Aleš Iglič

MEHANIKA

Kinematika

Kinematika (gr. kinematos = gibanje) se ukvarja z opisom gibanja teles po prostoru ne glede na vzroke gibanja.

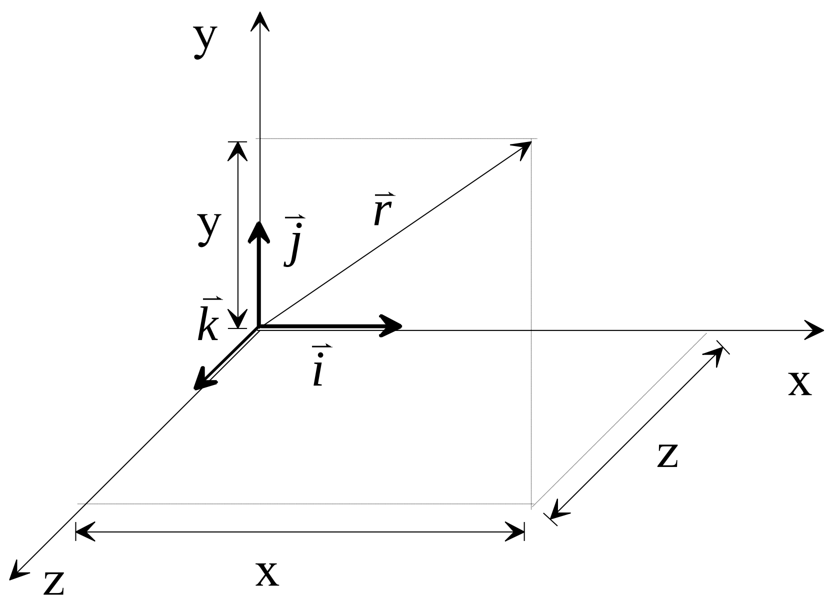
opazovalni sistem običajno vežemo na zemeljsko površje

PREDPOSTAVKA v okviru klasične fizike : čas je zvezen ter isti vsepovsod hkrati (**absoluten**)

Gibanje točkastega telesa (premiki telesa **veliko** večji kot razsežnost telesa)

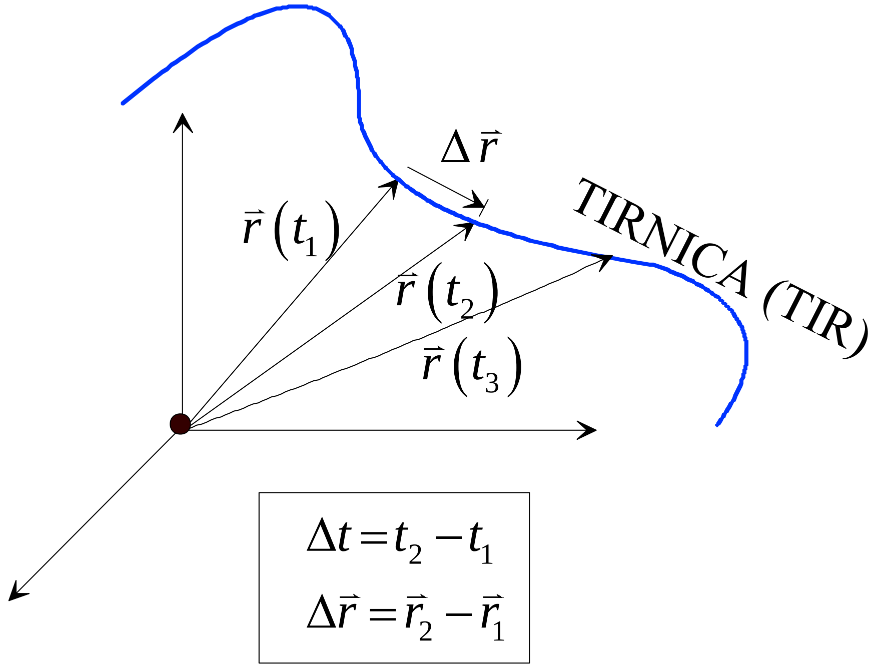
- **Lego točkastega telesa opišemo s krajevnim vektorjem** $\vec{r}\,(x, y, z)$ v kartezičnem koordinatnem sistemu (x, y, z), ki ima paroma pravokotne koordinatne osi x, y in z

$$\vec{r} = x\vec{i} + y\,\vec{j} + z\,\vec{k}$$



če se $\vec{r}$ spreminja s časom $\Rightarrow$ **gibanje**

- krivuljo, ki določa $\vec{r}(t)$ imenujemo **tirnica** ali **trajektorija**



$$\vec{r}(t) = \big(x(t),\ y(t),\ z(t)\big)$$

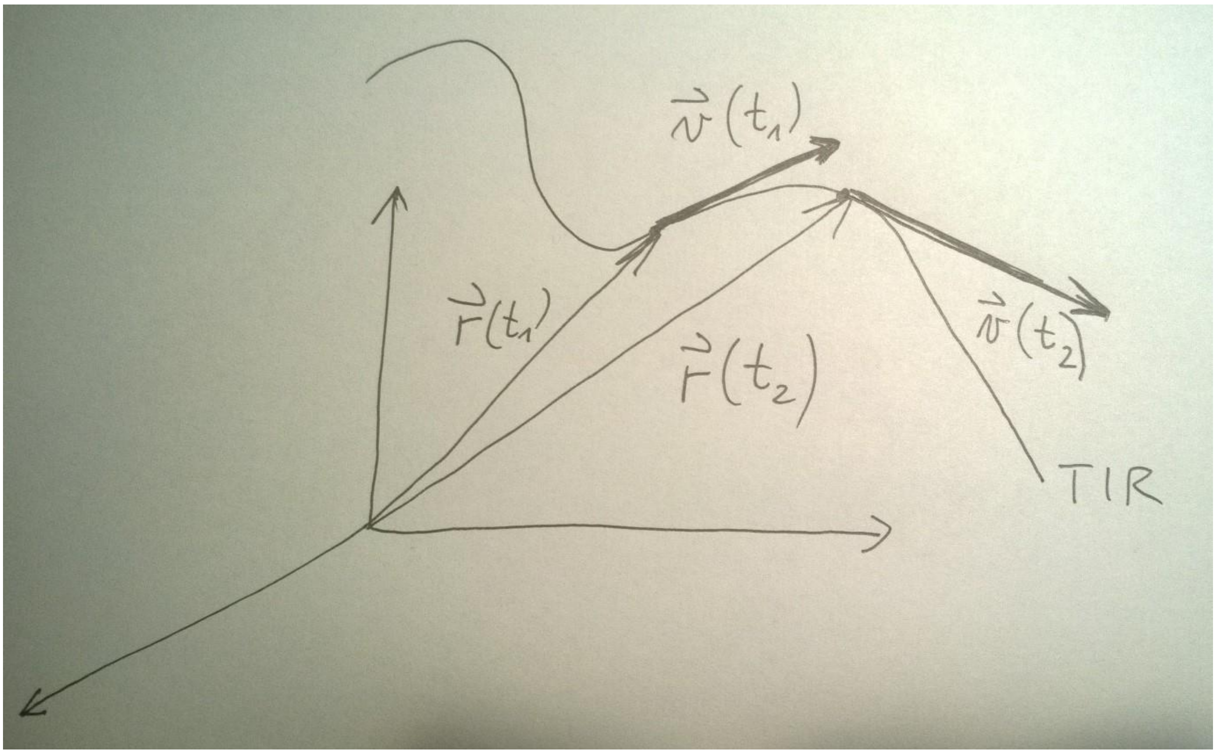
HITROST : kako hitro se lega delca spreminja s časom

vektor **hitrosti**

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = (v_x, v_y, v_z)$$

$$v = |\vec{v}| = \left(v_x^2 + v_y^2 + v_z^2\right)^{1/2}$$

POSPEŠEK : pove kako se s časom spreminja hitrost $\vec{v}(t)$



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \left(\frac{\mathrm{d}v_\mathrm{x}}{\mathrm{d}t}, \frac{\mathrm{d}v_\mathrm{y}}{\mathrm{d}t}, \frac{\mathrm{d}v_\mathrm{z}}{\mathrm{d}t} \right)$$

$$\vec{a} = \frac{\mathrm{d}}{\mathrm{d}t}(\vec{v}) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \right) = \left(\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} \right) = \left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}, \frac{\mathrm{d}^2y}{\mathrm{d}t^2}, \frac{\mathrm{d}^2z}{\mathrm{d}t^2} \right)$$

$$\vec{r}(t) = \left(x(t), y(t), z(t)\right)$$

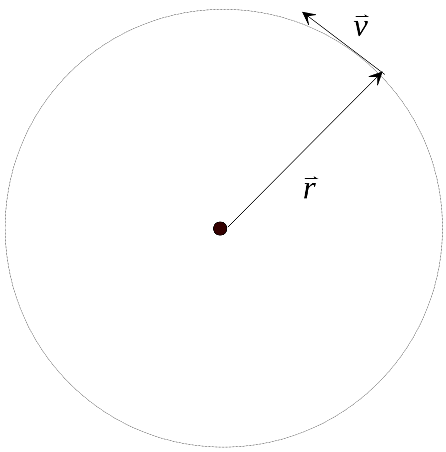
$$\vec{v}(t) = \left(v_x, v_{y,} v_z\right) = \left(\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}z}{\mathrm{d}t}\right)$$

$$\vec{a}(t) = \left(a_x, a_{y,} a_z\right) = \left(\frac{\mathrm{d}v_x}{\mathrm{d}t}, \frac{\mathrm{d}v_y}{\mathrm{d}t}, \frac{\mathrm{d}v_z}{\mathrm{d}t}\right)$$

