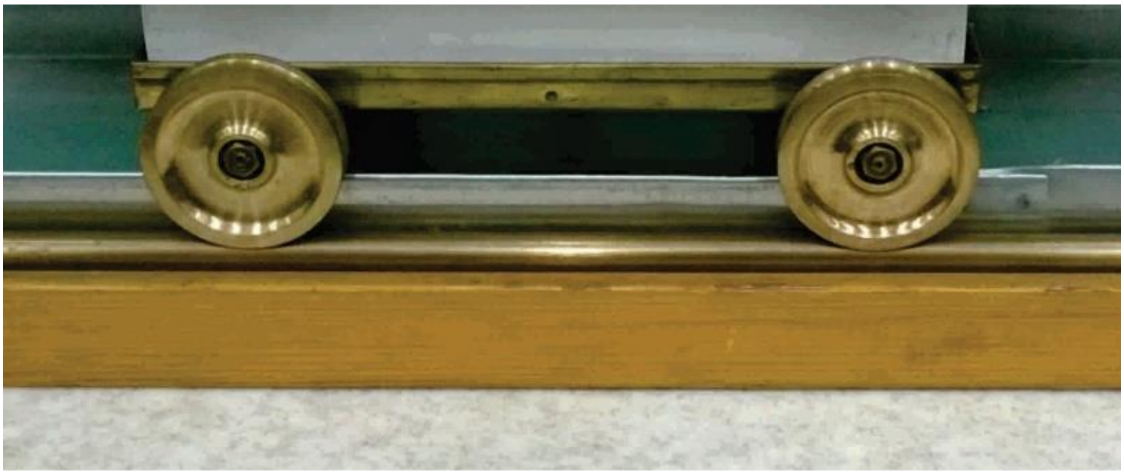
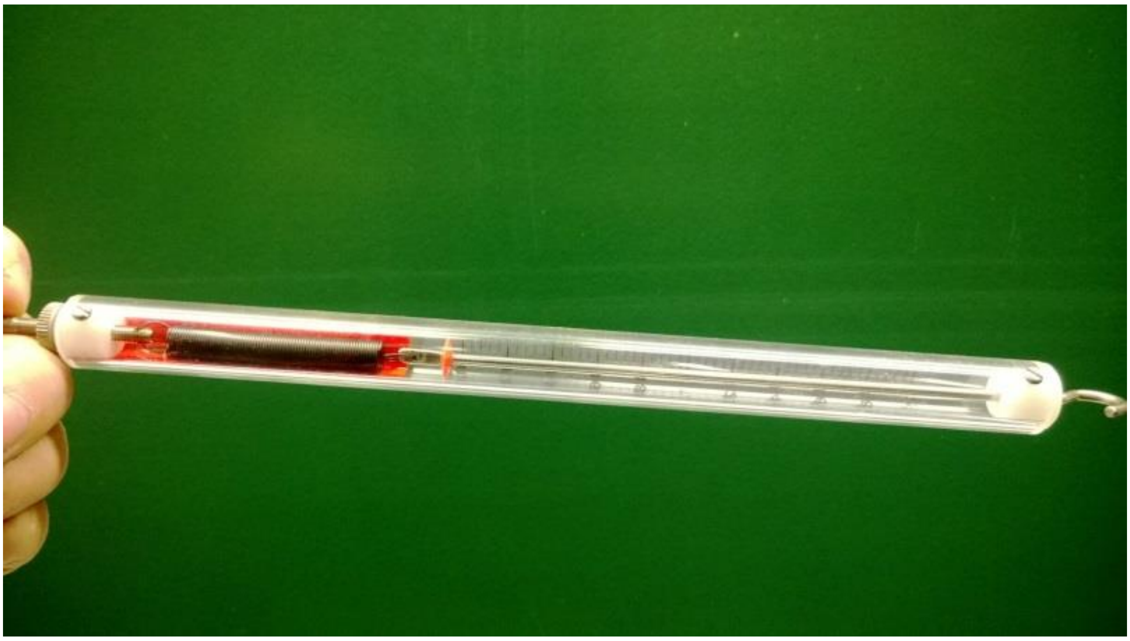
KINETIČNA ENERGIJA

Delo

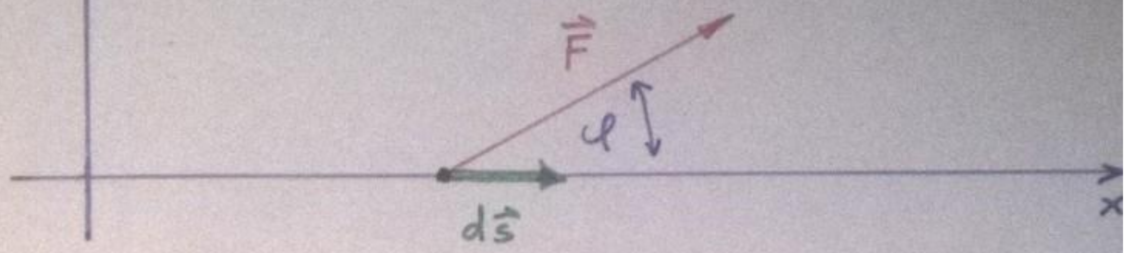




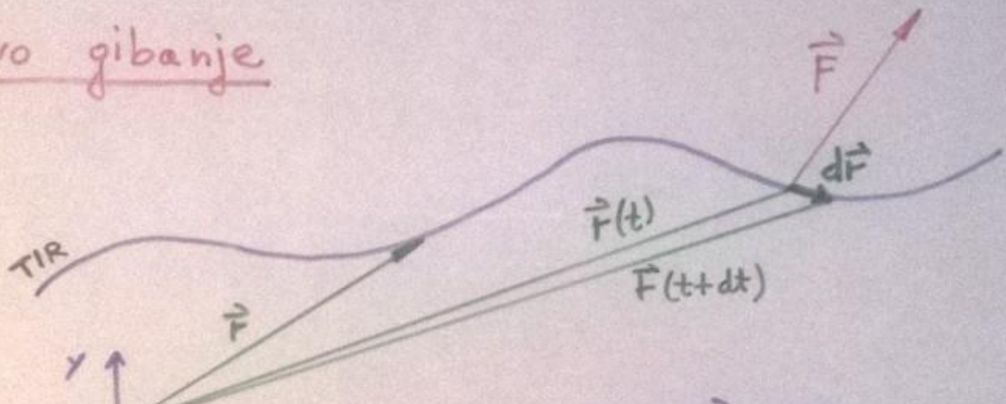
Delo (za točkasto telo)

- premo gibanje

$$dA = \vec{F} \cdot d\vec{s} = F\,ds\cos\varphi$$



- krivo gibanje



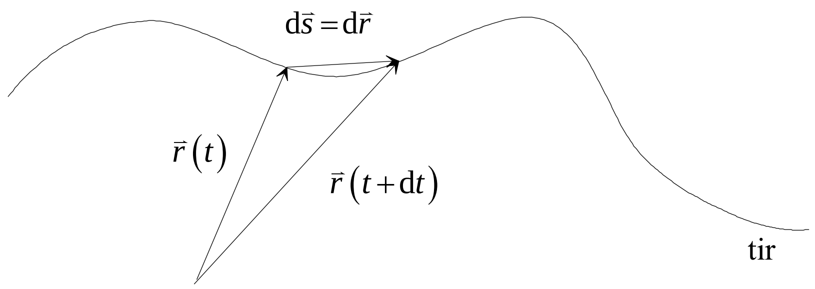
$$dA = \vec{F} \cdot d\vec{r}$$

enota za delo: $1\,Nm \equiv 1\,J$ (joule)

