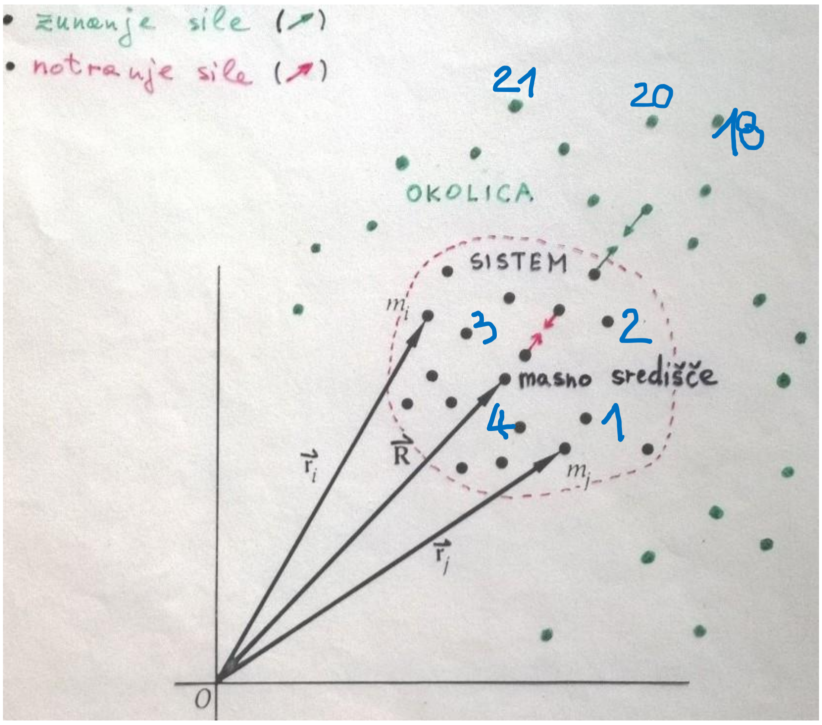
II. Newtonov zakon za sistem točkastih teles

točkasto telo

$$\vec{F} = \sum_i \vec{F}_i = m\,\vec{a}$$

sistem točkastih teles



II. Newtonov zakon za i-to točkasto maso v sistemu:

$$\sum_j \vec{F}_{ji} + \vec{F}_i = m_i \frac{\mathrm{d}^2 \vec{r}_i}{\mathrm{d}t^2} \quad , \qquad i = 1, 2, \ldots N,$$

$\vec{F}_{ji}$ so **notranje sile** med delci (n.pr. $\vec{F}_{23}$ je sila 2. delca na 3. delec v sistemu)

$\vec{F}_i$ je rezultanta vseh zunanji sil na i-to točkasto maso v sistemu

$$\sum_j \vec{F}_{j1} + \vec{F}_1 = m_1 \frac{\mathrm{d}^2 \vec{r}_1}{\mathrm{d}t^2}$$

$$\sum_j \vec{F}_{j2} + \vec{F}_2 = m_2 \frac{\mathrm{d}^2 \vec{r}_2}{\mathrm{d}t^2}$$

.

.

.

$$\sum_j \vec{F}_{jN} + \vec{F}_N = m_N \frac{\mathrm{d}^2 \vec{r}_N}{\mathrm{d}t^2}$$

$$\sum_{\substack{i,j\\ i\neq j}} \vec{F}_{ji} + \sum_i \vec{F}_i = \sum_i \left(m_i \frac{\mathrm{d}^2 \vec{r}_i}{\mathrm{d}t^2} \right) \qquad \sum_i \vec{F}_i = \frac{\mathrm{d}^2}{\mathrm{d}t^2} \left(\sum_i m_i \vec{r}_i \right)$$

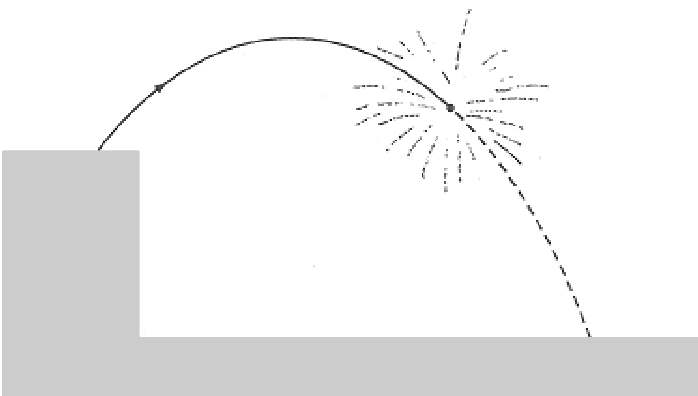
upoštevamo smo $\vec{F}_{ij} + \vec{F}_{ji} = \vec{0}$

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{m} \qquad \boxed{\sum_i \vec{F}_i = m \frac{\mathrm{d}^2 \vec{R}}{\mathrm{d}t^2}} \qquad m = \sum_i m_i$$