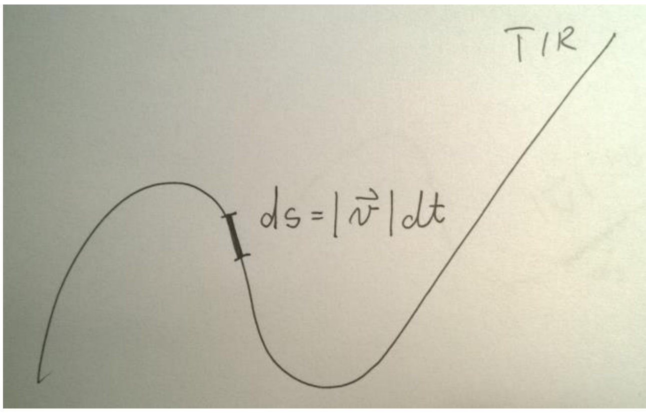
primer: enakomerno kroženje je pospešeno gibanje

$$|\vec{r}| = r = \text{konst.}$$

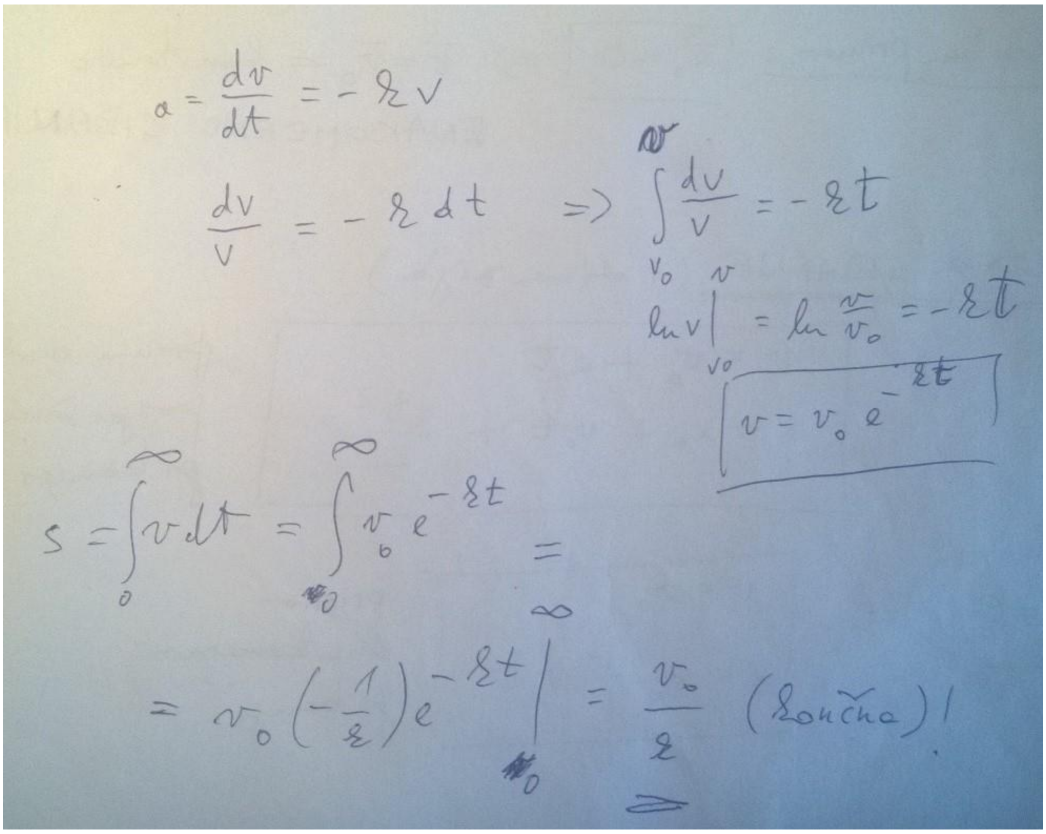
$$|\vec{v}| = v = \text{konst.}$$



Računanje poti: $$s = \int_{t_1}^{t_2} |\vec{v}| \, \mathrm{d}t$$



Primer:



Poseben primer: enakomerno pospešeno gibanje

$$\vec{a}(t) = \vec{a}_0 = \text{konstanta}$$

izberemo začetne pogoje: $\vec{v}_0 = \vec{v}(t=0)$

$\vec{r}_0 = \vec{r}(t=0)$

- $\vec{a}(t) \equiv \vec{a}_0 = \text{konstanta}$

izberemo začetne pogoje: $\vec{v}_0 = \vec{v}(t=0)$, $\vec{r}_0 = \vec{r}(t=0)$

$$\vec{a}_0 = \frac{d\vec{v}}{dt} \Rightarrow \int_{\vec{v}_0}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}_0\, dt$$

$$\boxed{\vec{v} = \vec{v}_0 + \vec{a}_0 t}$$

$\vec{v}_0 = \vec{v}(t=0)$

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow d\vec{r} = \vec{v}\, dt$$

$$\int_{\vec{r}_0}^{\vec{r}} d\vec{r} = \int_0^t (\vec{v}_0 + \vec{a}_0 t)\, dt$$

$\vec{r}_0 = \vec{r}(t=0)$

$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \vec{a}_0 \frac{t^2}{2}$$

$$\boxed{\vec{r} = \vec{r}_0 + \vec{v}_0 t + \vec{a}_0 \frac{t^2}{2}}$$

ZAPIS PO KOMPONENTAH:

$$\begin{cases} x = x_0 + v_{0x} t + a_{0x} \frac{t^2}{2} \\ y = y_0 + v_{0y} t + a_{0y} \frac{t^2}{2} \\ z = z_0 + v_{0z} t + a_{0z} \frac{t^2}{2} \end{cases}$$

poseben primer: $\boxed{\vec{a}_0 = 0} \Rightarrow \vec{v} = \vec{v}_0 = \text{konstanta}$

ENAKOMERNO GIBANJE

PREMO GIBANJE (1 dimenzija)