Točkasto telo – kinetična energija

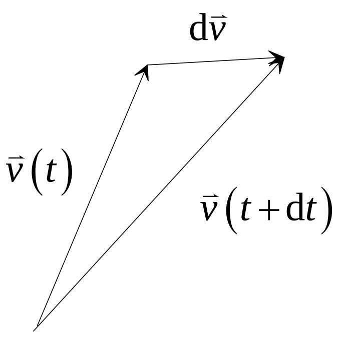
delo rezultante zunanjih sil ($\vec{F}$) na točkasti maso m

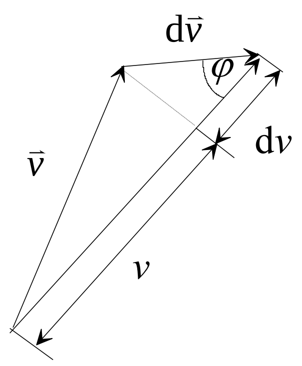
$$A=\int \vec{F}\cdot \mathrm{d}\vec{s}=\int \vec{F}\cdot \mathrm{d}\vec{r} \qquad \vec{F}=m\vec{a}=m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$



$$\mathrm{d}\vec{r} = \vec{v}\,\mathrm{d}t$$

$$A=\int \vec{F}\cdot \mathrm{d}\vec{r}=\int m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\cdot \mathrm{d}\vec{r}=\int m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\cdot \vec{v}\,\mathrm{d}t=\int m\,\mathrm{d}\vec{v}\cdot\vec{v}=m\int \vec{v}\cdot \mathrm{d}\vec{v}$$





$|\vec{v}|=v=\left(\vec{v}\cdot\vec{v}\right)^{\frac{1}{2}}$

$$\vec{v}\cdot \mathrm{d}\vec{v}=|\vec{v}|\underbrace{|\mathrm{d}\vec{v}|\cos\varphi}_{\mathrm{d}v}=v\,\mathrm{d}v \qquad |\mathrm{d}\vec{v}|\neq \mathrm{d}\left(|\vec{v}|\right)=\mathrm{d}v$$

$$\mathrm{d}\left(\vec{v}\cdot\vec{v}\right)=\mathrm{d}\vec{v}\cdot\vec{v}+\vec{v}\cdot\mathrm{d}\vec{v}=2\vec{v}\cdot\mathrm{d}\vec{v},$$

$$\vec{v}\cdot\mathrm{d}\vec{v}=\frac{1}{2}\mathrm{d}\left(\vec{v}\cdot\vec{v}\right)=\frac{1}{2}\mathrm{d}\left(v^2\right)=v\mathrm{d}v$$

$$A=\int \vec{F}\cdot \mathrm{d}\vec{r}=\int m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\cdot \vec{v}\,\mathrm{d}t=\int m\,\mathrm{d}\vec{v}\cdot \vec{v}=m\int \vec{v}\cdot \mathrm{d}\vec{v}=m\int v\mathrm{d}v$$

$$A = m\int \vec{v}\cdot d\vec{v} = m\int_{v_1}^{v_2} v\,\mathrm{d}v = \frac{1}{2}mv^2\Big|_{v_1}^{v_2} = \frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2\,.$$

kinetična energija **točkastega** telesa:

$$\boxed{W_k=\frac{1}{2}mv^2}$$

$$\boxed{A=\int \vec{F}\cdot d\vec{s}=\Delta W_k}$$

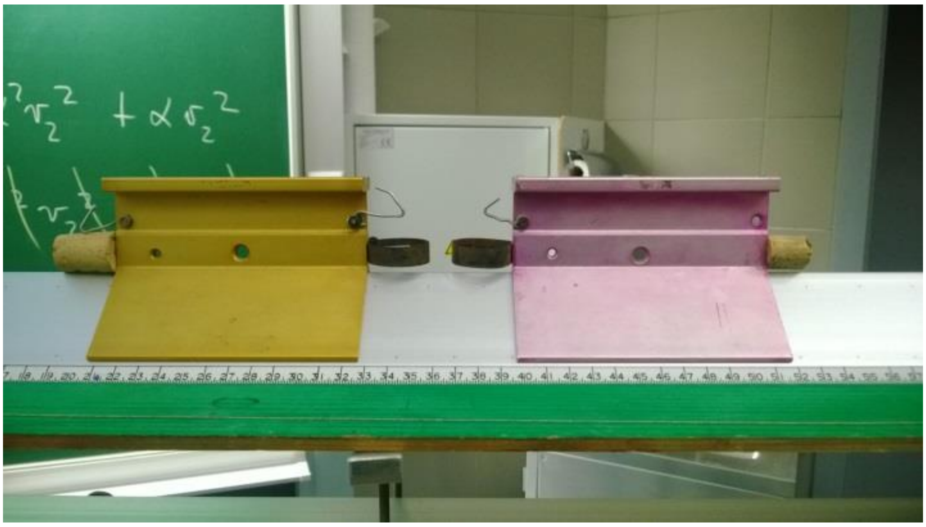
TOGO telo, ki se giblje po premici (premo gibanje) :

$$\boxed{W_k = \frac{1}{2} m v^2}$$

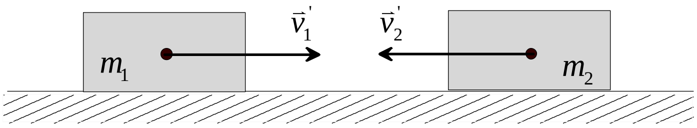
Zakon o ohranitvi kinetične energije

$$0 = A = \int \vec{F} \cdot d\vec{s} = \Delta W_k \qquad \Delta W_k = 0 \qquad W_{k,zač} = W_{k,kon}$$

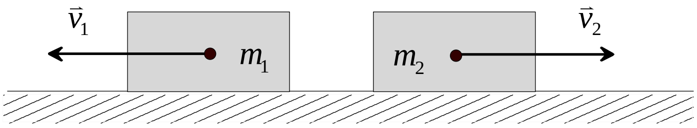
Zgled: **prožni (elastični) trk** dveh enako težkih jahačev (na zračni klopi)



pred trkom:



po trku:



$$m_1 \vec{v}_1' + m_2 \vec{v}_2' = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$\frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

poseben primer : $v_2' = 0$:

$$m_1 v_1' = m_1 v_1 + m_2 v_2 \qquad v_1' = v_1 + \left(\frac{m_2}{m_1}\right) v_2$$

$$\frac{1}{2} m_1 v_1'^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \qquad v_1'^2 = v_1^2 + \left(\frac{m_2}{m_1}\right) v_2^2$$

(1) $\Rightarrow v_1 = v_1' - \alpha v_2$ vstavi v (2) $\Rightarrow$

$$v_1'^2 = (v_1' - \alpha v_2)^2 + \alpha v_2^2$$

$$v_1'^2 = v_1'^2 - 2 v_1' \alpha v_2 + \alpha^2 v_2^2 + \alpha v_2^2$$

$$2 v_1' v_2 = \alpha v_2^2 + v_2^2$$

$$2 v_1' = \alpha v_2 + v_2$$

$$\boxed{v_2 = v_1' \frac{2}{(1+\alpha)}}$$

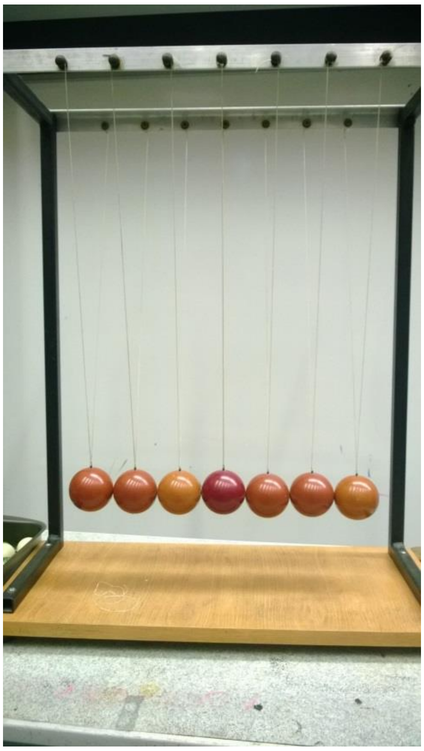
$$v_1 = v_1' - \frac{\alpha v_1' 2}{(1+\alpha)} = \frac{v_1' + \alpha v_1' - 2\alpha v_1'}{(1+\alpha)} = v_1' \frac{(1-\alpha)}{(1+\alpha)}$$

