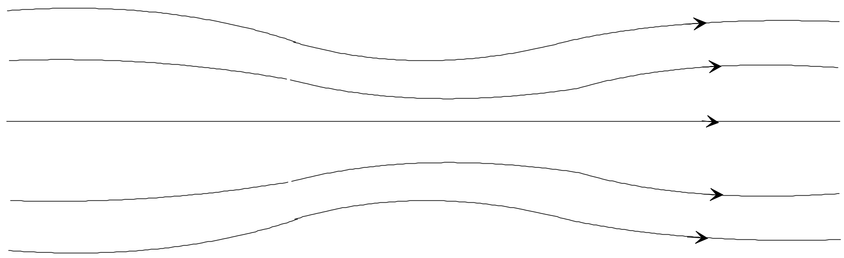
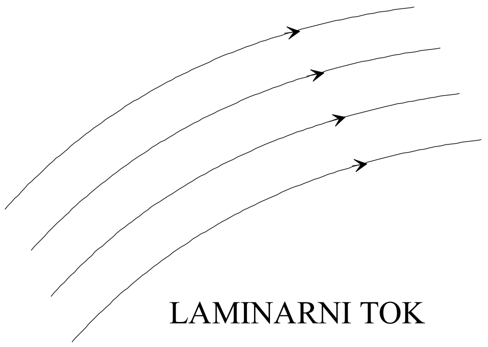
Pretakanje tekočin

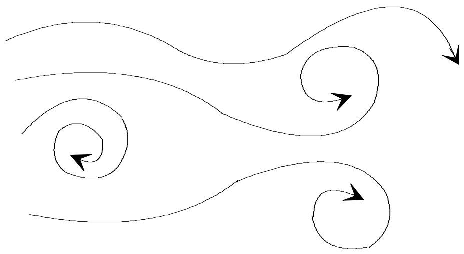
Tokovnica je krivulja, ki jo narišemo na trenutni sliki toka, če na primer s kratkim ekspozicijskim časom slikamo tok vode z aluminijastim prahom



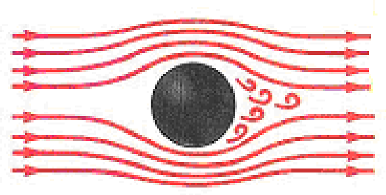
tangente na tokovnico kažejo v smer gibanja delčkov tekočine