$$v = v_0 + a_0 t$$
$$x = x_0 + v_0 t + \frac{a_0 t^2}{2}$$

premo enakomerno pospešeno gibanje

$a = 0 \Rightarrow$

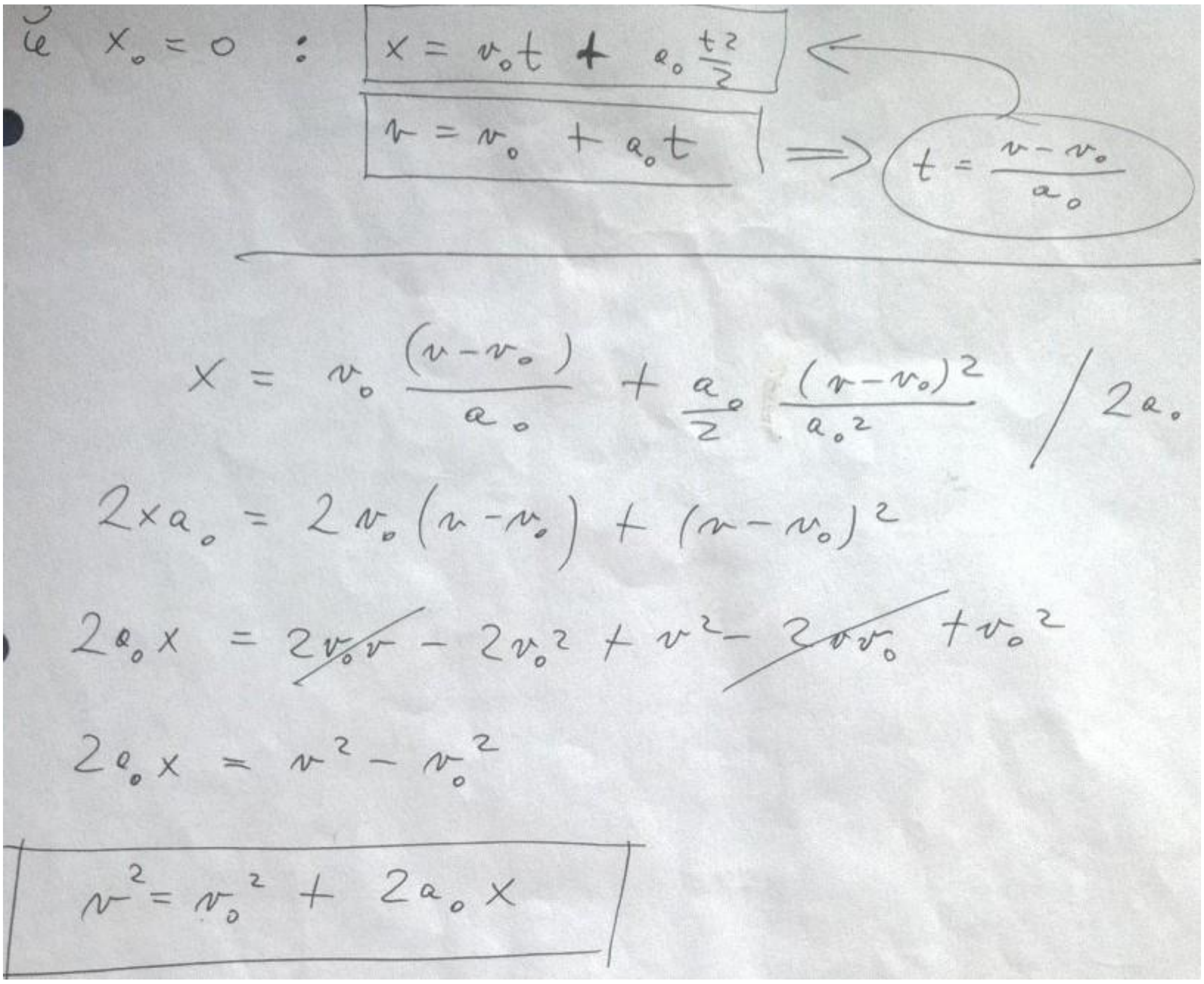
$$v = v_0$$
$$x = x_0 + v_0 t$$

premo enakomerno gibanje

Premo enakomerno pospešeno premo gibanje (gibanje po premici)

$$v = v_0 + a_0 t$$
$$x = x_0 + v_0 t + \frac{a_0 t^2}{2}$$

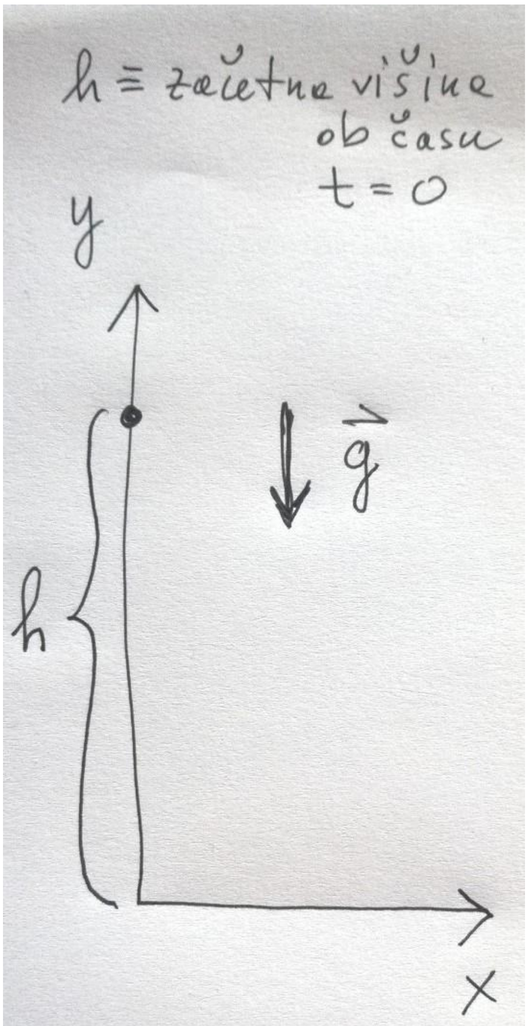
če $x_0 = 0$:



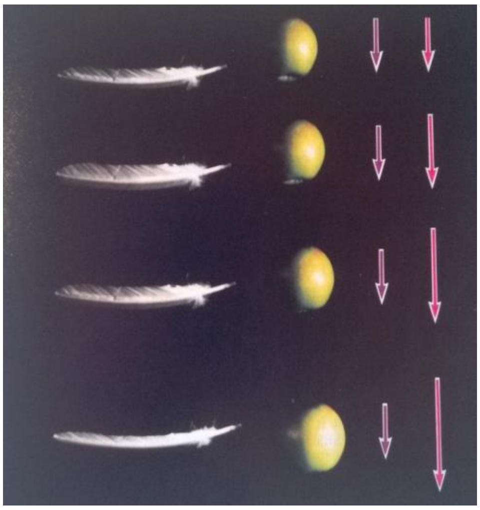
$$v^2 = v_0^2 + 2a_0\, x$$

Prosti pad (upor zraka zanemarimo)

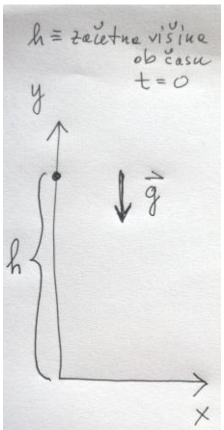
Evakuirna cev z gosjim peresom in rumeno kroglico

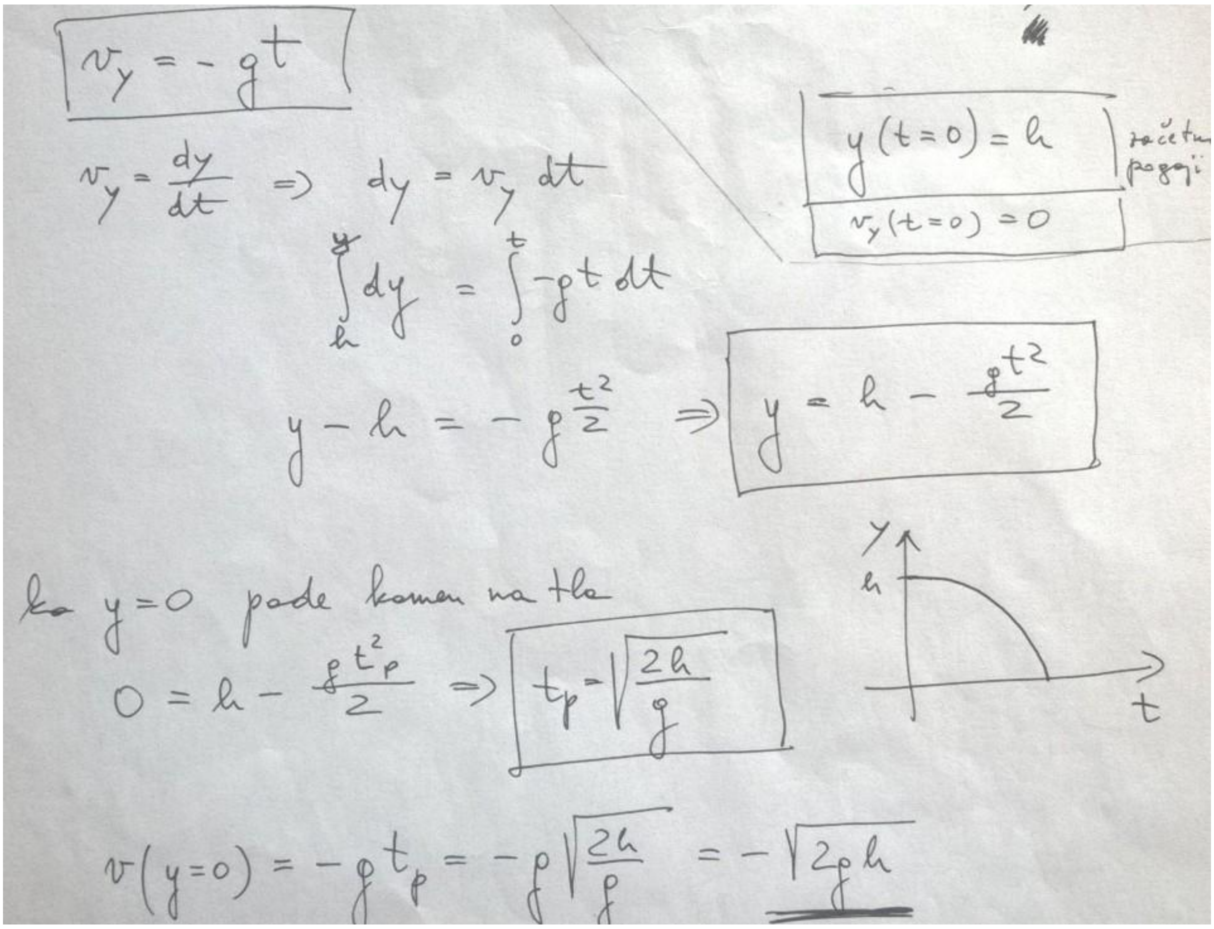


gravitacijski pospešek: $g \approx 9.8\,\mathrm{ms}^{-2}$



Spletna učilnica (Mehanika in termodinamika), **film**: Prosti_Pad_Krogla_Pero_primerjava