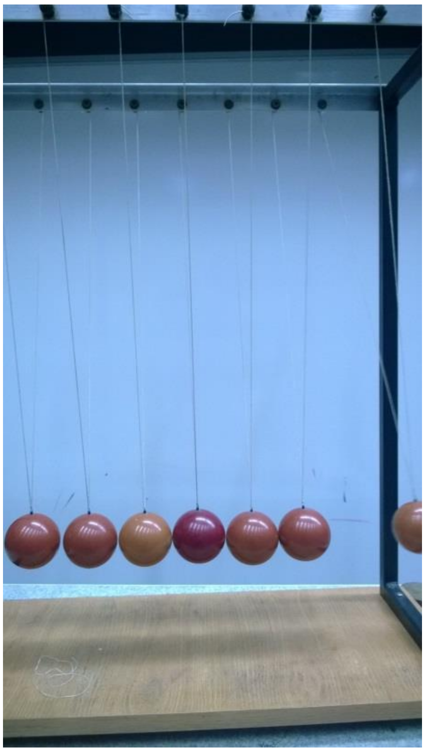
$$\boxed{v_1 = v_1' \cdot \frac{(1-\alpha)}{(1+\alpha)}}$$

$$v_1 = \frac{v_1'\left(1 - \frac{m_2}{m_1}\right)}{\left(1 + \frac{m_2}{m_1}\right)} \qquad v_2 = \frac{2v_1'}{1 + \frac{m_2}{m_1}}$$

$m_1 = m_2$: $v_1 = 0$ $v_2 = v_1'$

Zgled: biljarde krogle





Elastični in neelastični trki:

https://www.youtube.com/watch?v=jRliH0jVilM

https://www.youtube.com/watch?v=51IFubnEAsU

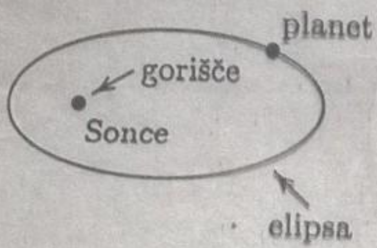
Biljardne krogle (elastični trki):

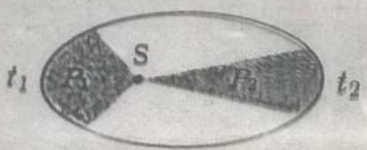
https://www.youtube.com/watch?v=ofgeRSCLyXc

GRAVITACIJA

KEPLERJEVI ZAKONI

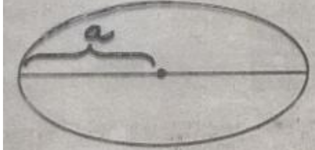
Johannes Kepler (1571–1630)