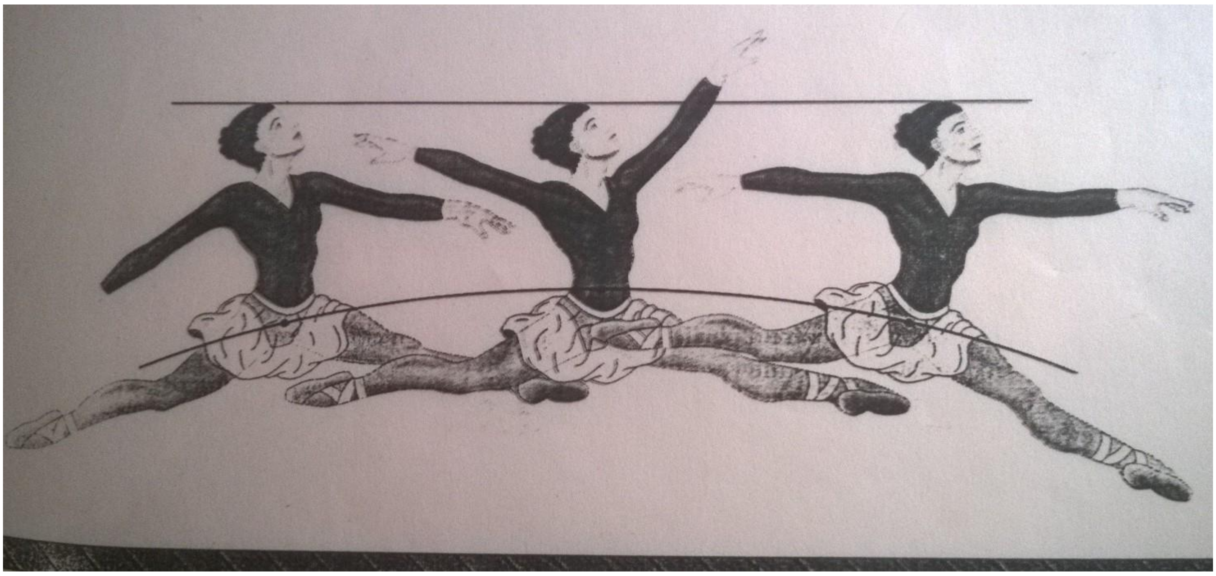
$$\vec{a}_R = \frac{\mathrm{d}^2 \vec{R}}{\mathrm{d}t^2}$$

Opomba: v izpeljavi nismo nikjer predpostavili, da so razdalje med delci, ki sestavljajo sistem konstantne.

Gibanje masnega centra rakete pred (polna črta) in po eksploziji (črtkana črta)



skok: grand jeté

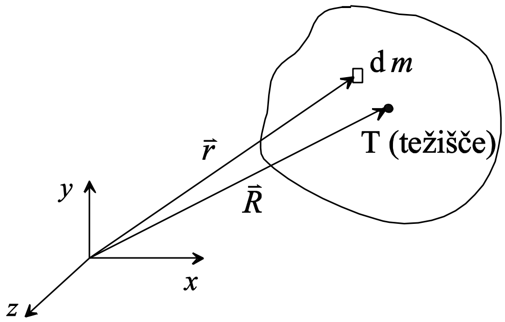


II. Newtonov zakon za togo telo

razdalje med posameznimi deli telesa konstantne

kontinuumski opis - uvedemo gostoto togega telesa

$$\rho(\vec{r}) = \frac{\mathrm{d}m}{\mathrm{d}V}$$



$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \vec{r}_i}{m}$$

transformacije:

$$\sum_i \to \int$$

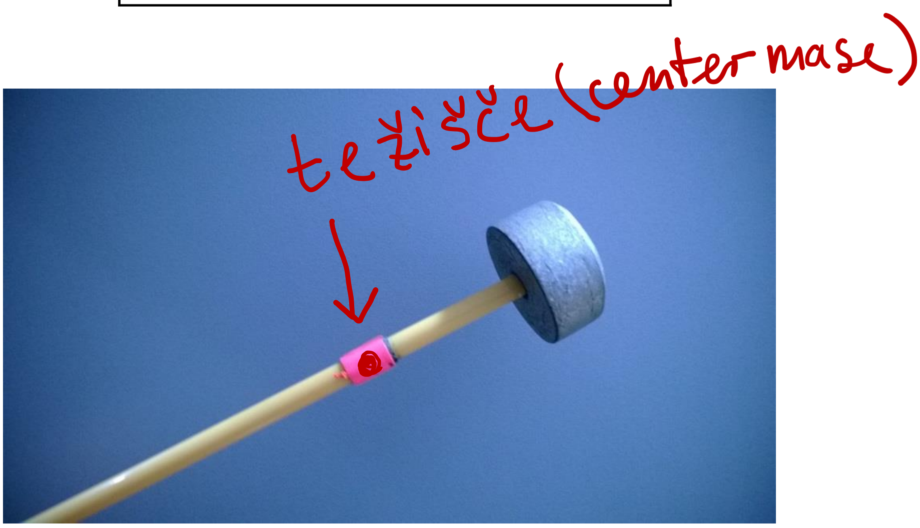
$$m_i \to \mathrm{d}m$$

$$\vec{r}_i \to \vec{r}$$

$$\vec{R} = \frac{\int \vec{r}\,\mathrm{d}m}{\int \mathrm{d}m} = \frac{\int \vec{r}\,\mathrm{d}m}{m} = \frac{\int \vec{r}\,\rho\,\mathrm{d}V}{\int \rho\,\mathrm{d}V}$$

II. Newtonov zakon za togo telo :

$$\vec{F} = m\vec{a}_R = m\frac{\mathrm{d}\vec{v}_R}{\mathrm{d}t} = m\frac{\mathrm{d}^2\vec{R}}{\mathrm{d}t^2}$$



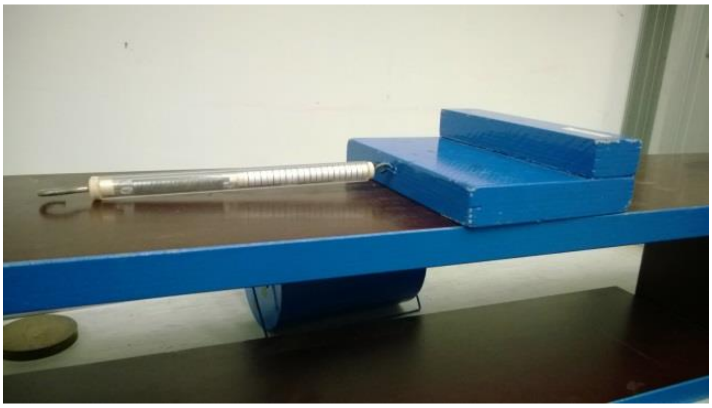
Konstantna gostota :