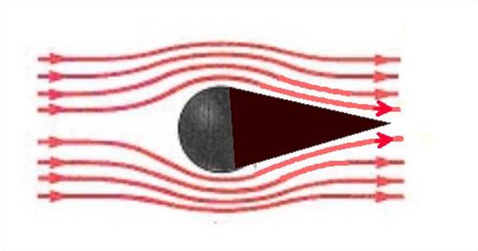


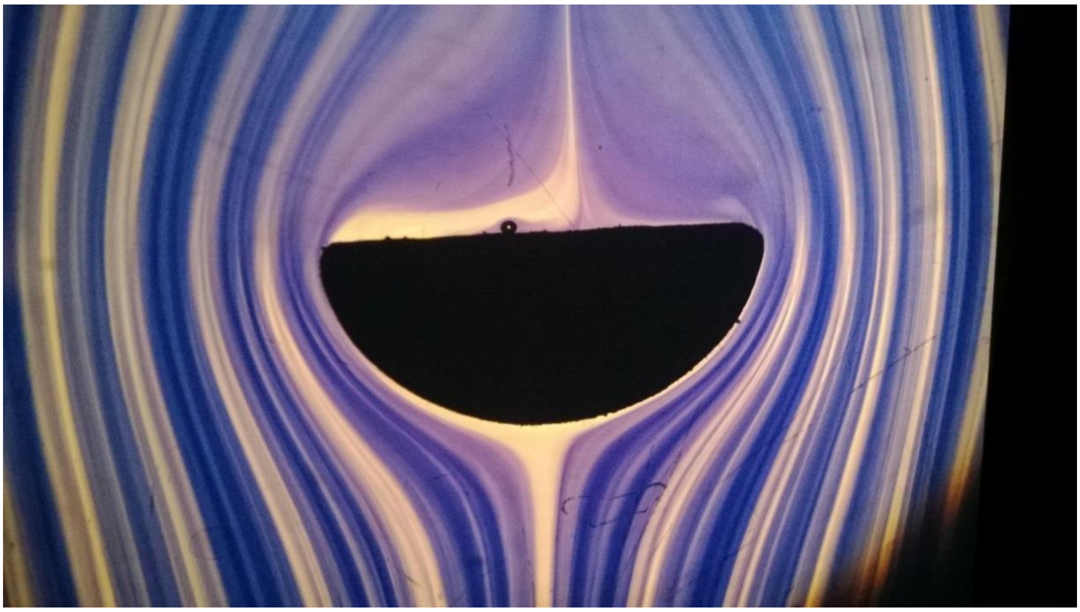
LAMINARNI TOK



TURBULENTNI TOK



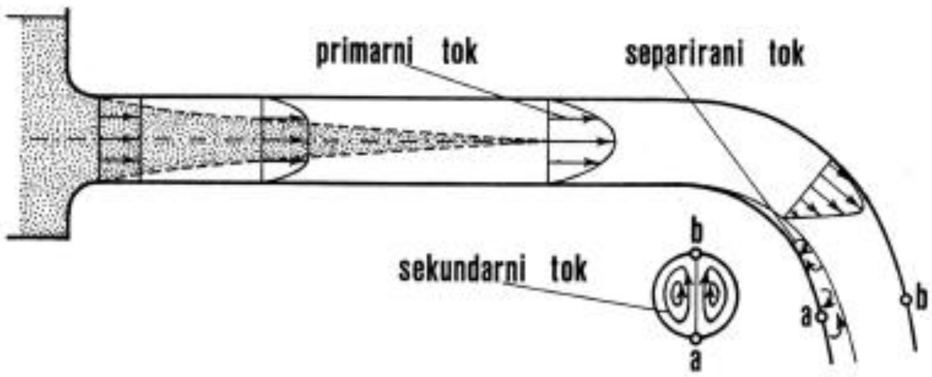




TURBULENTNI TOK PO CEVI

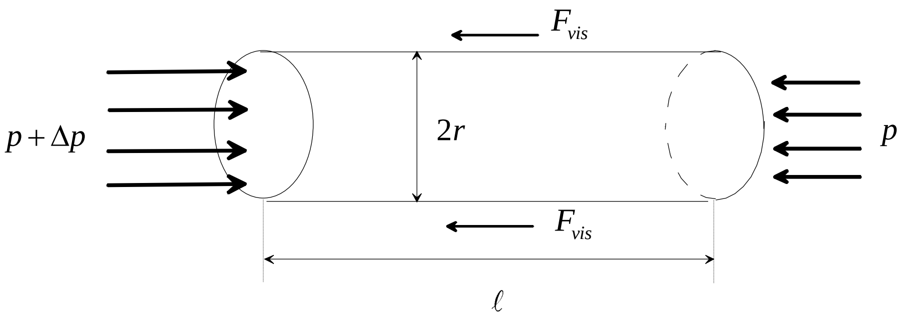
Pretakanje viskozne tekočine po cevi

primer: tekočina ima ob vstopu v cev enakomeren hitrostni profil :



Poiseuille – Hagenov zakon

profil hitrosti pri laminarnem **razvitem** toku kapljevine skozi dolgo in tanko valjasto cev



$$\left[\left(p+\Delta p\right)-p\right]\pi r^2=\Delta p\,\pi r^2$$

$$\frac{F_{vis}}{S}=-\eta\frac{\mathrm{d}v}{\mathrm{d}r} \qquad S=2\pi r\,\ell \qquad F_{vis}=-\eta\left(2\pi r\,\ell\right)\frac{\mathrm{d}v}{\mathrm{d}r}$$

$$\pi r^2\,\Delta p=-\eta\left(2\pi r\ell\right)\frac{\mathrm{d}v}{\mathrm{d}r}$$

$$\pi r^2 \Delta p = -\eta (2\pi r \ell) \frac{\mathrm{d}v}{\mathrm{d}r}$$

$$\frac{\Delta p}{\eta 2\ell} \int_r^R r\,\mathrm{d}r = -\int_v^0 \mathrm{d}v \qquad \frac{\Delta p}{\eta 2\ell} \frac{r^2}{2} \Big|_r^R = v$$

