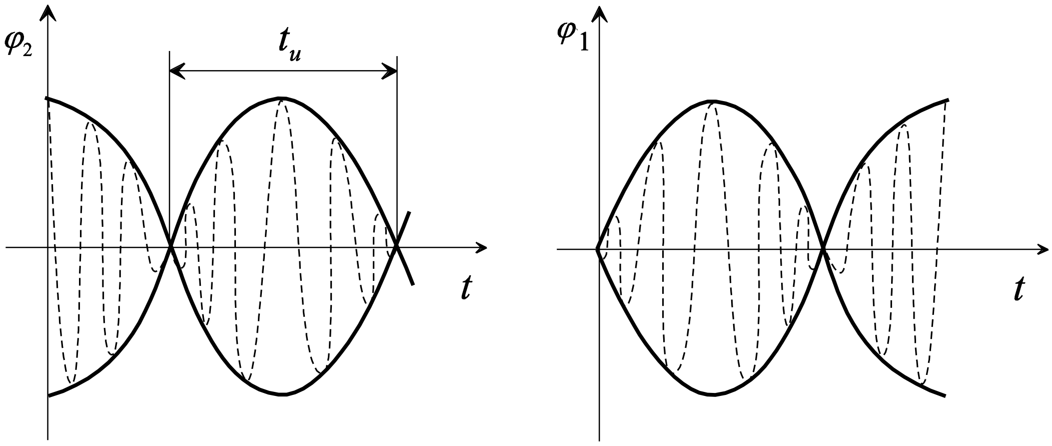
$$\cos\alpha - \cos\beta = -2 \cdot \sin\left(\frac{\alpha - \beta}{2}\right)\sin\left(\frac{\alpha + \beta}{2}\right)$$

$$\varphi_2 = \varphi_0 \cos\left[\frac{\omega_1 - \omega_2}{2} t\right] \cdot \cos\left[\frac{\omega_1 + \omega_2}{2} t\right]$$

$$\varphi_1 = -\varphi_0 \sin\left[\frac{\omega_1 - \omega_2}{2} t\right] \cdot \sin\left[\frac{\omega_1 + \omega_2}{2} t\right]$$

$$\varphi_1 = \underbrace{-\,\varphi_0 \sin\left[\frac{\omega_1-\omega_2}{2}\,t\right]}_{\text{amplituda}} \cdot \sin\left[\frac{\omega_1+\omega_2}{2}\,t\right]$$

$$\varphi_2 = \underbrace{\varphi_0 \cos\left[\frac{\omega_1-\omega_2}{2}\,t\right]}_{\text{amplituda}} \cdot \cos\left[\frac{\omega_1+\omega_2}{2}\,t\right]$$



Utripanje:

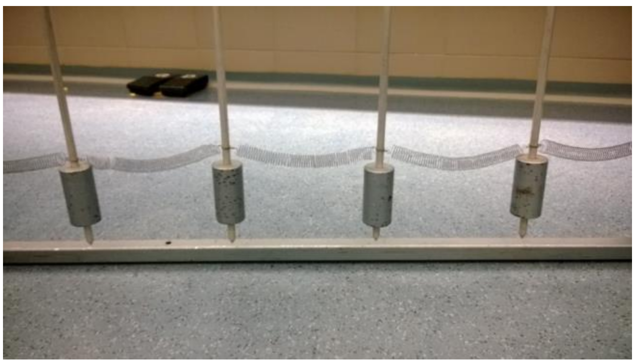
$$t_u \equiv \text{čas utripanja: } \frac{\omega_2-\omega_1}{2}\cdot t_u = \pi \;\Rightarrow\; \boxed{t_u = \frac{2\pi}{\omega_2-\omega_1}}$$

$$\varphi_1 = \underbrace{-\,\varphi_0 \sin\left[\frac{\omega_1-\omega_2}{2}\,t\right]}_{\text{amplituda}} \cdot \sin\left[\frac{\omega_1+\omega_2}{2}\,t\right]$$