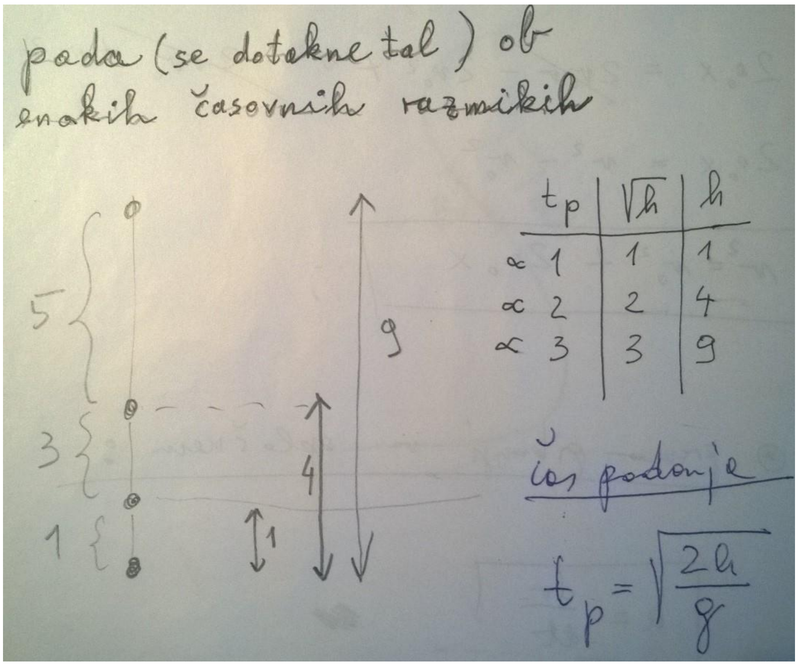


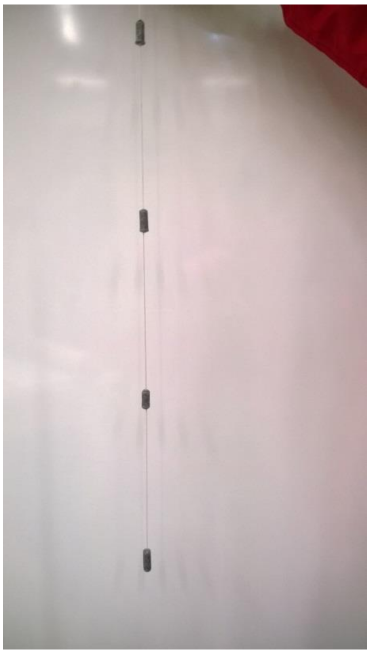
je hitrost pri tleh - tik preden se telo dotakne tal

PROSTI PAD v vakuumu :

$$t_p = \sqrt{\frac{2h}{g}}$$ **čas padanja**

prosti pad: kroglice na vrvici (test, da a = g = konst.)

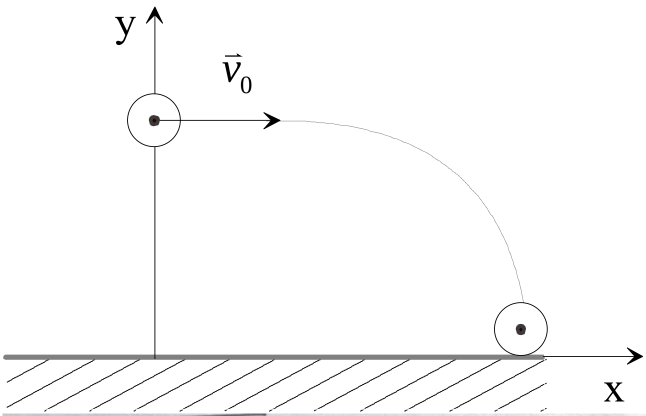


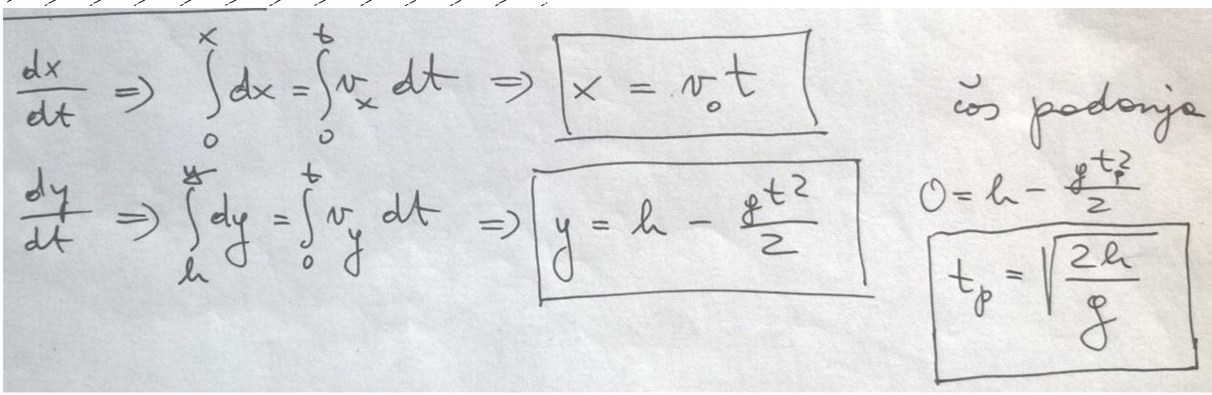


Vodoravni met (v vakuumu)

začetni pogoji:

$$v_y(t=0)=0$$
$$v_x(t=0)=v_0$$
$$y(t=0)=h$$



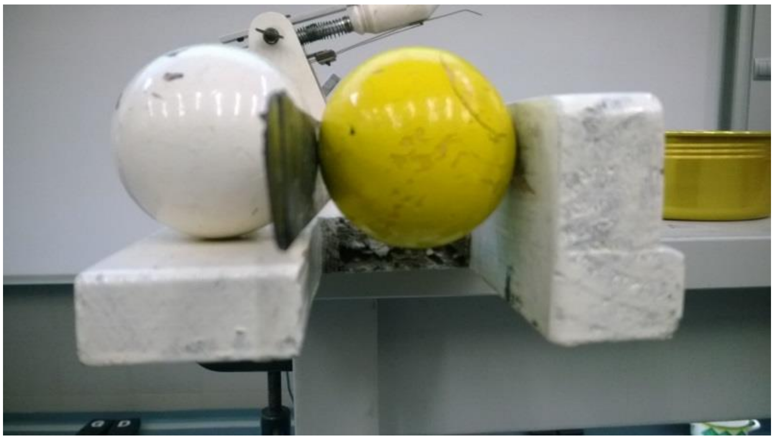


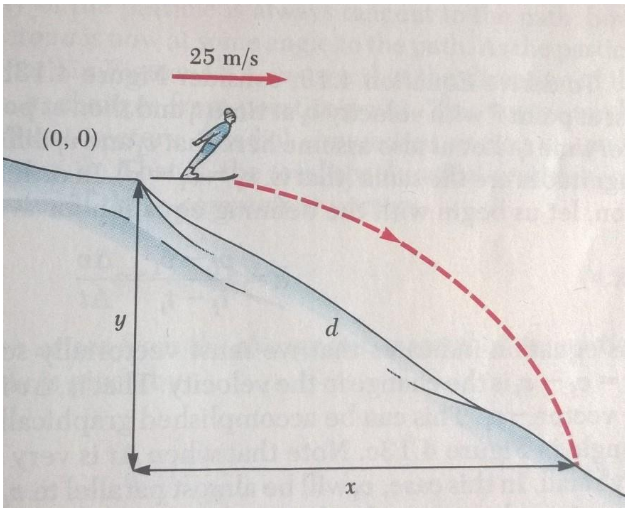
PROSTI PAD v vakuumu :

$$t_p = \sqrt{\frac{2h}{g}}$$

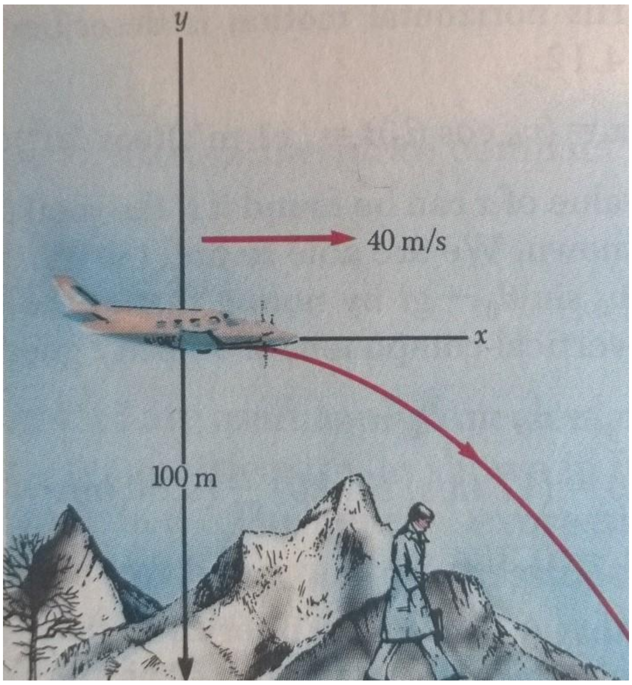
čas padanja

PROSTI PAD + VODORAVNI MET : primerjava padanja dveh kroglic



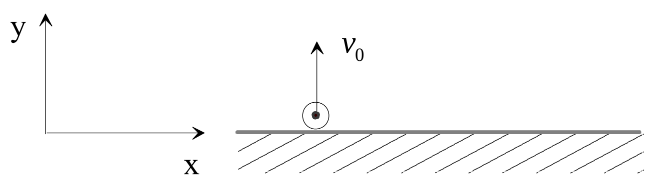


nagib klanca je 35°   VPR : x,y ?