$$\frac{P_1}{t_1} = \frac{P_2}{t_2}$$

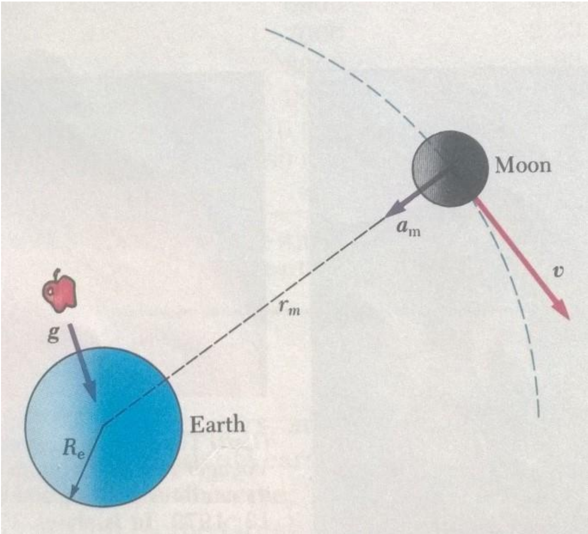
t = čas



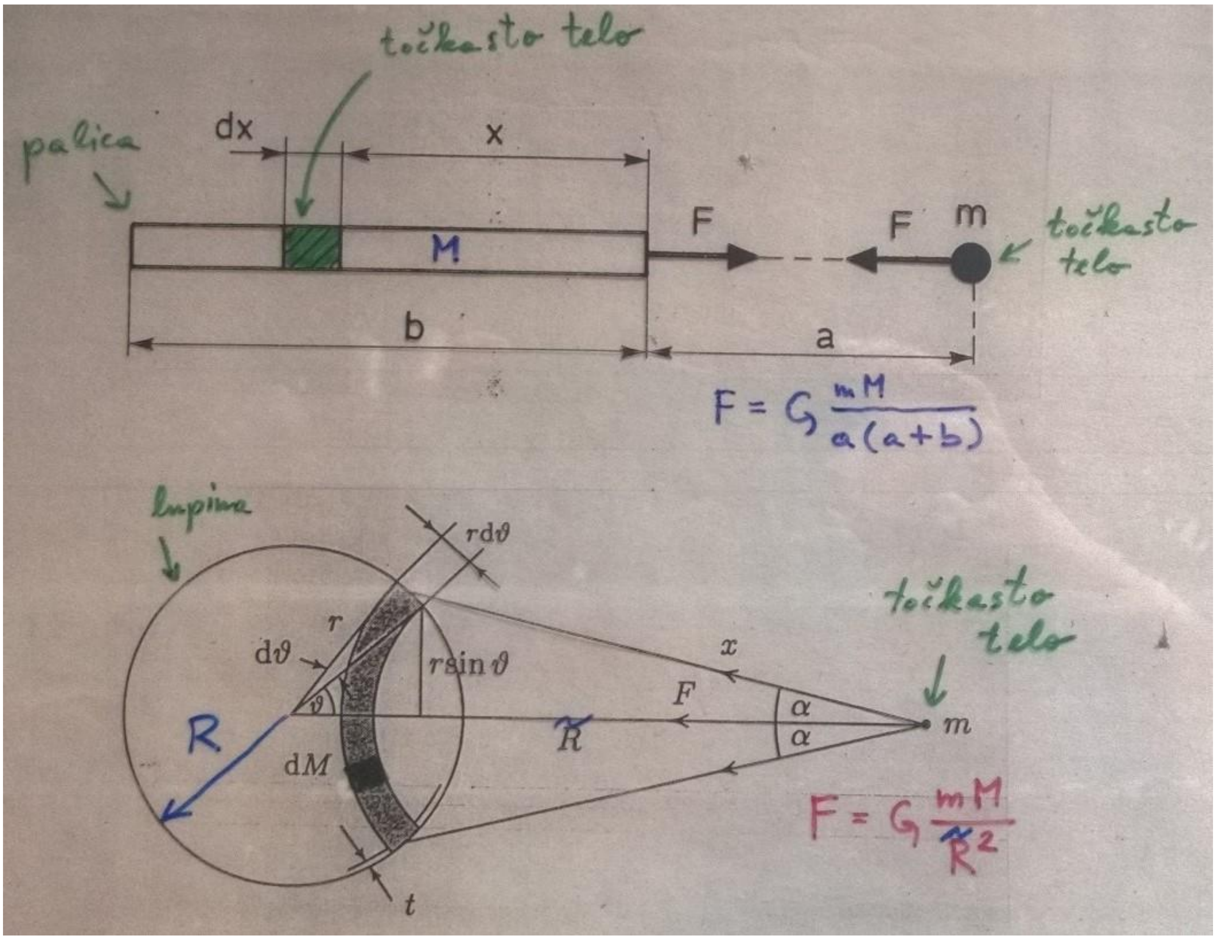
$$\frac{a^3}{t_0^2} = \kappa$$

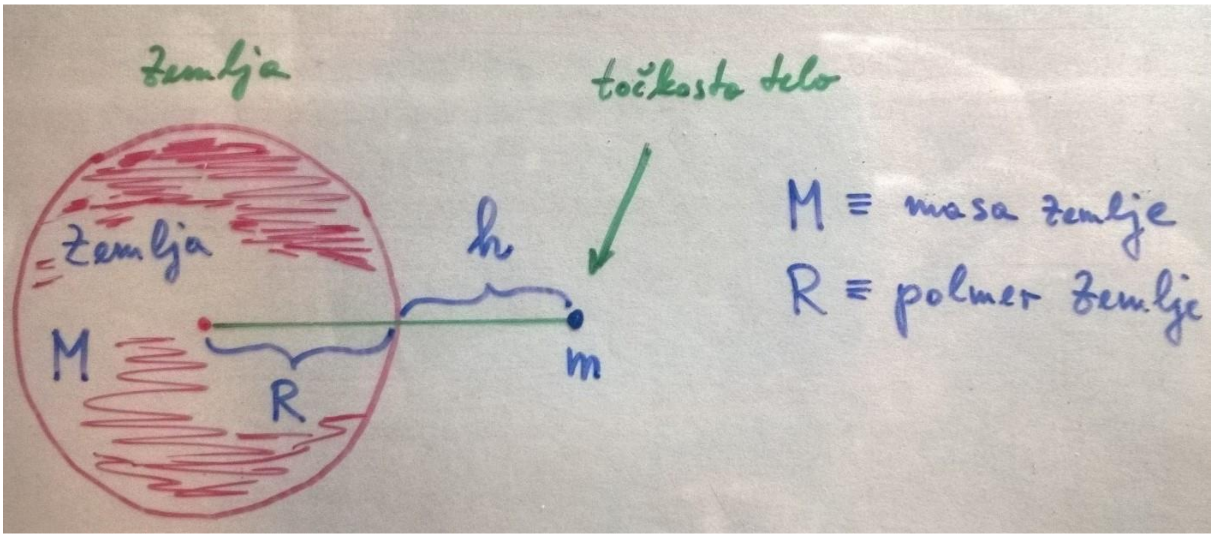
Planet	r [UA]	t_0 [leto]	$\frac{r^3}{t_0^2}$ $\left[\frac{UA^3}{leto^2}\right]$
Merkur	0,387	0,241	0,998
Venera	0,723	0,616	0,996
Zemlja	1,000	1,000	1,000
Mars	1,524	1,88	1,001
Saturn	5,20	11,86	1,000
Jupiter	9,54	29,46	1,000
Uran	19,18	84,0	1,001
Neptun	30,06	164,8	1,001
Pluton	39,5	247,7	1,001

Newtonov gravitacijski zakon

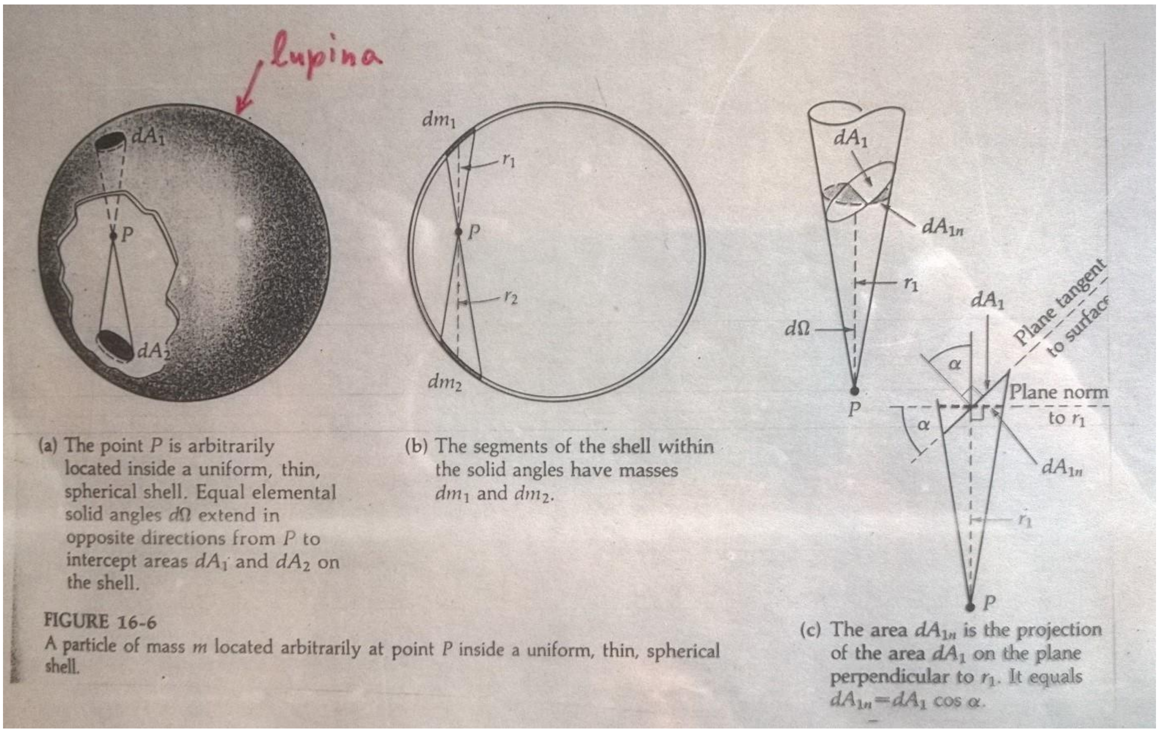


$$\vec{F}_{1,2} = G\frac{m_1\, m_2}{r^2}$$





$$\vec{F}_g = -G\frac{mM_z}{r^2}\frac{\vec{r}}{r}$$



(a) The point P is arbitrarily located inside a uniform, thin, spherical shell. Equal elemental solid angles $d\Omega$ extend in opposite directions from P to intercept areas dA_1 and dA_2 on the shell.

(b) The segments of the shell within the solid angles have masses dm_1 and dm_2.

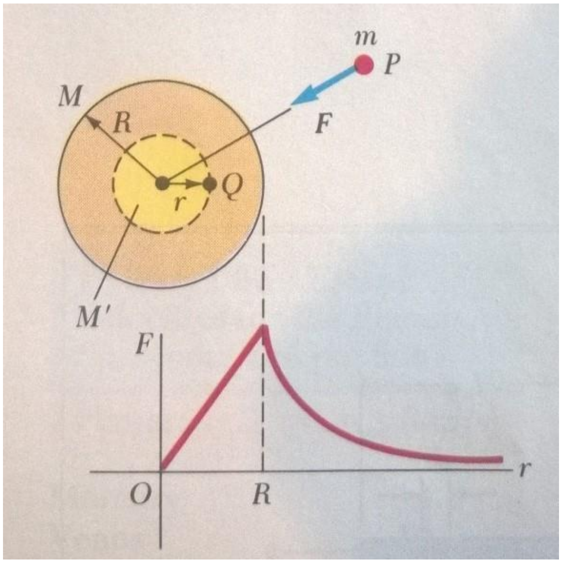
(c) The area dA_{1n} is the projection of the area dA_1 on the plane perpendicular to r_1. It equals $dA_{1n}=dA_1 \cos\alpha$.