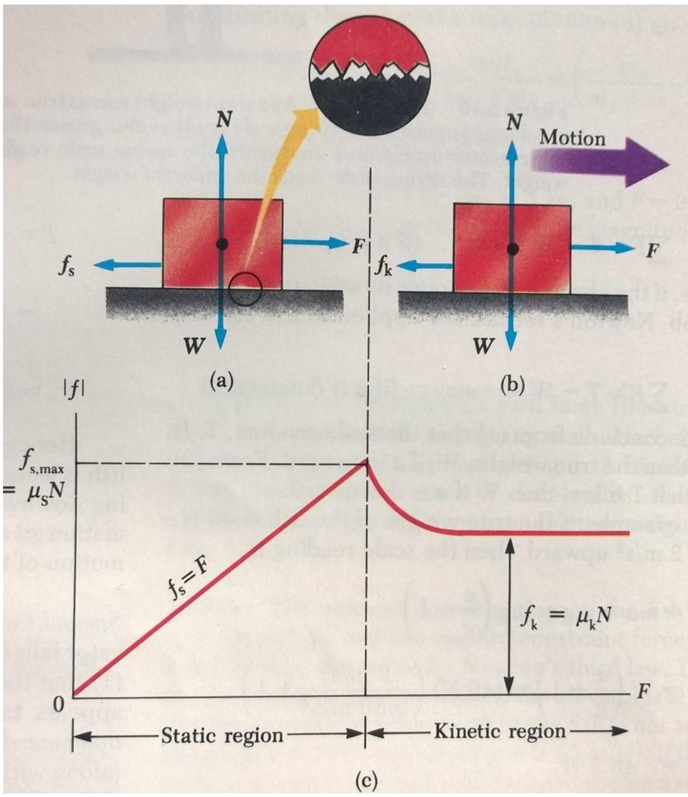
$$\vec{R} = \frac{\int \vec{r}\,\rho\,\mathrm{d}V}{\int \rho \mathrm{d}V} = \frac{\int \vec{r}\,\mathrm{d}V}{\int \mathrm{d}V} = \frac{\int \vec{r}\,\mathrm{d}V}{V}$$

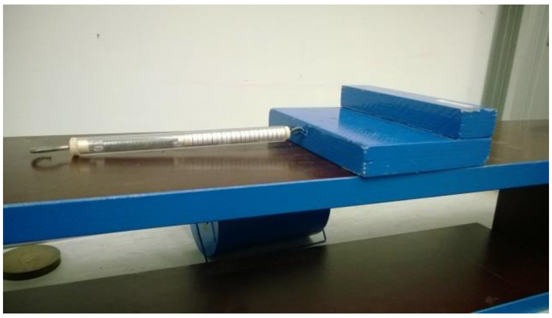
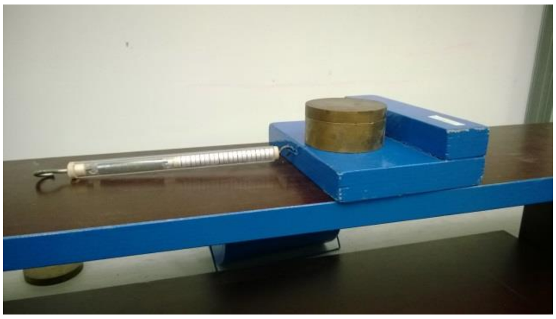
$$\boxed{\vec{F} = m\,\vec{a}_R = m\frac{\mathrm{d}\,\vec{v}_R}{\mathrm{d}t} = m\frac{\mathrm{d}^2\,\vec{R}}{\mathrm{d}\,t^2}}$$

DOLOČANJE LEGE TEŽIŠČA Z OBEŠANJEM TELES :

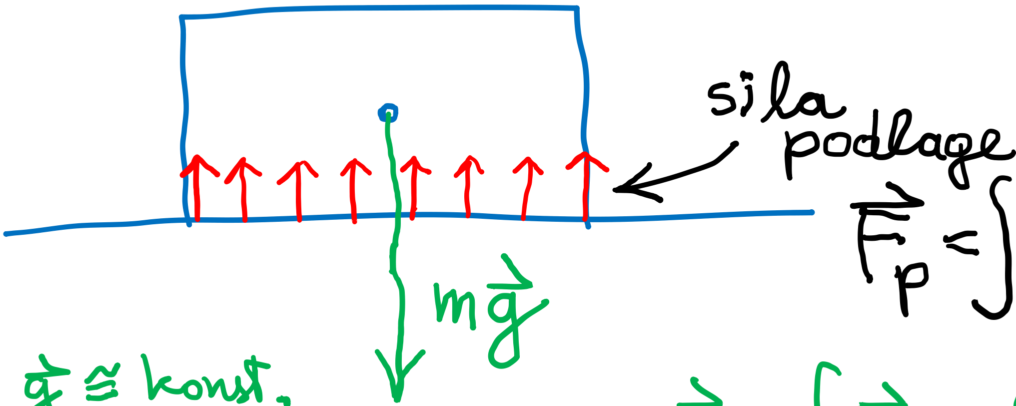
SILI TRENJA IN LEPENJA





MIROVANJE



$$\vec{F}_p = \int p\, d\vec{S}$$

$\vec{g} \approx$ konst.
$\rho \approx$ konst.