VPR : x = ?

Navpični met



$$a_g = -g$$

začetni pogoji:

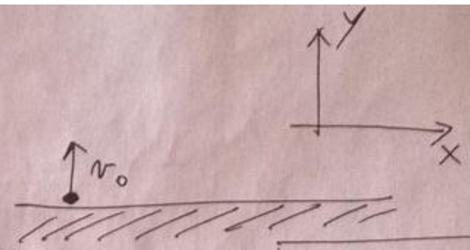
$$y(t=0)=0$$
$$v_y(t=0)=v_0$$

- navpični met

$$a_y = -g$$

$$\frac{dv_y}{dt} = -g\,, \quad \int_{v_0}^{v} dv_y = -\int_0^t g\,dt$$

$$v_y = v_0 - gt$$



začetni pogoji:

$$y(t=0) = 0$$

$$v_y(t=0) = v_0$$

$$v_y = \frac{dy}{dt} \Rightarrow \int_0^y dy = \int_0^t v_y\,dt$$

$$y = \int_0^t (v_0 - gt)\,dt = v_0 t - \frac{gt^2}{2}$$

$$y = v_0 t - \frac{gt^2}{2}$$

- najvišja točka je dosežena, ko je $v_y = 0$:

$$v_y = v_0 - g t_d = 0$$

$$t_d = \frac{v_0}{g}$$

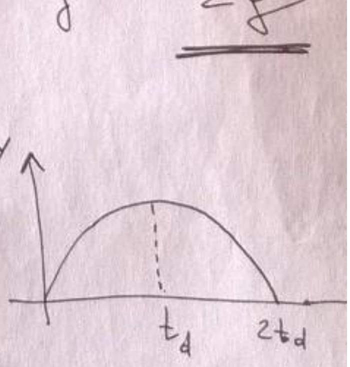
- pri tem doseže telo višino

$$h = y(t_d) = v_0 \frac{v_0}{g} - \frac{g}{2}\frac{v_0^2}{g^2} = \frac{v_0^2}{2g}$$

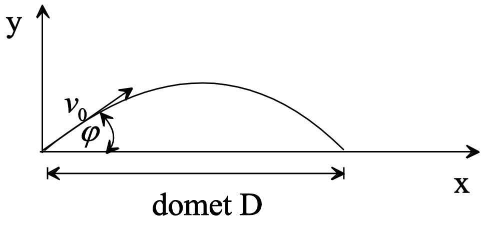
- Kdaj pade telo na tla?

$$y = 0 = v_0 t - \frac{gt^2}{2}$$

$$t = \frac{2v_0}{g} = 2t_d$$



Poševni met (v vakuumu)



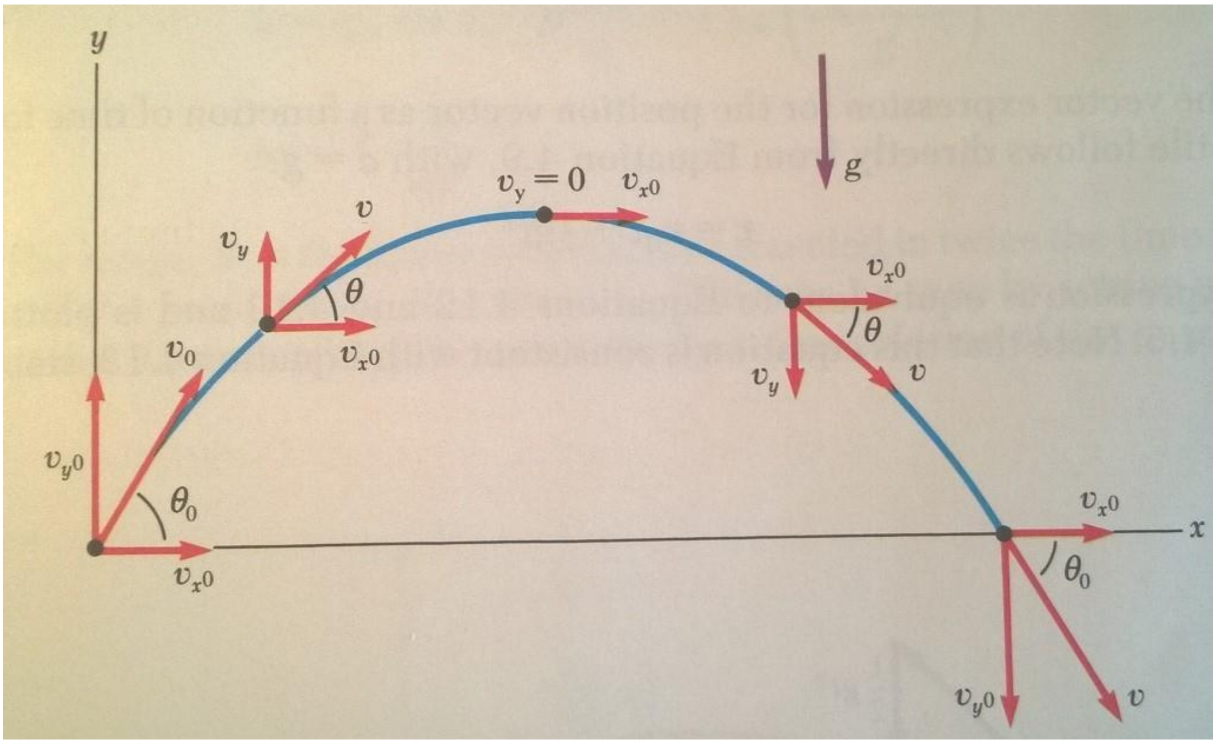
začetni pogoji:

$a_x = 0$, $v_x(t=0) = v_0 \cos\varphi$
$a_y = -g$, $v_y(t=0) = v_0 \sin\varphi$

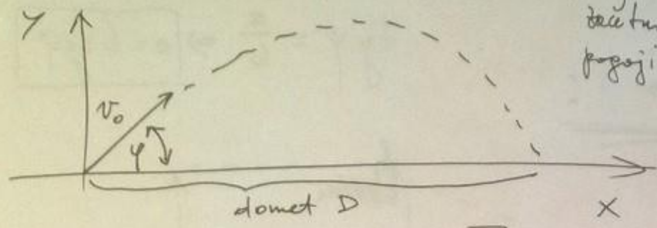
$x(t=0) = 0$
$y(t=0) = 0$

komponente vektorja hitrosti:

$$v_x = v_0 \cos\varphi$$
$$v_y = v_0 \sin\varphi - gt$$



poševni met



začetni pogoji

$$a_x = 0 \qquad a_y = -g$$

$$v_x(t=0) = v_0 \cos\varphi \qquad v_y(t=0) = v_0 \sin\varphi$$

$$x(t=0) = 0 \qquad y(t=0) = 0$$

$$\left[\begin{aligned} v_x &= v_0 \cos\varphi \\ v_y &= v_0 \sin\varphi - gt \end{aligned}\right]$$

$$\left[\begin{aligned} x &= v_0 \cos\varphi \, t \\ y &= v_0 \sin\varphi \, t - \frac{g t^2}{2} \end{aligned}\right]$$

$$x = v_0 \cos\varphi \, t \Rightarrow t = \frac{x}{v_0 \cos\varphi}$$

$$y = v_0 \sin\varphi \, t - \frac{g t^2}{2}$$

$$y = \frac{v_0 \sin\varphi \, x}{v_0 \cos\varphi} - \frac{g}{2} \frac{x^2}{v_0^2 \cos^2\varphi}$$

$$\boxed{y = x \cdot \mathrm{tg}\,\varphi - \frac{g}{2 v_0^2 \cos^2\varphi} x^2}$$

To je enačba parabole

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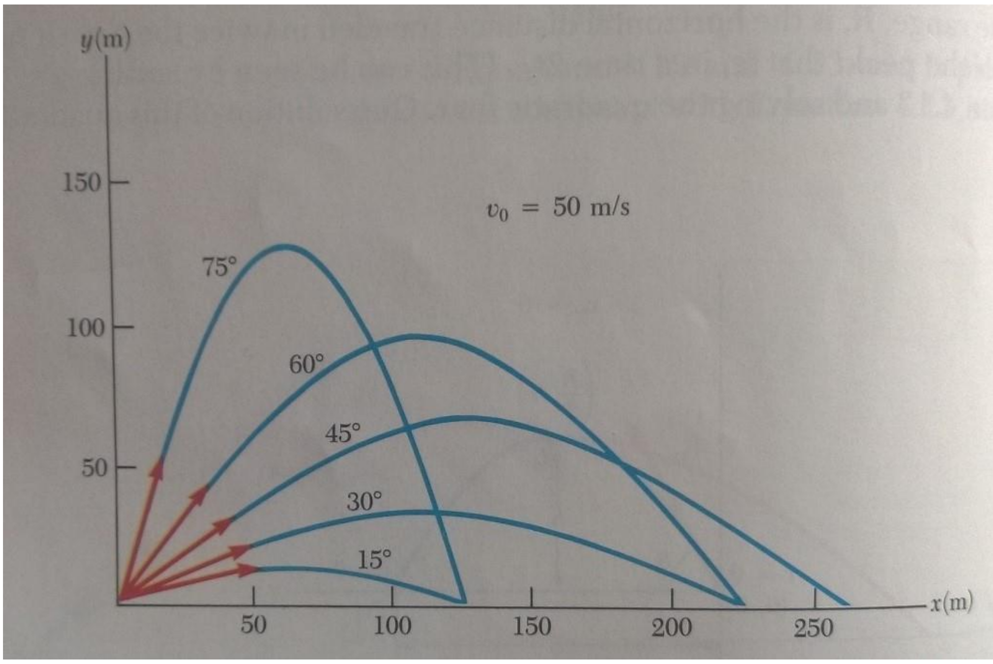
domet:

$$0 = x \,\mathrm{tg}\,\varphi - \frac{g}{2 v_0^2 \cos^2\varphi} x^2$$

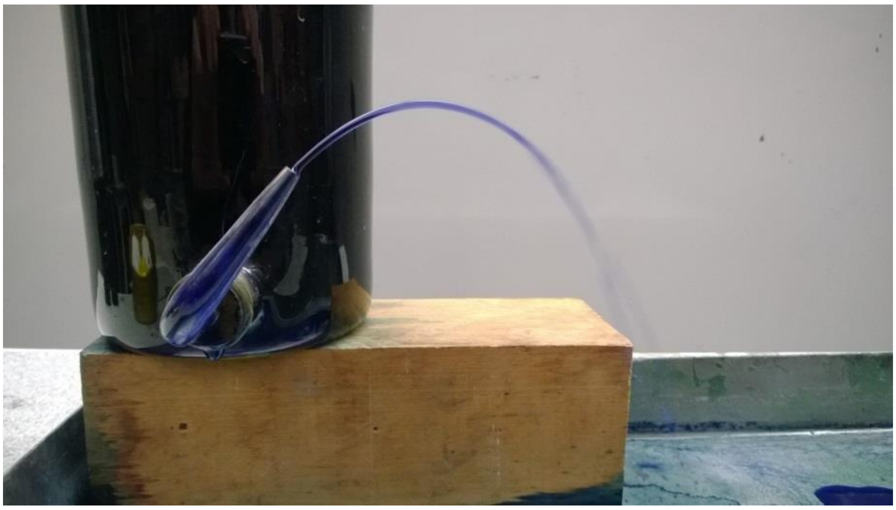
$$\mathrm{tg}\,\varphi = \frac{g}{2 v_0^2 \cos^2\varphi} x \Rightarrow x = \frac{2 v_0^2 \sin\varphi \cos\varphi}{g} = \frac{v_0^2}{g} \sin(2\varphi)$$

maksimalen domet, ko je $2\varphi = 90^\circ \Rightarrow \varphi = 45^\circ$

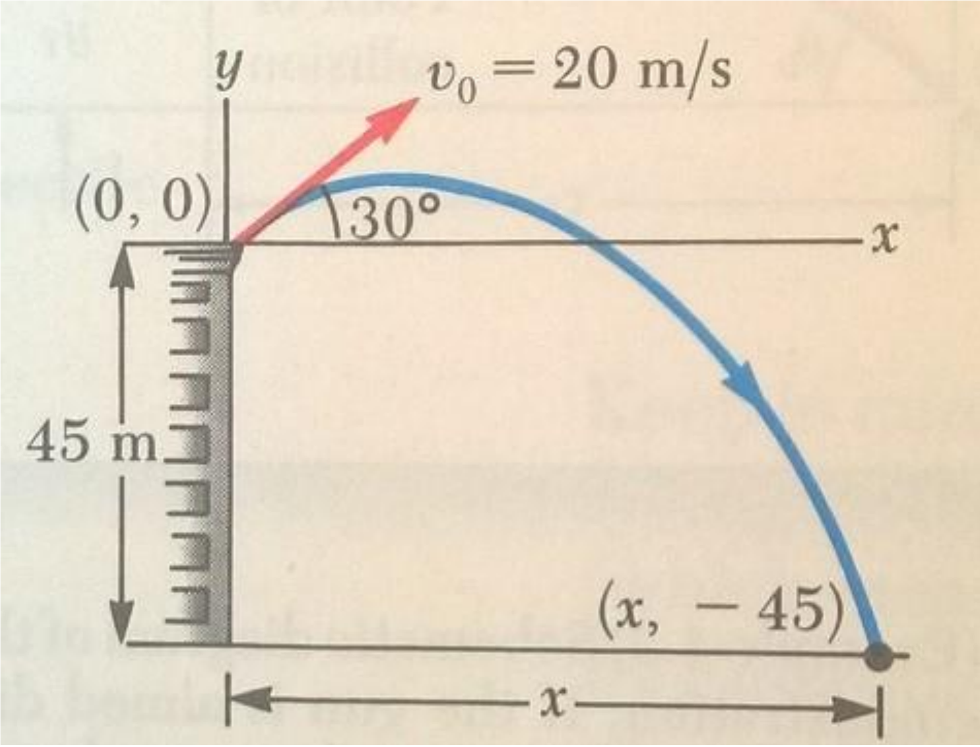
DOMET



curek vode z dna posode

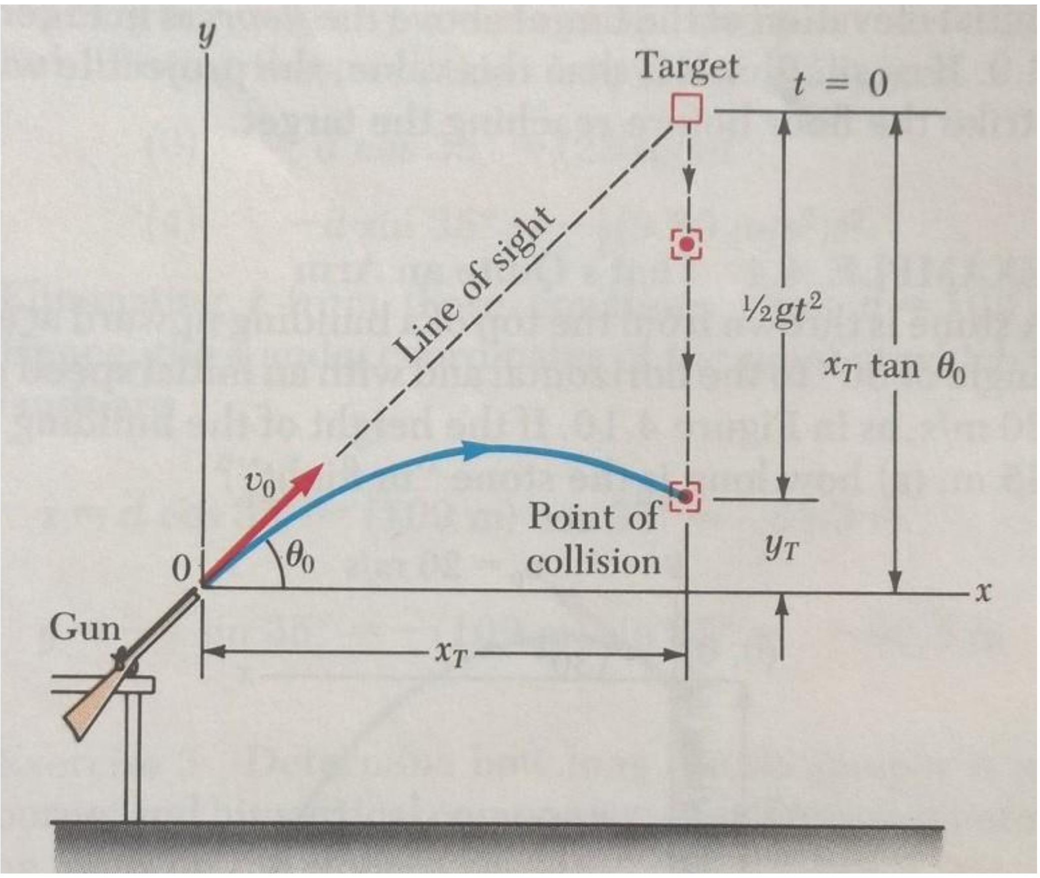


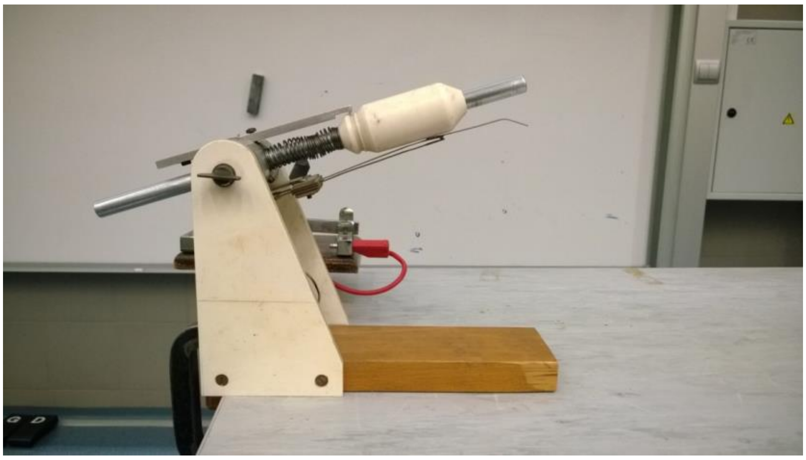
domet = ?