FIGURE 16-6
A particle of mass m located arbitrarily at point P inside a uniform, thin, spherical shell.

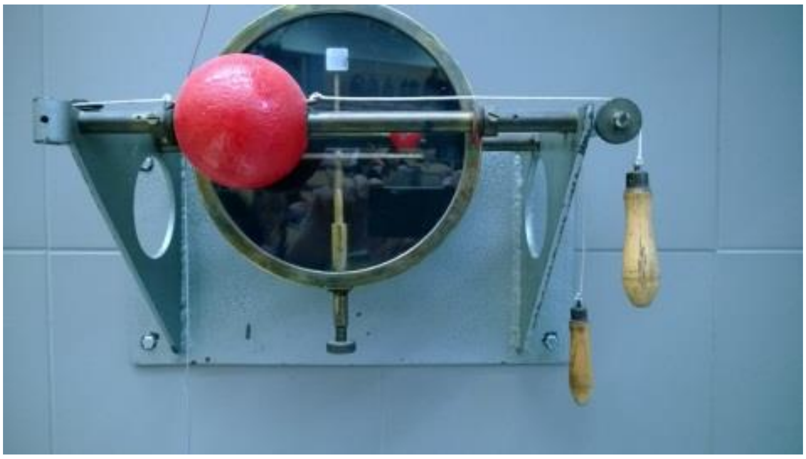
$$dm_1 = \sigma\, dA_1 = \frac{\sigma\, d\Omega\, r_1^2}{\cos\alpha} \qquad \text{and} \qquad dm_2 = \sigma\, dA_2 = \frac{d\Omega\, r_2^2}{\cos\alpha}$$

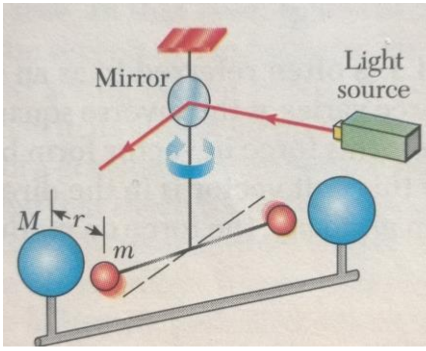
These two elements of mass exert gravitational forces on the particle m. The two forces are in opposite directions, and the ratio of their magnitudes is

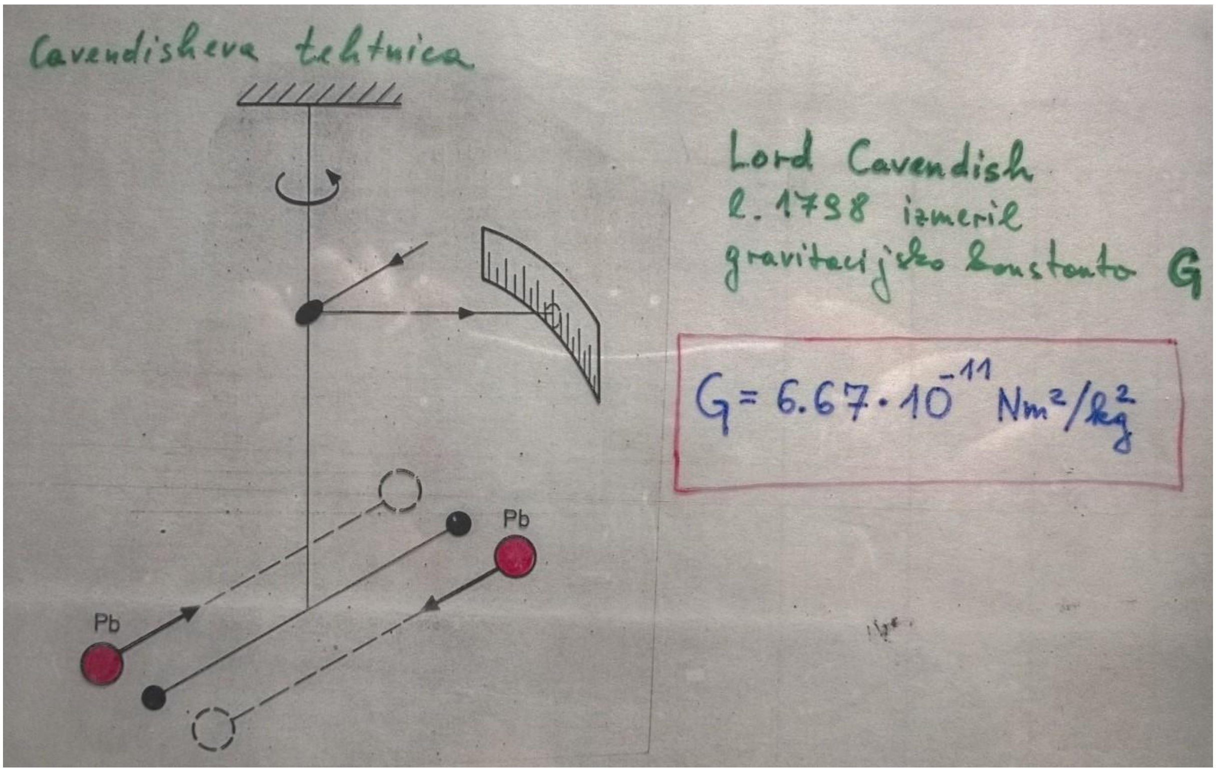
$$\frac{dF_1}{dF_2} = \frac{\left(\dfrac{Gm\, dm_1}{r_1^2}\right)}{\left(\dfrac{Gm\, dm_2}{r_2^2}\right)} = \frac{\left(\dfrac{\sigma\, d\Omega\, r_1^2}{\cos\alpha r_1^2}\right)}{\left(\dfrac{\sigma\, d\Omega\, r_2^2}{\cos\alpha r_2^2}\right)} = 1$$



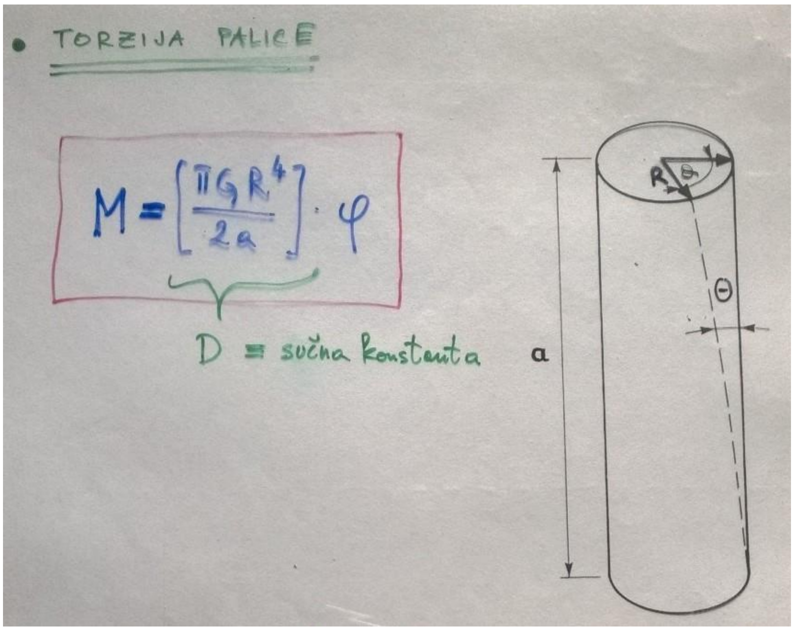
MERJENJE GRAVITACIJSKE KONSTANTE IN POSPEŠKA (Cavendishova gravitacijska tehtnica)