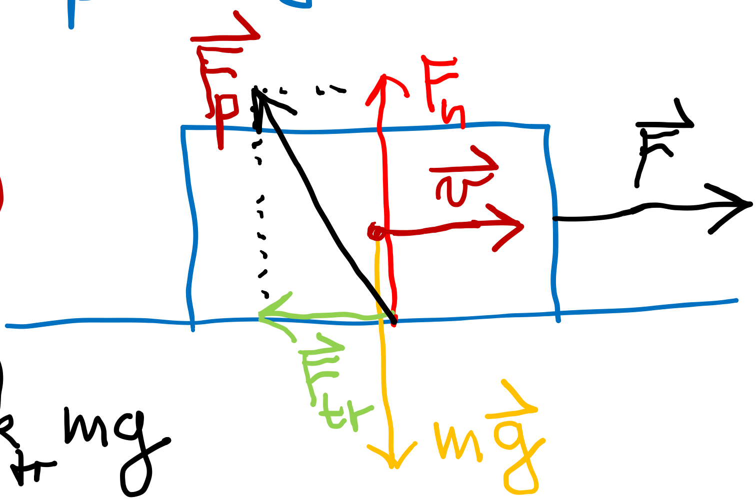
$$m\vec{g} = \int \vec{g} \rho\, dV$$

ravnovesje: $\vec{F}_p + m\vec{g} = \vec{0}$

$$F_p = mg$$

GIBANJE

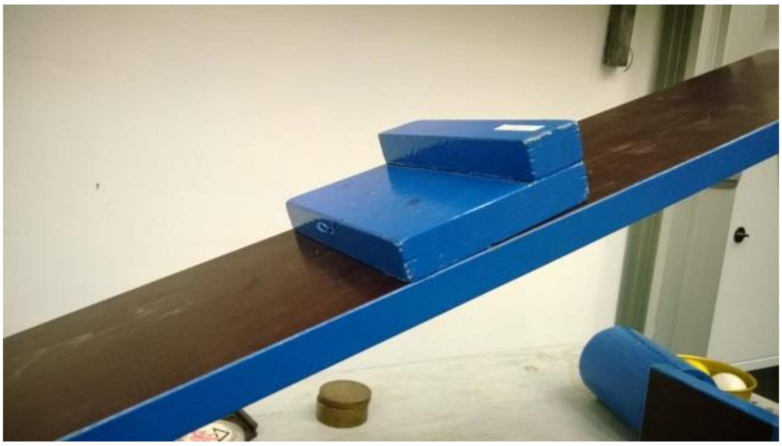
$$\vec{F}_p = (-F_{tr}, F_n, 0)$$



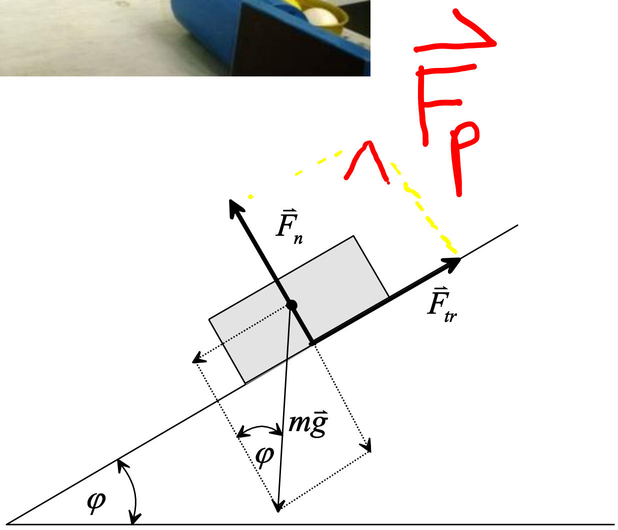
$$F_{tr} = k_{tr} F_n = k_{tr} mg$$

$$F_l = k_l F_n = k_l mg$$

$$k_{tr} < k_l$$



$\vec{F}_n$ = normalna komponenta sile podlage
m = masa klade
φ = nagib klanca
$\vec{F}_{tr}$ = sila trenja
$m\vec{g}$ = sila teže



II. Newtonov zakon za gibanje težišča togega telesa (klade)

$$mg\sin\varphi - F_{tr} = ma$$

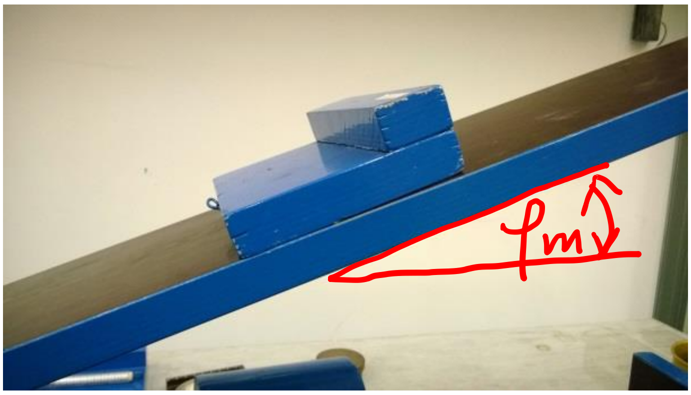
sila trenja F_{tr} je sorazmerna $F_n = m\,g\cos\varphi$