VPR : x = ?

POŠEVNI MET: streljanje v padajočo tarčo

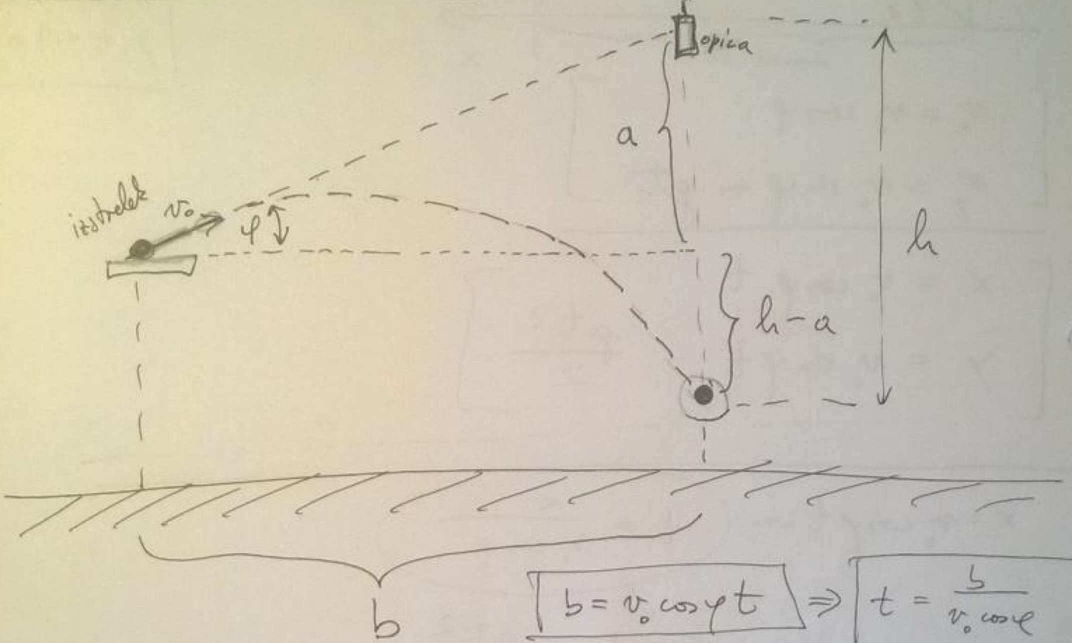




Zadetek vedno (ne glede na hitrost v_0)

↓ :

$$\mathrm{tg}\,\varphi = \frac{a}{b} \Rightarrow \boxed{a = b\,\mathrm{tg}\,\varphi}$$



$$\boxed{b = v_0\cos\varphi\, t} \Rightarrow \boxed{t = \frac{b}{v_0\cos\varphi}}$$

izstrelek: $y = x\,\mathrm{tg}\,\varphi - \dfrac{g}{2v_0^2\cos^2\varphi}x^2$

$|x \equiv b|$: $\boxed{y = b\,\mathrm{tg}\,\varphi - \dfrac{g}{2v_0^2\cos^2\varphi}b^2}$

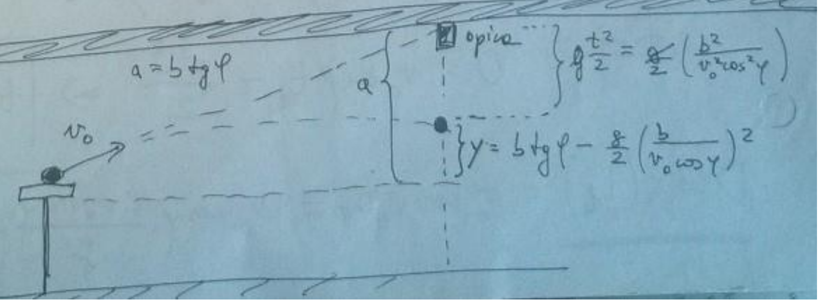
opica: $h = g\dfrac{t^2}{2} = \dfrac{g}{2}\left(\dfrac{b}{v_0\cos\varphi}\right)^2 = \dfrac{gb^2}{2v_0^2\cos^2\varphi}$

$$\boxed{h-a = \frac{gb^2}{2v_0^2\cos^2\varphi} - b\,\mathrm{tg}\,\varphi}$$

$$\Rightarrow |y| = h-a$$

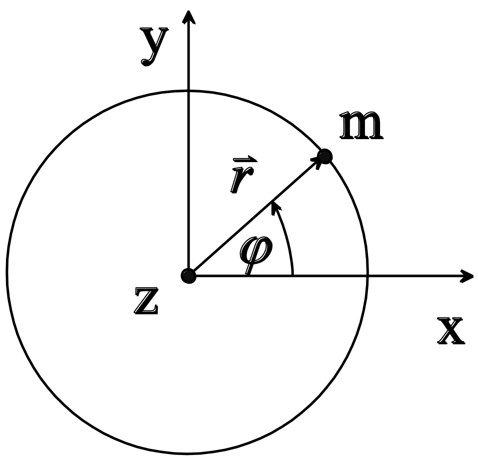
če večja hitrost v_0:

zadane opico že nad nivojem mize



Kroženje točkastega telesa

Obravnavamo gibanje po krožnici v ravnini x,y (z = 0):



$$x = r\cos\varphi$$
$$y = r\sin\varphi$$
$$z = 0$$

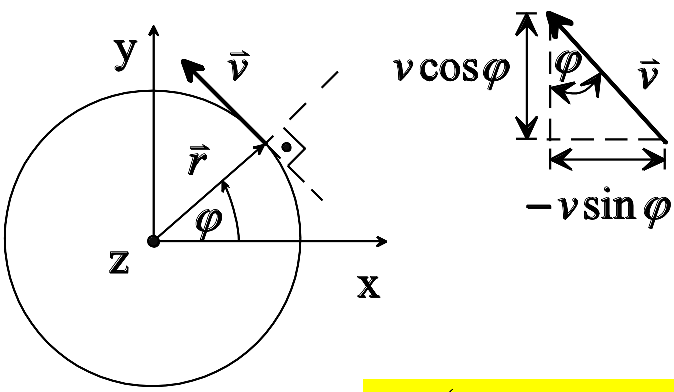
$$\varphi = \varphi(t)$$

$$\vec{r} = (r\cos\varphi, r\sin\varphi, 0)$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \left(-r\sin\varphi\frac{d\varphi}{dt},\ r\cos\varphi\frac{d\varphi}{dt}, 0\right)$$

kotna hitrost : $\omega = \dfrac{d\varphi}{dt}$

$$\vec{v} = \omega r(-\sin\varphi,\ \cos\varphi, 0)$$



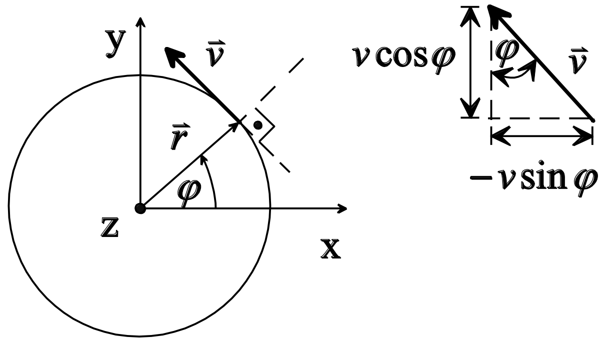
$$\vec{v} = v(-\sin\varphi,\ \cos\varphi, 0)$$

iz primerjave 2 enačb :

$$\vec{v}=\omega r\left(-\sin\varphi,\ \cos\varphi,0\right)$$

$$\vec{v}=v\left(-\sin\varphi,\ \cos\varphi,0\right)$$

sledi $\boxed{v=\omega r}$



enotni vektor v smeri hitrosti (t.j. v smeri tangente na tir) : $\vec{e}_t=\left(-\sin\varphi,\cos\varphi,0\right)$.

$$\left|\vec{e}_t\right|=e_t=\left(\vec{e}_t\cdot\vec{e}_t\right)^{\frac{1}{2}}=\left[\left(-\sin\varphi\right)^2+\cos^2\varphi\right]^{\frac{1}{2}}=1.$$

$$\vec{v}=\omega r\left(-\sin\varphi,\ \cos\varphi,0\right)=\omega r\,\vec{e}_t=v\vec{e}_t$$

- enotni vektor v smeri osi z : $\vec{e}_z=\left(0,0,1\right)$

- enotni vektor v smeri krajevnega vektorja $\vec{r}$:

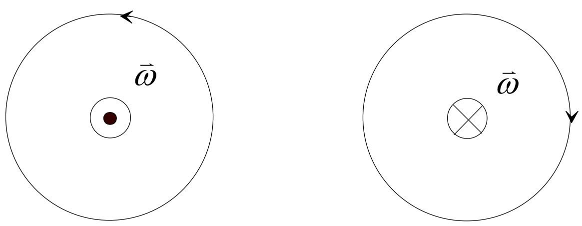
$\vec{e}_r=\left(\cos\varphi,\sin\varphi,0\right)$ torej $\vec{r}=r\,\vec{e}_r$