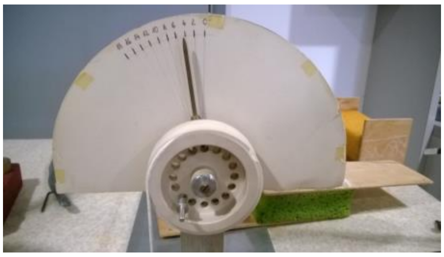






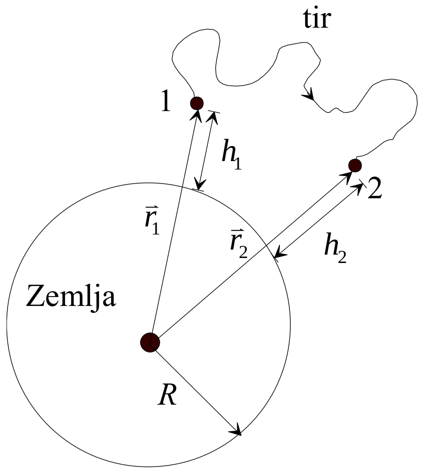
Torzijska tehtnica na žico (torzija kovinske tanke palice)





Gravitacijska potencialna energija

$$\vec{F}_g = -G\frac{m M_z}{r^2}\frac{\vec{r}}{r}$$



R = polmer Zemlje
h = nadmorska višina

$$\vec{F} = \vec{F}_{ost} + \vec{F}_g$$

$$A = \int\left(\vec{F}_{ost} + \vec{F}_g\right)\cdot d\vec{s} = \int\left(\vec{F}_{ost} + \vec{F}_g\right)\cdot d\vec{r} = \Delta W_k$$

$$A_{ost} = \int \vec{F}_{ost}\cdot \mathrm{d}\vec{r} = \Delta W_k - \int \vec{F}_g \cdot \mathrm{d}\vec{r}$$

$$-\int \vec{F}_g \cdot \mathrm{d}\vec{r} = \int G\frac{m M_z}{r^2}\frac{\vec{r}}{r}\cdot \mathrm{d}\vec{r} = G\,m\,M_z\int\frac{1}{r^3}\vec{r}\cdot \mathrm{d}\vec{r} =$$

$$= G\,m\,M_z\int\frac{1}{r^3}r\,\mathrm{d}r = G\,m\,M_z\int_{r_1}^{r_2}\frac{\mathrm{d}r}{\mathrm{r}^2} = G\,m\,M_z\left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} =$$

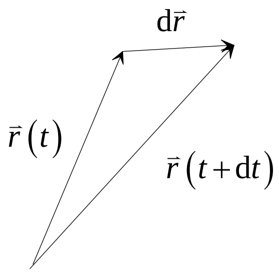
$$= -G\frac{m M_z}{r}\Big|_{r_1}^{r_2} = -G\frac{m M_z}{r_2} - \left(-G\frac{m M_z}{r_1}\right) = \Delta W_p$$

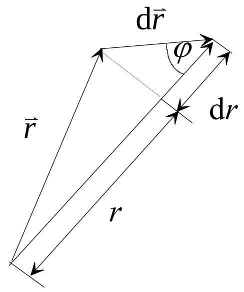
$$\boxed{W_p = -G\frac{mM_z}{r}} \qquad \Delta W_p = W_{p,2} - W_{p,1}$$

$$A_{ost} = \int \vec{F}_{ost} \cdot \mathrm{d}\vec{r} = \Delta W_k - \int \vec{F}_g \cdot \mathrm{d}\vec{r}$$

$$\boxed{A_{ost} = \Delta W_k + \Delta W_p}$$

izrek o kinetični in potencialni energiji za točkasto telo





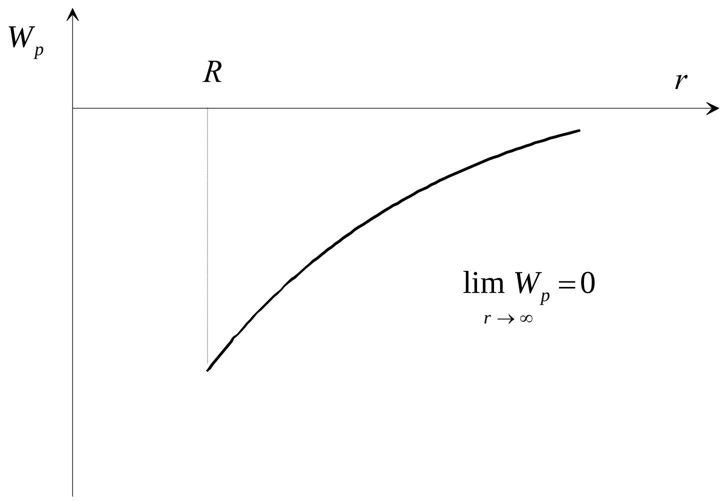
$$\mathrm{d}(\vec{r}\cdot\vec{r}) = \mathrm{d}\vec{r}\cdot\vec{r} + \vec{r}\cdot\mathrm{d}\vec{r} = 2\vec{r}\cdot\mathrm{d}\vec{r}$$

$$\vec{r}\cdot\mathrm{d}\vec{r} = \frac{1}{2}\mathrm{d}(\vec{r}\cdot\vec{r}) = \frac{1}{2}\mathrm{d}(\mathrm{r}^2) = r\,\mathrm{d}r$$

$$W_p = -G\frac{mM_z}{r}$$

$$r = R + h$$

$$g_0 = G\frac{M_z}{R^2} \cong 9.8\,\mathrm{m\,s^{-2}}$$



majhne nadmorske višine *h* :

$$W_p = -G\frac{mM_z}{r} = -G\frac{mM_z}{(R+h)} = -G\frac{mM_z}{R\left(1+\frac{h}{R}\right)} = -G\frac{mM_z}{R}\left(1-\frac{h}{R}+\frac{h^2}{R^2}-\cdots\right)$$

$$\frac{h}{R}\to 0 \qquad W_p \cong -G\frac{mM_z}{R}\left(1-\frac{h}{R}\right) = -G\frac{mM_z}{R} + G\frac{M_z}{R^2}mh$$

$$g_0 = G\frac{M_z}{R^2} \cong 9.8\,\mathrm{m\,s^{-2}}$$

$$\boxed{W_p = m g_0 h}$$

UBEŽNA HITROST (groba ocena)

$$A_{od} = 0 = \Delta W_p + \Delta W_k$$

$$\Delta(W_p + W_k) = 0 \Rightarrow W_p + W_k = \text{konst.}$$

$$(W_p + W_k)_{r=R} = (W_p + W_k)_{r \to \infty}$$

$$-G\frac{mM}{R} + \frac{m v_0^2}{2} = 0 + 0 \quad \Big\} \text{ ker pri } r \to \infty: \; W_k \to 0, \; W_p \to 0$$