$$F_{tr} = k_t\, m\, g\cos\varphi\,,$$

$$mg\sin\varphi - k_t\, m\, g\cos\varphi = ma$$

$$a = g\left(\sin\varphi - k_t\cos\varphi\right)$$

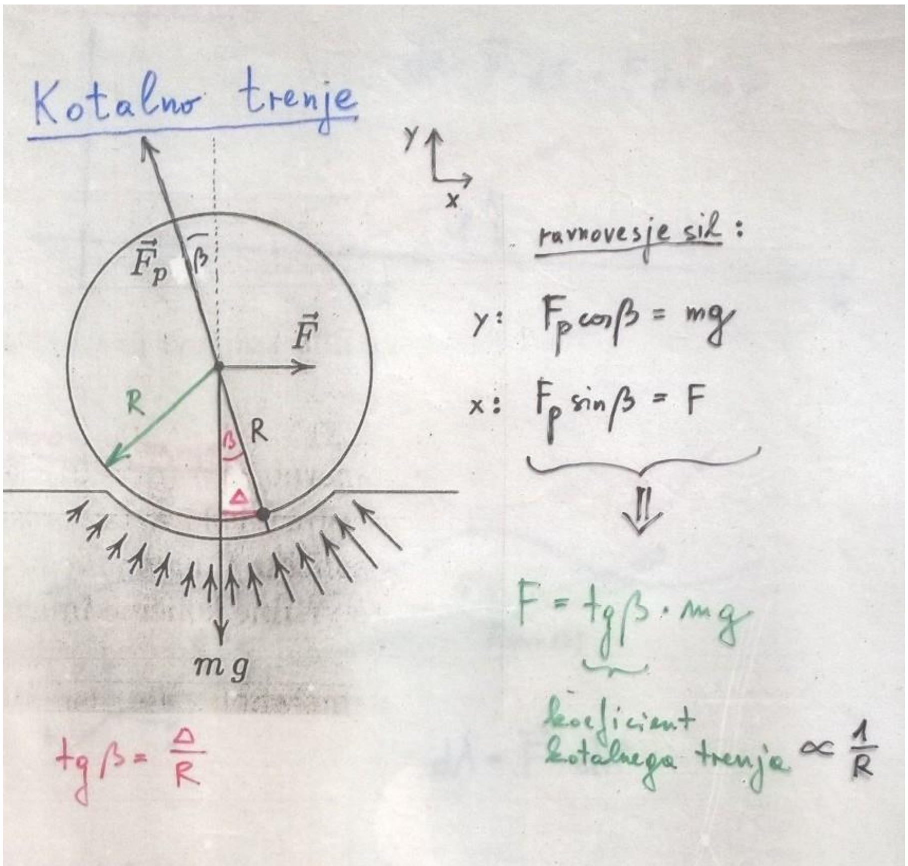
MEJNI KOT **za drsenje klade po klancu navzdol:**



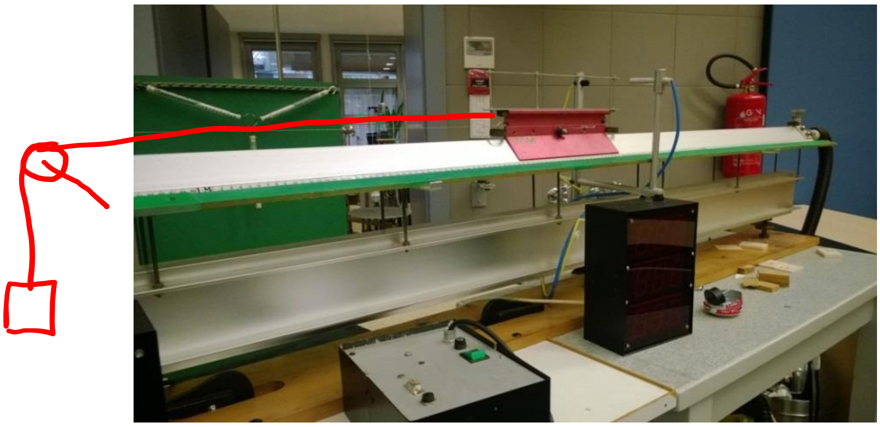
$$a = 0 = g\left(\sin\varphi_m - k_\ell \cos\varphi_m\right)$$

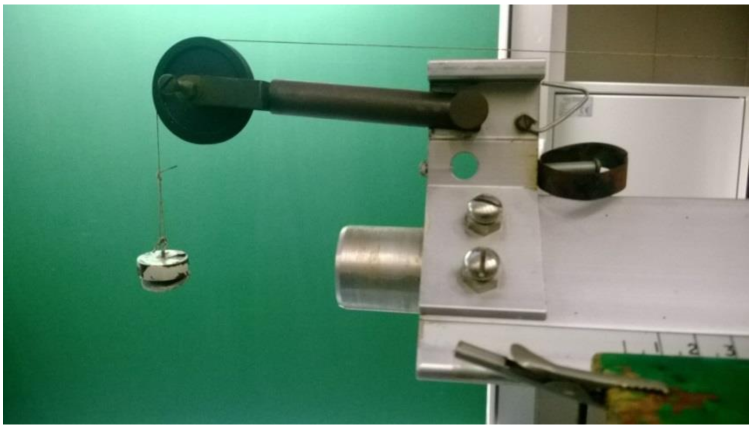
$$tg\,\varphi_m = k_\ell$$

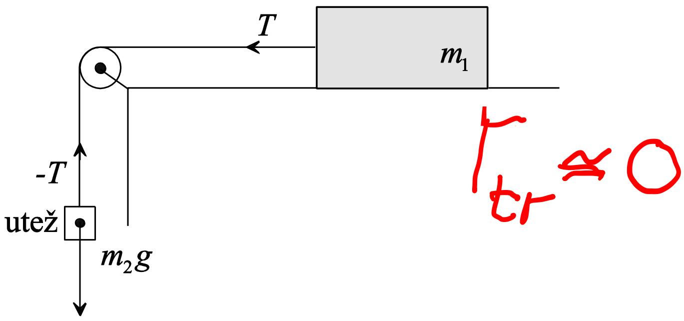
k_ℓ = koeficient lepenja



Zgled: klada (na zračni klopi), pripeta z zelo lahko vrvico na utež







$$T = m_1 a$$

$$m_2 g - T = m_2 a$$

$$a = \frac{m_2 g}{(m_1 + m_2)}$$

IZREK O SUNKU SILE

Newtonov zakon za gibanje težišča togega telesa

$$\boxed{\vec{F} = m\,\vec{a}_R = m\,\frac{\mathrm{d}\,\vec{v}_R}{\mathrm{d}t}}$$