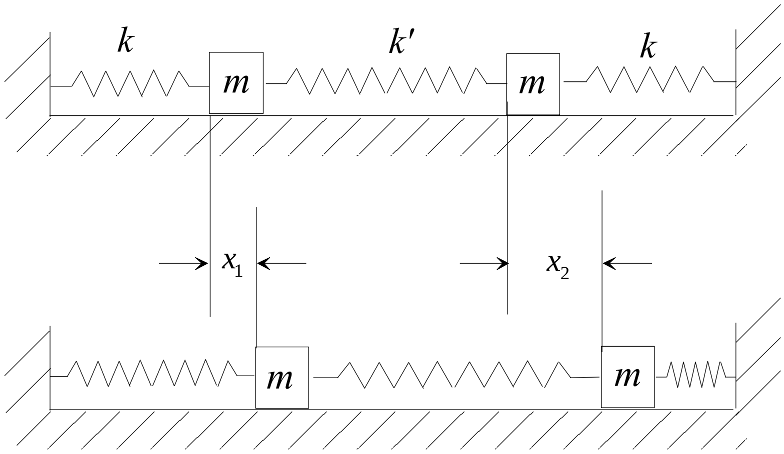
$$\varphi_2 = \underbrace{\varphi_0 \cos\left[\frac{\omega_1-\omega_2}{2}\,t\right]}_{\text{amplituda}} \cdot \cos\left[\frac{\omega_1+\omega_2}{2}\,t\right]$$

- nihali nihata s frekvenco $\frac{\omega_1+\omega_2}{2}$
- amplituda se spreminja s frekvenco $\frac{\omega_1-\omega_2}{2}$

$$\omega_1 = \omega_0 = \sqrt{\frac{m\,g\,\ell}{J}} \qquad \omega_2 = \sqrt{\omega_0^2 + 2D} \qquad D = \frac{k\,b^2}{J}$$



Nihanje dveh sklopljenih nihal na vijačno vzmet (poglejte si v skripti ali v učbenikih - katerih PDF-ji so na Spletni učilnici ali na domači strani Laboratorija za fiziko FE)



Zapišemo II. Newtonov zakon za masi m:

$$m\ddot{x}_1 = -k\,x_1 - k'(x_1 - x_2), \tag{100}$$

$$m\ddot{x}_2 = -k\,x_2 - k'(x_2 - x_1). \tag{101}$$

Seštejemo enačbi (101) in (102):

$$m(\ddot{x}_1 + \ddot{x}_2) = -k\,(x_1 + x_2). \tag{102}$$

Odštejemo enačbi (101) in (102):

$$m(\ddot{x}_1 - \ddot{x}_2) = -k\,(x_1 - x_2) - 2k'(x_1 - x_2) = -(k + 2k')(x_1 - x_2)\ . \tag{103}$$

Izberemo začetna pogoja: $x_1 = x_0$ in $x_2 = 0$ (ob času $t = 0$)
Ob času $t = 0$ tako velja:

$$x_1 + x_2 = x_0\,, \tag{104}$$

$$x_1 - x_2 = x_0\,. \tag{105}$$

Ob upoštevanju enačb (104) in (105) napišemo rešitvi enačb (102) in (103) v obliki:

$$x_1 + x_2 = x_0 \cos\omega_1 t\,, \tag{106}$$

$$x_1 - x_2 = x_0 \cos\omega_2 t\,, \tag{107}$$

kjer sta lastni krožni frekvenci: $\omega_1^2 = \dfrac{k}{m}$ in $\omega_2^2 = \dfrac{k+2k'}{m}$.

Iz enačb (106) in (107) sledi:

$$x_1 = \frac{x_0}{2}\left(\cos\omega_1 t + \cos\omega_2 t\right) = x_0 \cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\cdot\cos\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right], \quad (108)$$

$$x_2 = \frac{x_0}{2}\left(\cos\omega_1 t - \cos\omega_2 t\right) = -x_0 \sin\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\cdot\sin\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right]. \quad (109)$$

Upoštevali smo:

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right),$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right).$$

Dobili smo **utripanje**:

$$x_1 = x_0 \cos\left[\frac{\omega_1 - \omega_2}{2}t\right]\cdot\cos\left[\frac{\omega_1 + \omega_2}{2}t\right], \quad (110)$$

$$x_2 = -x_0 \sin\left[\frac{\omega_1 - \omega_2}{2}t\right]\cdot\sin\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right], \quad (111)$$

kjer sta dve lastni krožni frekvenci:

$$\omega_1 = \omega_0 = \sqrt{\frac{k}{m}}, \quad (112)$$

$$\omega_2 = \omega_0\left(1 + 2\frac{k'}{k}\right)^{1/2}. \quad (113)$$

- **Šibka sklopitev:** $k' << \mathrm{k}$

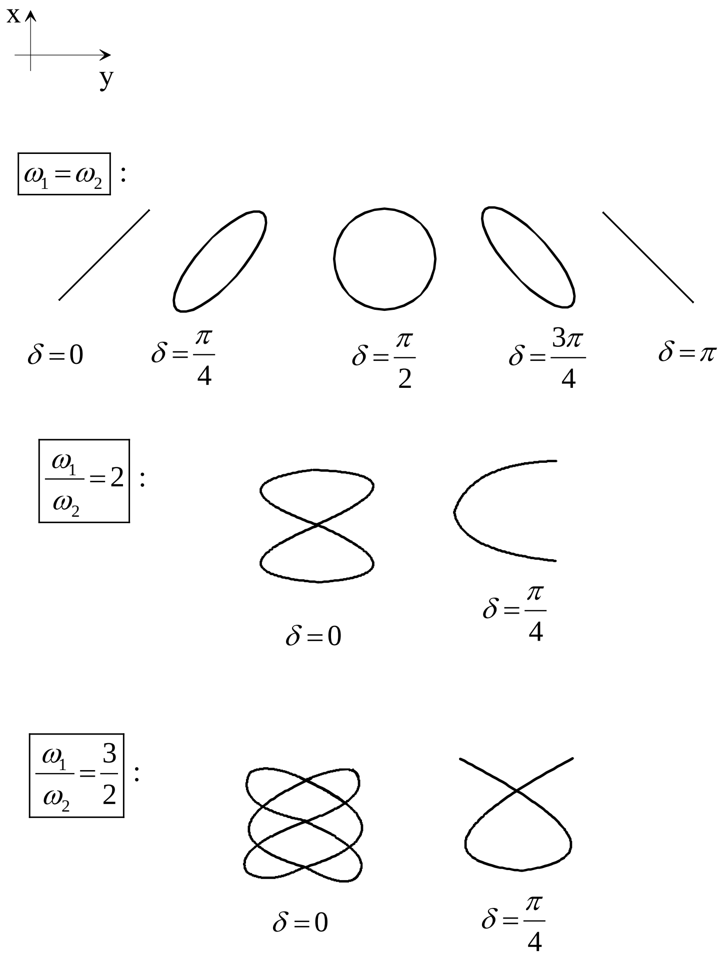
Če uporabimo $(1+x)^{1/2} \cong 1 + \frac{1}{2}x$ za x<<1 iz enačb (112) in (113) sledi:

$$\frac{\omega_1 + \omega_2}{2} \cong \omega_0\left(1 + \frac{k'}{2k}\right), \quad (114)$$

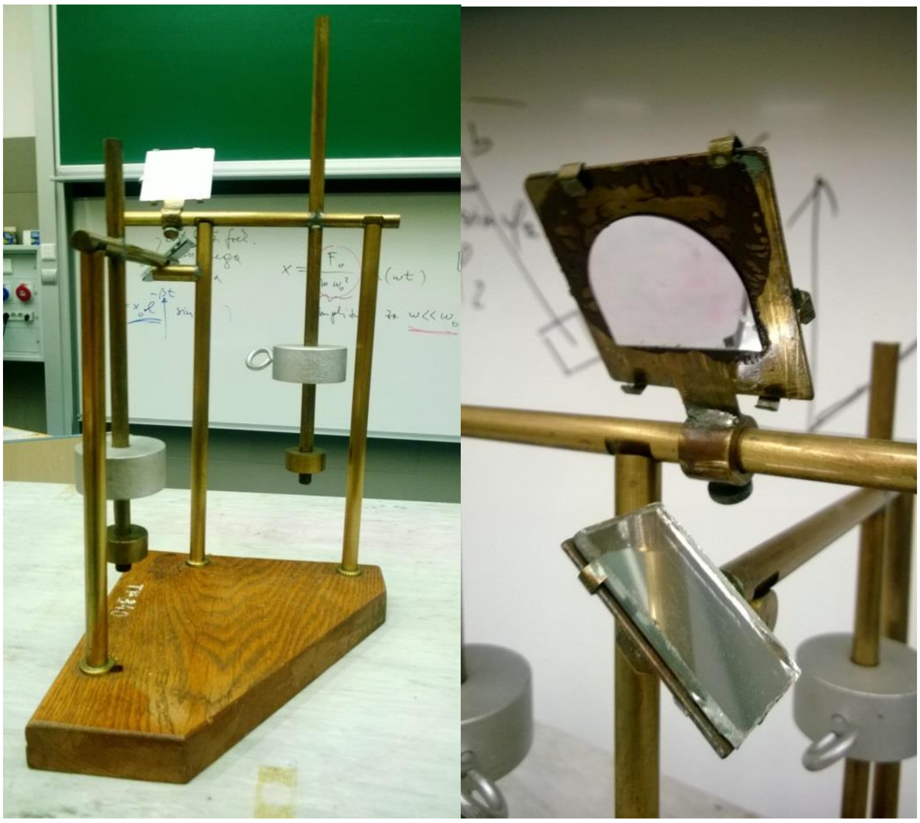
$$\frac{\omega_2 - \omega_1}{2} \cong \frac{k'\omega_0}{2k}. \quad (115)$$