$$\Downarrow$$

$$v_0 = \left(\frac{2GM}{R}\right)^{1/2} \approx \underline{11\ \text{km/s}}$$

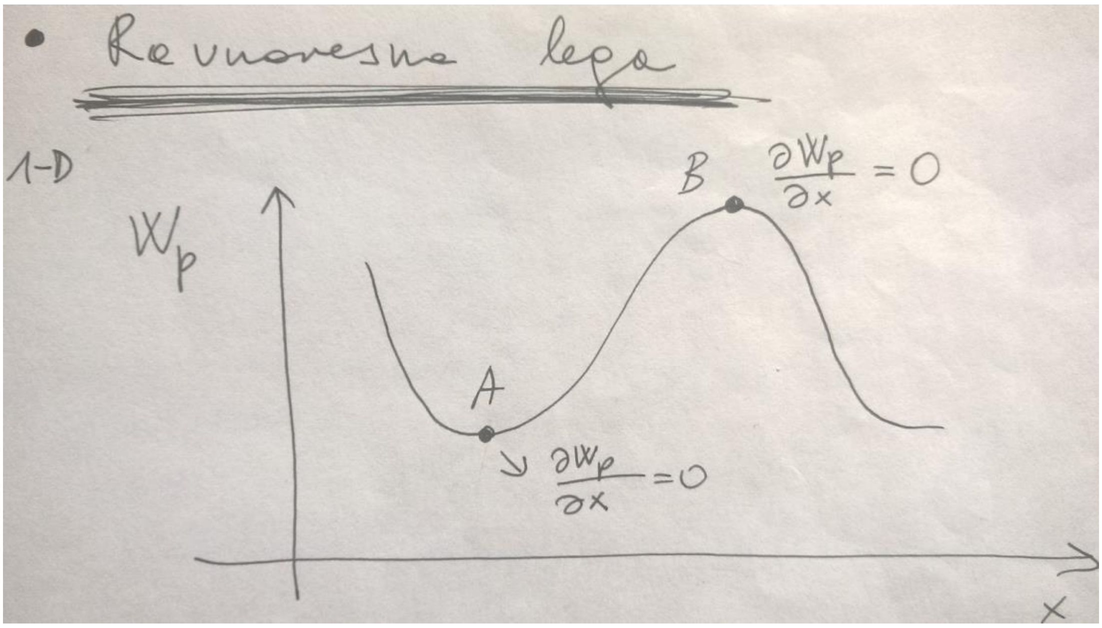
$$\left(\frac{2GM}{R}\right)^{1/2} = \Big(2\underbrace{\frac{GM}{R^2}}_{g_0}R\Big)^{1/2} = (2 g_0 R)^{1/2}$$

$$R = 6400\ \text{km}$$

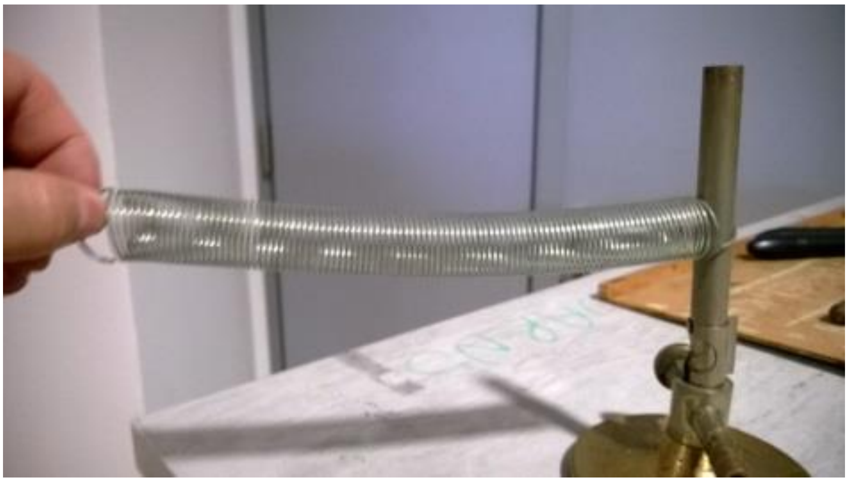
$$g_0 = 9.8\ \text{m s}^{-2}$$

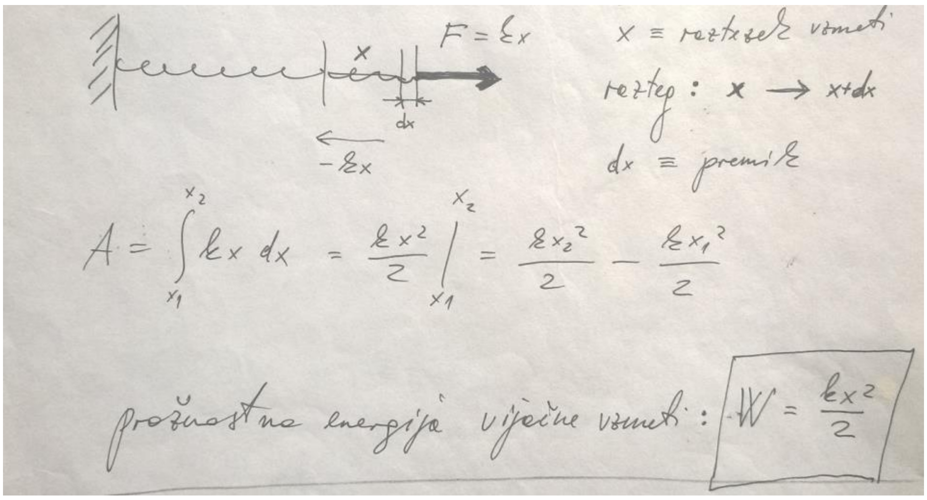
TABLE 14.3 Escape Velocities for the Planets, the Moon, and the Sun

Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5.0
Jupiter	60
Saturn	36
Uranus	22
Neptune	24
Pluto	1.1
Sun	618