$$\vec{G} = m\vec{v}_R \qquad\qquad \vec{F} = \frac{\mathrm{d}\,\vec{G}}{\mathrm{d}t}$$

$$\int_{t_1}^{t_2} \vec{F}\,\mathrm{d}t = \vec{G}\,\Big|_{\vec{G}_1}^{\vec{G}_2} = \vec{G}_2 - \vec{G}_1 = \Delta\vec{G}$$

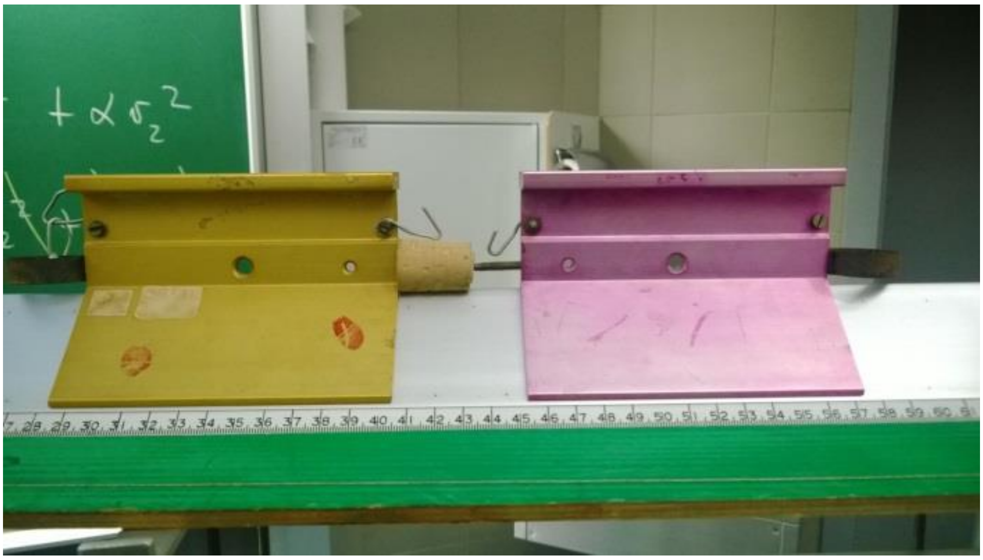
ZAKON O OHRANITVI GIBALNE KOLIČINE

$$\Delta\,\vec{G} = 0 \qquad\qquad \vec{G}_1 = \vec{G}_2$$

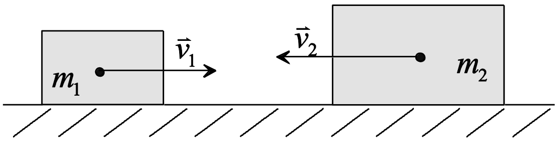
$$\vec{G}_1 = m\vec{v}_1 \quad \vec{G}_2 = m\vec{v}_2$$

možna posplošitev na več teles

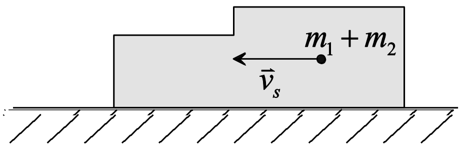
Zgled: neprožen trk dveh vozičkov na zračni blazini



pred trkom:



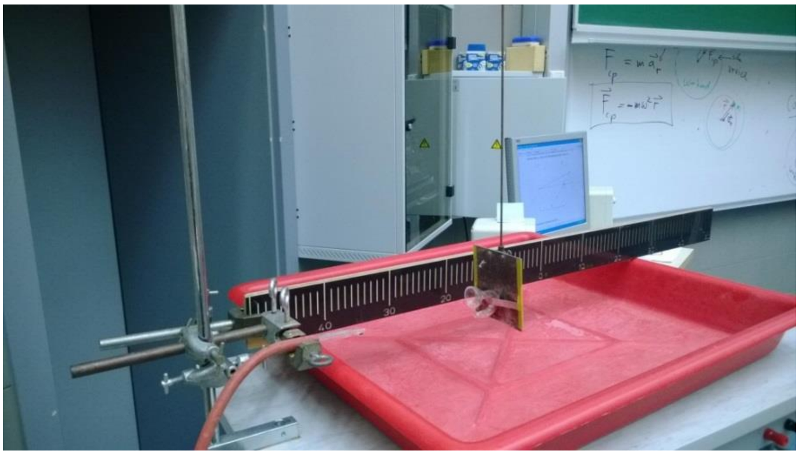
po trku:

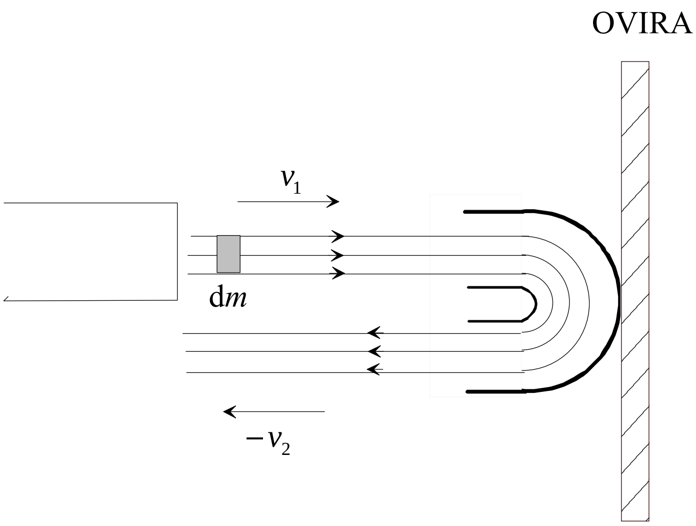


$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = \left(m_1 + m_2\right) \vec{v}_s$$

$$\vec{v}_s = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{\left(m_1 + m_2\right)}$$