$e_z=\left(\vec{e}_z\cdot\vec{e}_z\right)^{\frac{1}{2}}=1$ $\qquad$ $e_r=\left(\vec{e}_r\cdot\vec{e}_r\right)^{\frac{1}{2}}=\left(\cos^2\varphi+\sin^2\varphi\right)^{\frac{1}{2}}=1.$

$$\vec{e}_t=\vec{e}_z\times\vec{e}_r$$

$$\vec{v}=\omega r\,\vec{e}_t=\omega r\left(\vec{e}_z\times\vec{e}_r\right)=\omega\vec{e}_z\times r\,\vec{e}_r=\vec{\omega}\times\vec{r}$$

smer vektorja $\vec{\omega}$ določimo po pravilu desnosučnega vijaka:



$$\boxed{\vec{v}=\vec{\omega}\times\vec{r}}$$

POSPEŠKI pri kroženju točkastega telesa

$\vec{v} = \vec{\omega} \times \vec{r}$,

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{\omega} \times \vec{r}\right) = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} \times \vec{r} + \vec{\omega} \times \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

definirali vektor **kotnega pospeška**:

$$\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t}$$

Ker $\vec{\omega} = (0, 0, \omega)$

$$\vec{\alpha} = \frac{\mathrm{d}}{\mathrm{d}t}(0, 0, \omega) = \left(0, 0, \frac{\mathrm{d}\omega}{\mathrm{d}t}\right) = (0, 0, \alpha)$$

od prej $\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

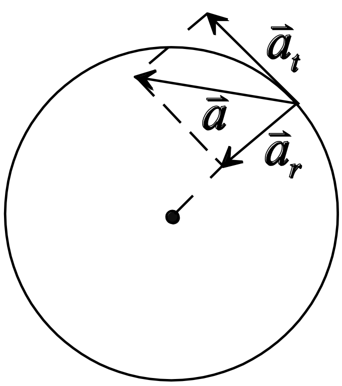
velja : $\vec{\omega} \times \vec{v} = \vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) = \vec{\omega}\left(\vec{\omega} \cdot \vec{r}\right) - \vec{r}\left(\vec{\omega} \cdot \vec{\omega}\right) = 0 - \omega^2 \vec{r}$

$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r} - \omega^2 \vec{r} = \vec{a}_t + \vec{a}_r}$$

kjer je

$$\vec{a}_r = -\omega^2 \vec{r}$$

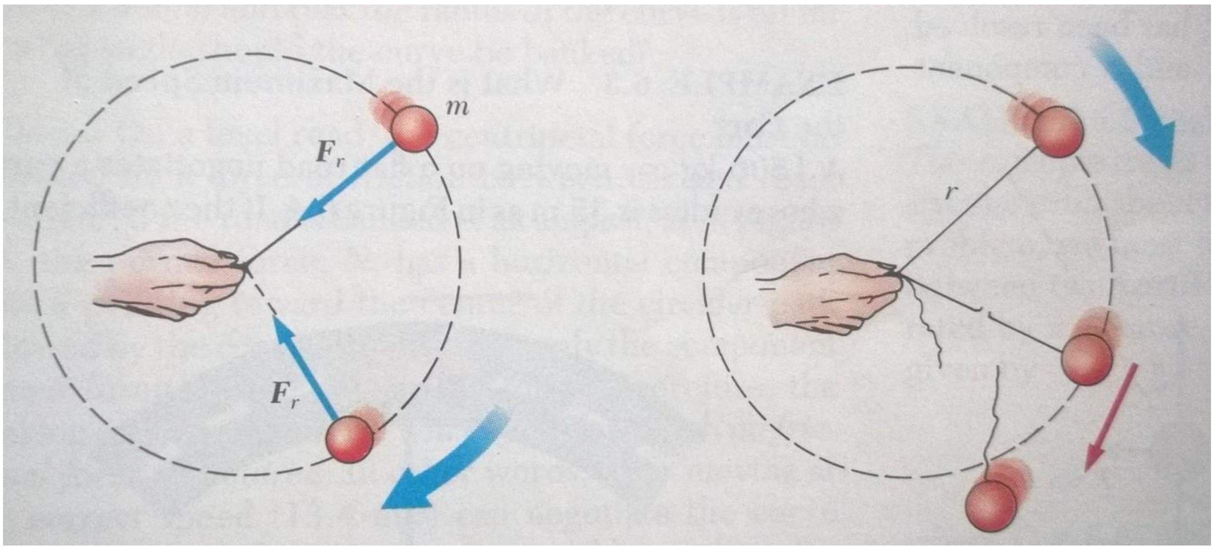
$$\vec{a}_t = \vec{\alpha} \times \vec{r} = \alpha \vec{e}_z \times r \vec{e}_r = \alpha r \left(\vec{e}_z \times \vec{e}_r\right) = \alpha r \vec{e}_t$$

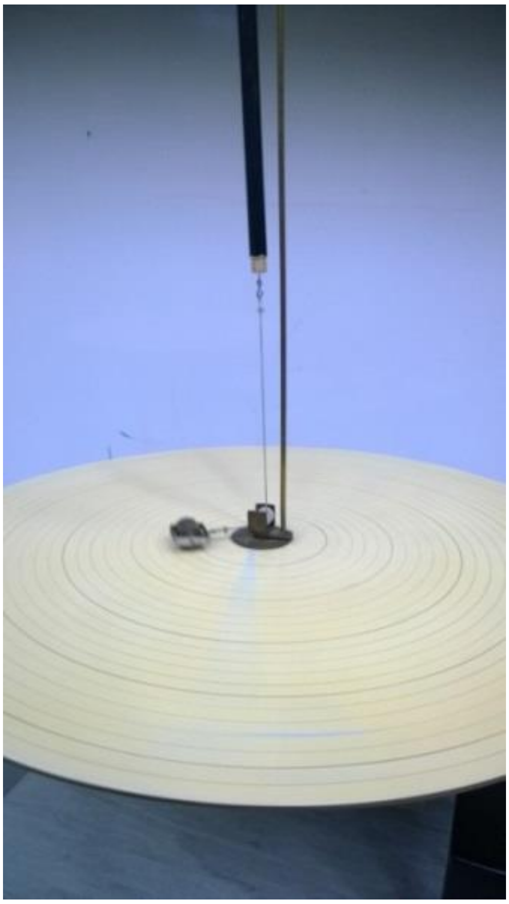


velikost celotnega pospeška

$$a=(\vec{a}\cdot\vec{a})^{\frac{1}{2}}=\left(a_t^2+a_r^2\right)^{\frac{1}{2}}=\left(\alpha^2 r^2+\omega^4 r^2\right)^{\frac{1}{2}}$$

Radialni pospešek je različen od nič tudi v primeru, ko je $v=|\vec{v}|=$ konst.





Poseben primer: **enakomerno pospešeno kroženje**

$$\alpha = \alpha_0 = \text{konst.}$$

$$\alpha_0 = \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

$$\mathrm{d}\omega = \alpha_0 \, \mathrm{d}t$$

$$\int_{\omega_0}^{\omega} \mathrm{d}\omega = \int_0^t \alpha_0 \, \mathrm{d}t \qquad \omega_0 = \omega(t=0)$$

$$\omega - \omega_0 = \alpha_0 \, t$$

$$\boxed{\omega = \omega_0 + \alpha_0 \, t}$$

$$\omega = \frac{\mathrm{d}\varphi}{\mathrm{d}t}$$

$$\mathrm{d}\varphi = \omega \, \mathrm{d}t$$

$$\int_{\varphi_0}^{\varphi} \mathrm{d}\varphi = \int_0^t (\omega_0 + \alpha_0 \, t) \mathrm{d}t \qquad \varphi_0 = \varphi(t=0)$$

$$\varphi - \varphi_0 = \omega_0 \, t + \alpha_0 \frac{t^2}{2}$$

$$\boxed{\varphi = \omega_0 \, t + \alpha_0 \frac{t^2}{2}}$$

kjer smo postavili/izbrali $\varphi_0 = 0$

Poseben primer**: enakomerno kroženje**

$$\alpha = 0$$

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = 0 \quad \Rightarrow \quad \boxed{\omega = \omega_0 = \text{konst.}}$$

$$\omega = \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \omega_0$$

$$\mathrm{d}\varphi = \omega_0 \,\mathrm{d}t$$

$$\int_{\varphi_0}^{\varphi} \mathrm{d}\varphi = \int_0^t \omega_0 \,\mathrm{d}t$$

$$\varphi - \varphi_0 = \omega_0 \, t \qquad \varphi_0 = \varphi(t=0)$$

$$\boxed{\varphi = \omega_0 \, t}$$

kjer smo izbrali $\varphi_0 = 0$

čas enega obhoda (obhodni čas):

$$t_0 = \frac{2\pi r}{v_0} = \frac{2\pi r}{\omega_0 r} = \frac{2\pi}{\omega_0}$$

od koder sledi:

$$\omega_0 = 2\pi \frac{1}{t_0} = 2\pi \upsilon_0$$

kjer smo definirali frekvenco

$$\boxed{\upsilon_0 = \frac{1}{t_0}}$$