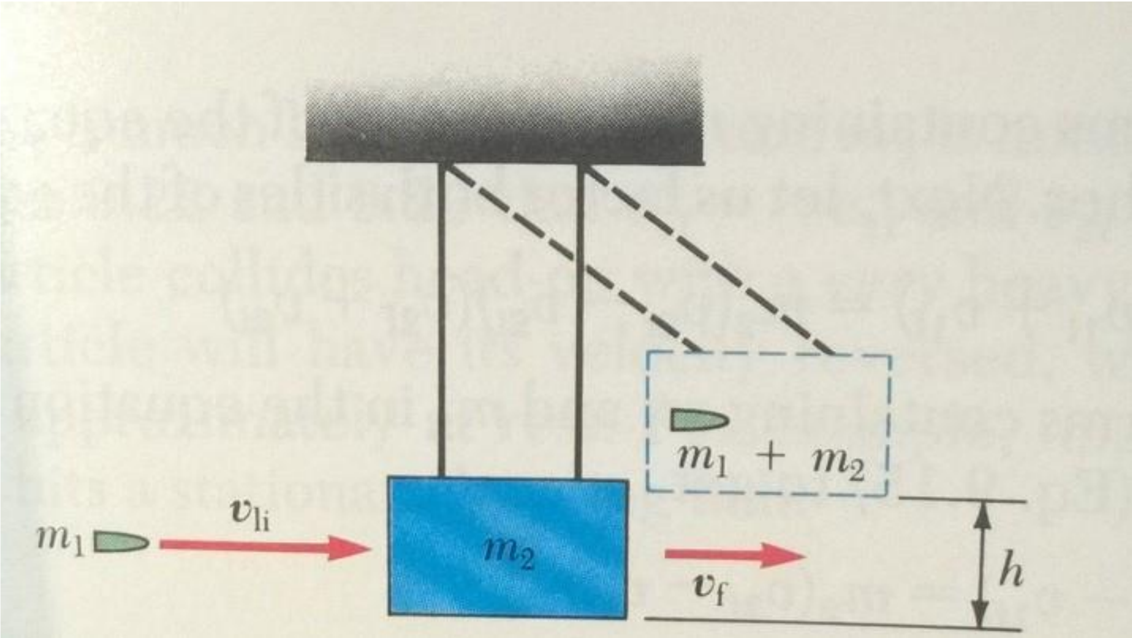


DRUGE ENERGIJE : PROŽNOSTNA (ELASTIČNA) ENERGIJA





ZGLED : balistično nihalo

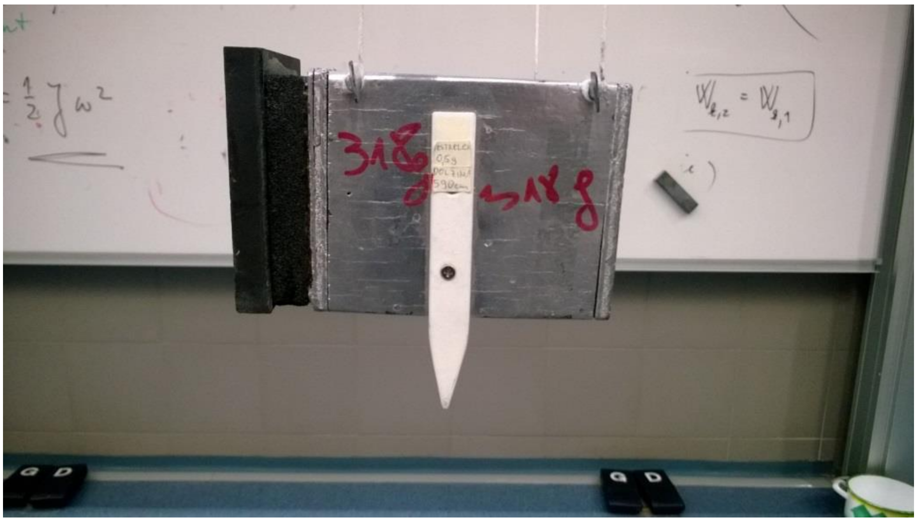


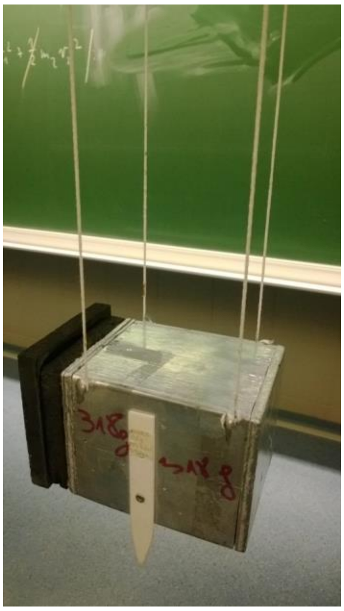
$$m_1 v_{li} = (m_1 + m_2) v_f \quad (\Delta G = 0)$$

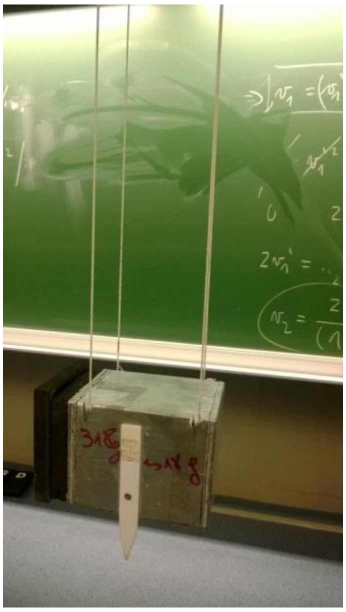
$$\frac{1}{2}(m_1 + m_2) v_f^2 = (m_1 + m_2) g_0 h$$

$$\Rightarrow v_f = \sqrt{2 g_0 h}$$

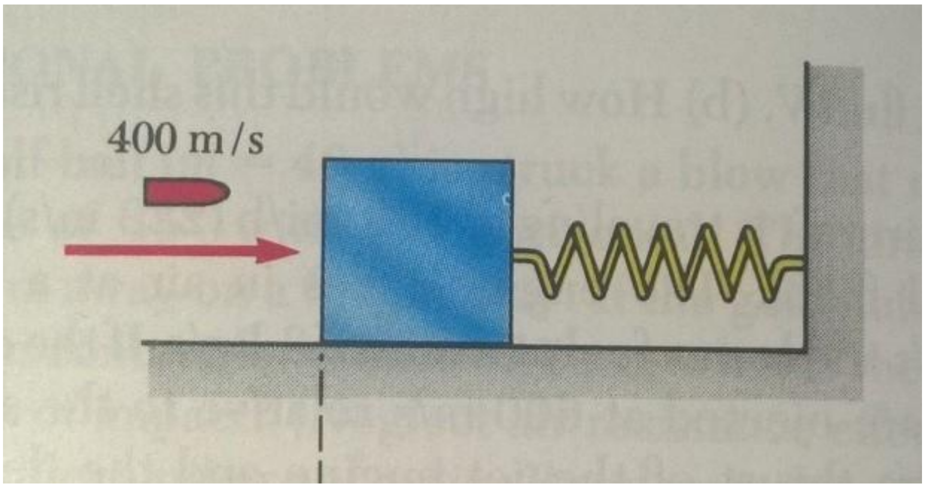
$$v_{li} = \frac{(m_1 + m_2)}{m_1} v_f = \frac{(m_1 + m_2)}{m_1} \sqrt{2 g_0 h}$$





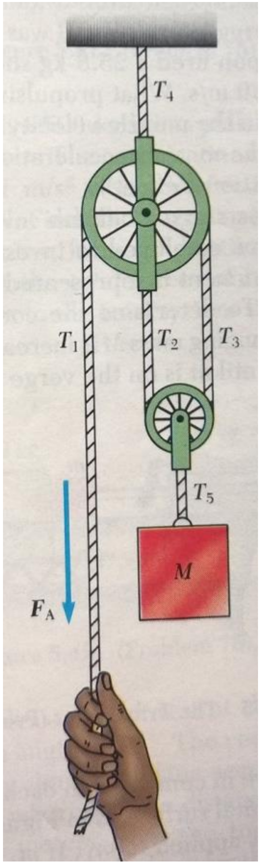


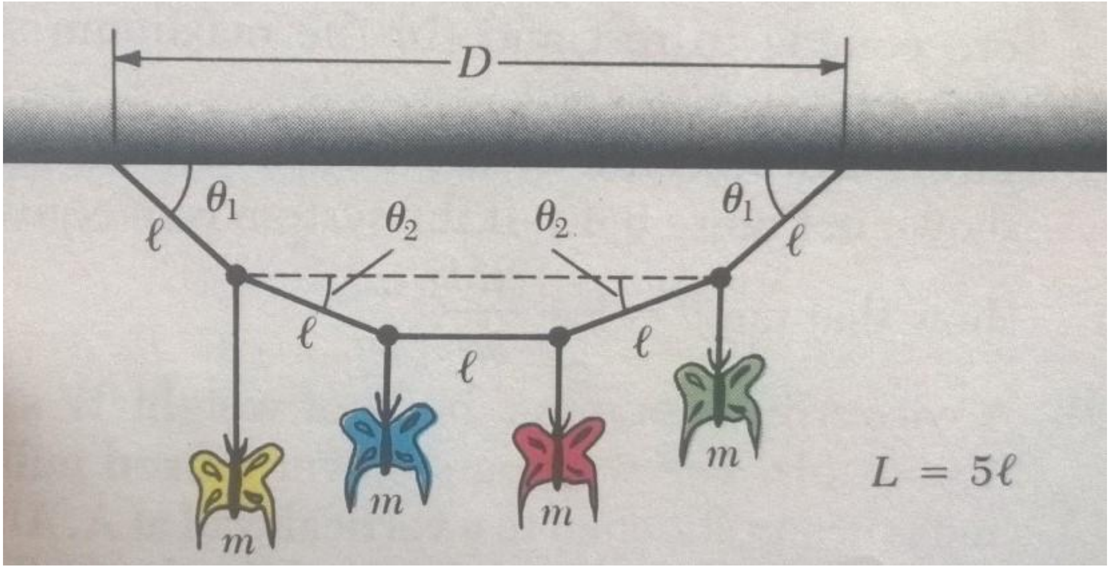
ZGLED: ne-elastičen trk, elastična energija



MEHANSKO RAVNOVESJE

$$\sum_i \vec{F}_i = 0 \qquad \sum_i \vec{M}_i = 0$$

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θ_2
θ_1
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