Zgled: sila curka vode na oviro





$$F_0\,\mathrm{d}t = -\mathrm{d}m\,v_2 - \mathrm{d}m\,v_1$$

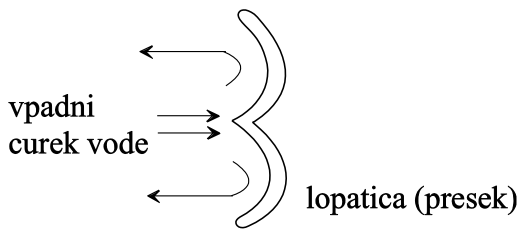
$F_0\,\mathrm{d}t$ = sunek sile ovire na maso dm

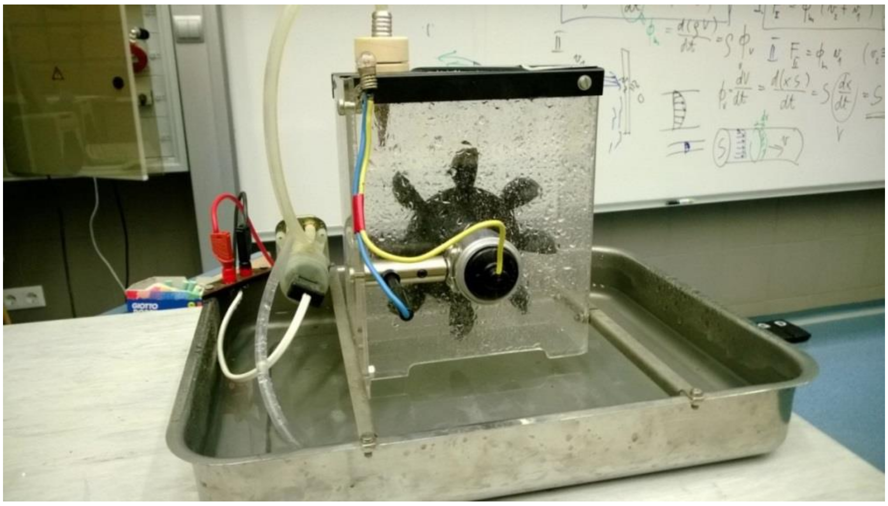
$$F_0 = -\Phi_m\left(v_1 + v_2\right) \qquad \Phi_m = \frac{\mathrm{d}m}{\mathrm{d}t}$$

$$F = -F_0 = \Phi_m\left(v_1 + v_2\right)$$

če $v_2 \approx 0$ je sila F minimalna !

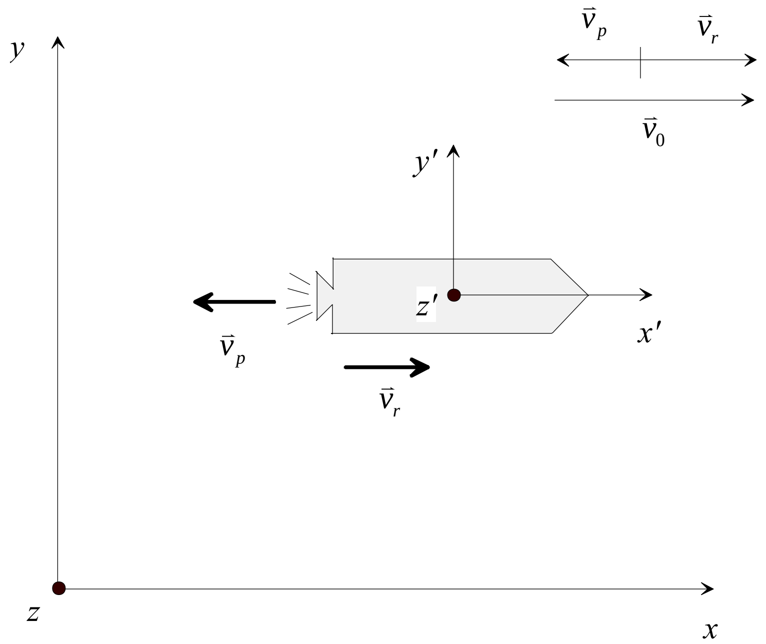
Zato :





Zgled: **vozilo na reaktivni pogon (raketa)**

vakuum, ni gravitacije :
$\vec{v}_r$ = hitrost rakete, $\vec{v}_p$ = hitrost iz rakete izhajajočih plinov



$$F_0\,\mathrm{d}t = -\mathrm{d}m\,v_p - \mathrm{d}m\,v_r$$

$F_0\,\mathrm{d}t$ = sunek sile na maso dm

$$F_0 = -\Phi_m\left(v_p + v_r\right)$$

masni tok plinov :

$$\Phi_m = \frac{\mathrm{d}m}{\mathrm{d}t}$$

$$F = -F_0 = \Phi_m\,v_0 \qquad v_0 = v_p + v_r$$

II. Newtonov zakon za gibanje težišča rakete v inercialnem sistemu (x, y, z) :

$$F = \Phi_m\,v_0 = m\frac{\mathrm{d}v}{\mathrm{d}t}$$

$$m = m_0 - \Phi_m t$$

m_p = celotna masa goriva enaka

raketa porabi vso gorivo ob času :