Sestavljanje dveh pravokotnih nihanj

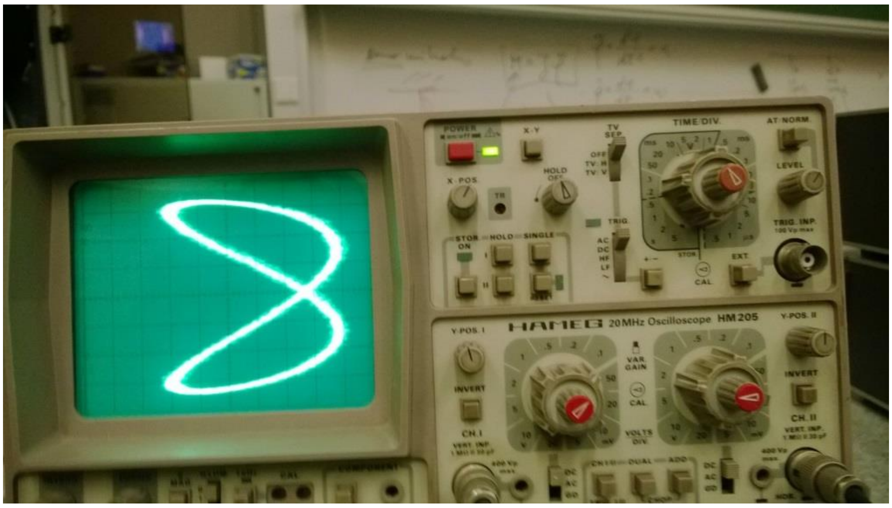
$$x = x_0 \sin(\omega_1 t) \qquad y = y_0 \sin(\omega_2 t - \delta)$$

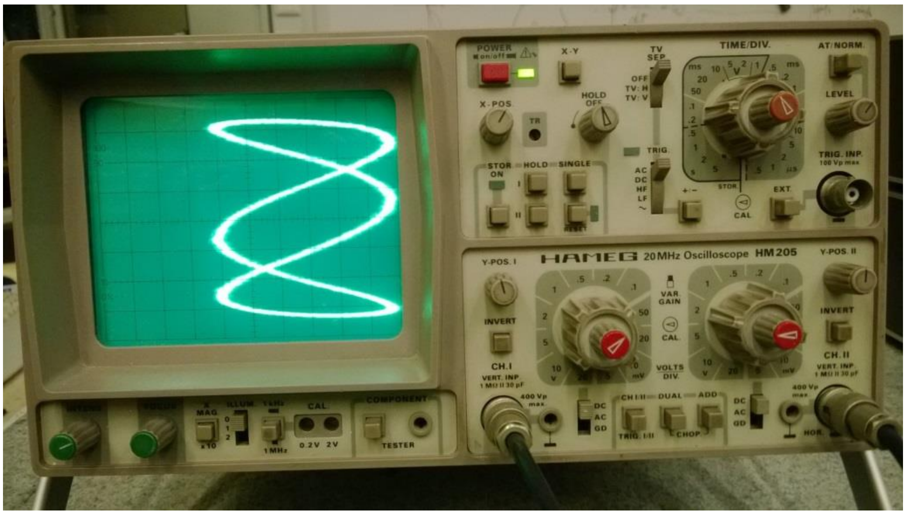


Lissajousove krivulje (dve nihajoči fizični nihali z zrcali)



Lissajousove krivulje na osciloskopu


HAMEG 20MHz Oscilloscope HM205
TIME/DIV.
X-Y
X-POS.
LEVEL
TRIG. INP.
Y-POS. I
Y-POS. II
INVERT
CH. I
CH. II
VAR. GAIN
CAL.
VOLTS DIV.
DC
AC
GD


POWER
X-Y
TV SEP
TIME/DIV.
AT/NORM.
X-POS.
HOLD OFF
LEVEL
TRIG.
TRIG. INP.
STOR. ON
HOLD
SINGLE
EXT.
CAL
HAMEG 20MHz Oscilloscope HM205
Y-POS. I
Y-POS. II
VAR. GAIN
INVERT
CH. I
CH. II
VOLTS DIV.
CH I/II
DUAL
ADD
CHOP
TRIG. I/II
DC
AC
GD
CAL.
0.2V
2V
COMPONENT
TESTER