$$t_0 = \frac{m_p}{\Phi_m}$$

$$\Phi_m v_0 = \left(m_0 - \Phi_m t\right) \frac{\mathrm{d}v}{\mathrm{d}t}$$

$$\int_0^v \frac{\mathrm{d}v}{v_0} = \int_0^t \frac{\Phi_\mathrm{m}}{\left(m_0 - \Phi_m t\right)} \mathrm{dt}$$

$$v = v_0 \ln\left[\frac{m_0}{m_0 - \Phi_m t}\right]$$

maksimalno hitrost ob času t_0 (ko je porabljeno vso gorivo):

$$v_{\max} = v_0 \ln\left[\frac{m_0}{m_0 - \Phi_m t_0}\right] = v_0 \ln\left[\frac{1}{1 - \frac{\Phi_m t_0}{m_0}}\right]$$

PREPROSTI MODELI REAKCIJSKEGA MOTORJA

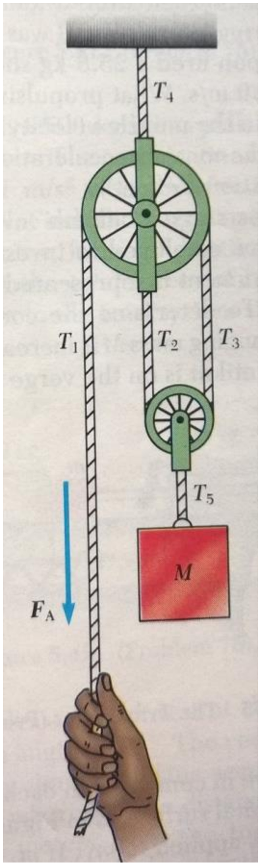
zračna proga: balonček na jahaču izpihuje zrak

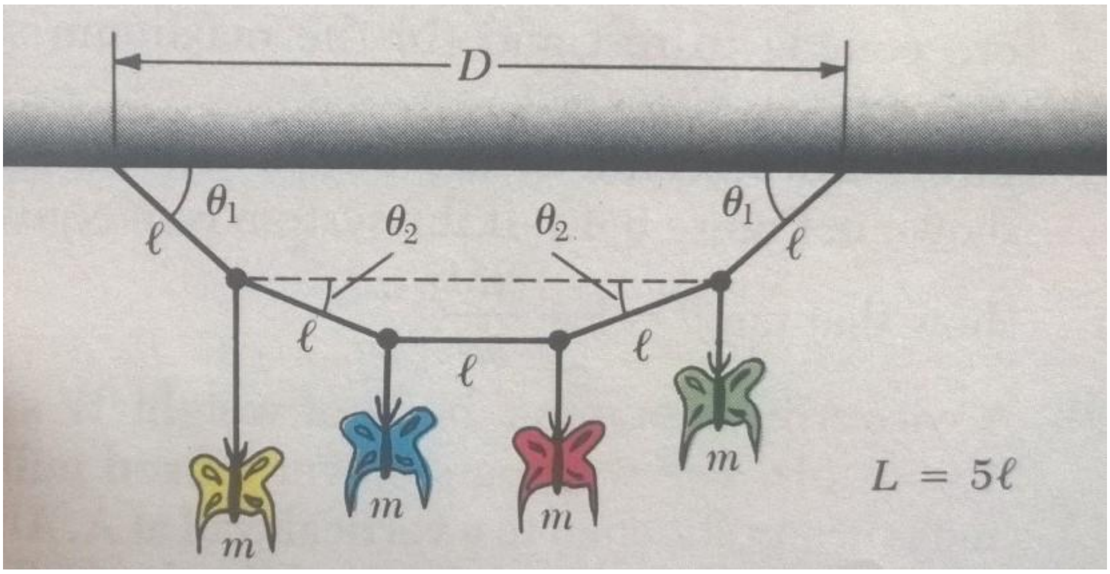
voziček na tiru (šibre)

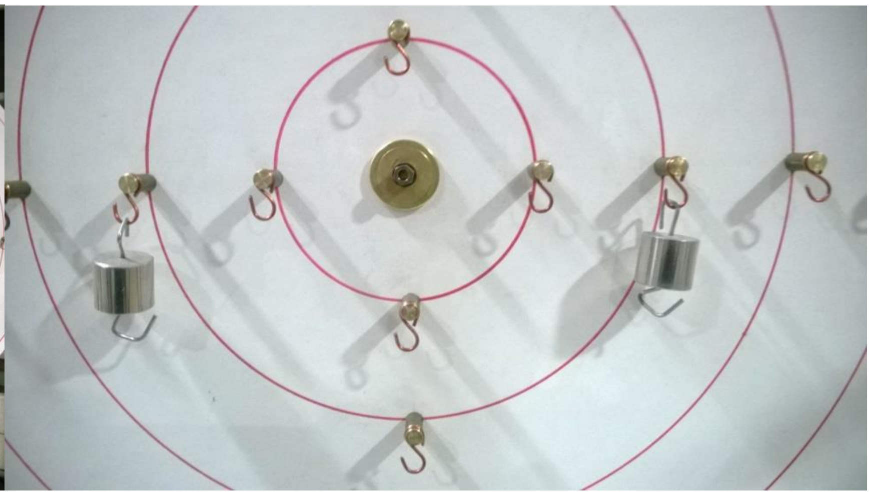
čolniček na vodi

MEHANSKO RAVNOVESJE

$$\sum_i \vec{F}_i = 0$$







NAVOR

$$\sum_i \vec{F}_i = 0 \qquad \sum_i \vec{M}_i = 0$$