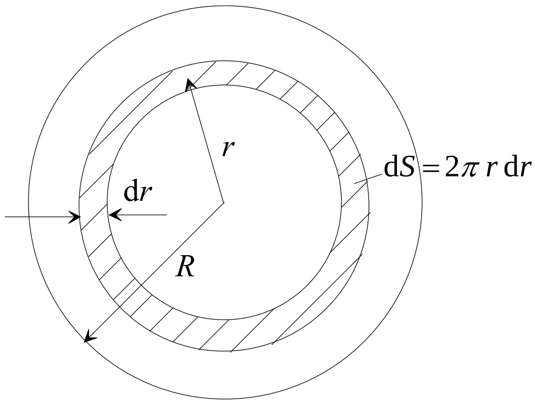
$$v = v_0 \left(1 - \frac{r^2}{R^2}\right) \qquad v_0 = \frac{\Delta p R^2}{4\eta \ell}$$

konstanten profil hitrosti : $\Phi_v = \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}(Sx)}{\mathrm{d}t} = Sv$

paraboličen profil : $\Phi_v = \int \mathrm{d}\Phi_v = \int v(r)\,\mathrm{d}S$

$$\Phi_v = \int \mathrm{d}\Phi_v = \int v(r)\,\mathrm{d}S = \int_0^R v(r)\, 2\pi r \mathrm{d}r = \int_0^R v_0 \left(1 - \frac{r^2}{R^2}\right) 2\pi r \mathrm{d}r = \frac{1}{2} v_0 \pi R^2$$

$\mathrm{d}S = 2\pi r\,\mathrm{d}r$



$$\Phi_v = \frac{1}{2} v_0 \pi R^2$$

$$v_0 = \frac{\Delta p R^2}{4\eta \ell}$$

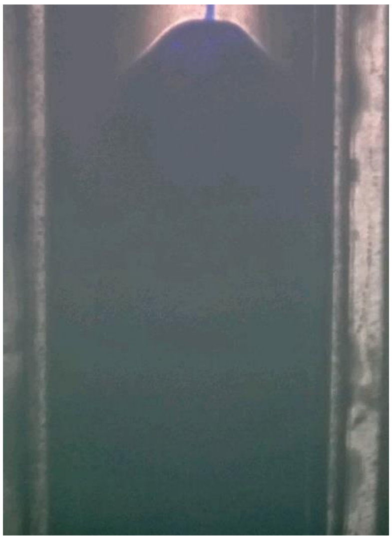
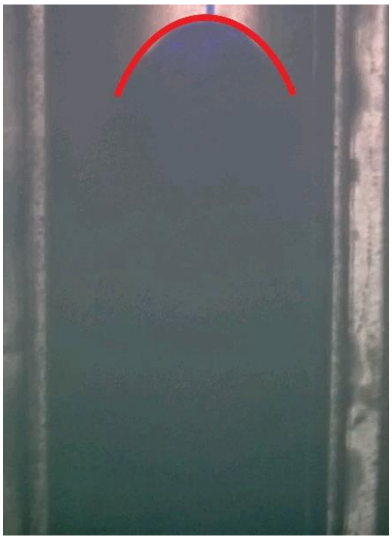
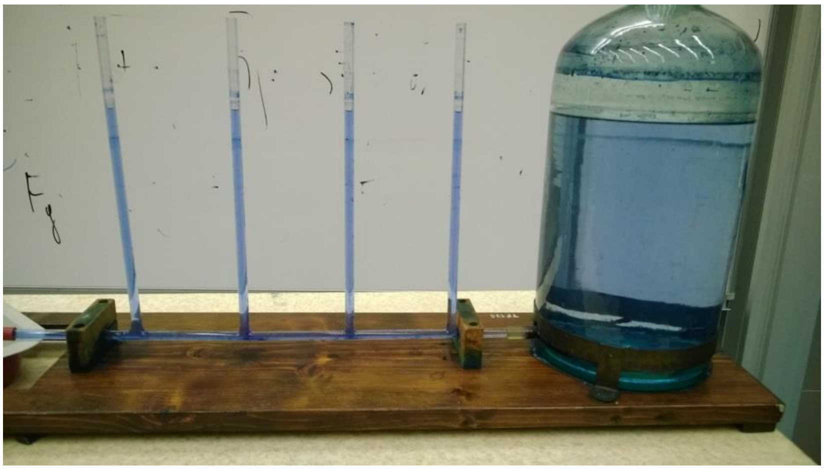
Poiseuille – Hagenov zakon :

$$\Phi_V = \frac{\pi R^4}{8\eta \ell} \Delta p$$

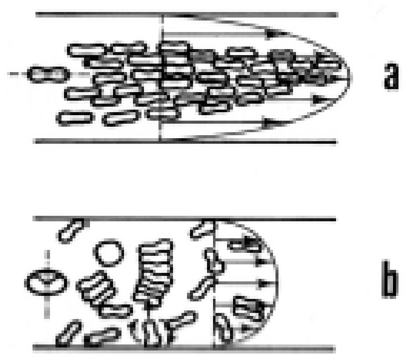
»Elektrotehniški zapis« ($I = \frac{U}{R}$) :

$$\Phi_V = \frac{\Delta p}{R_V}$$

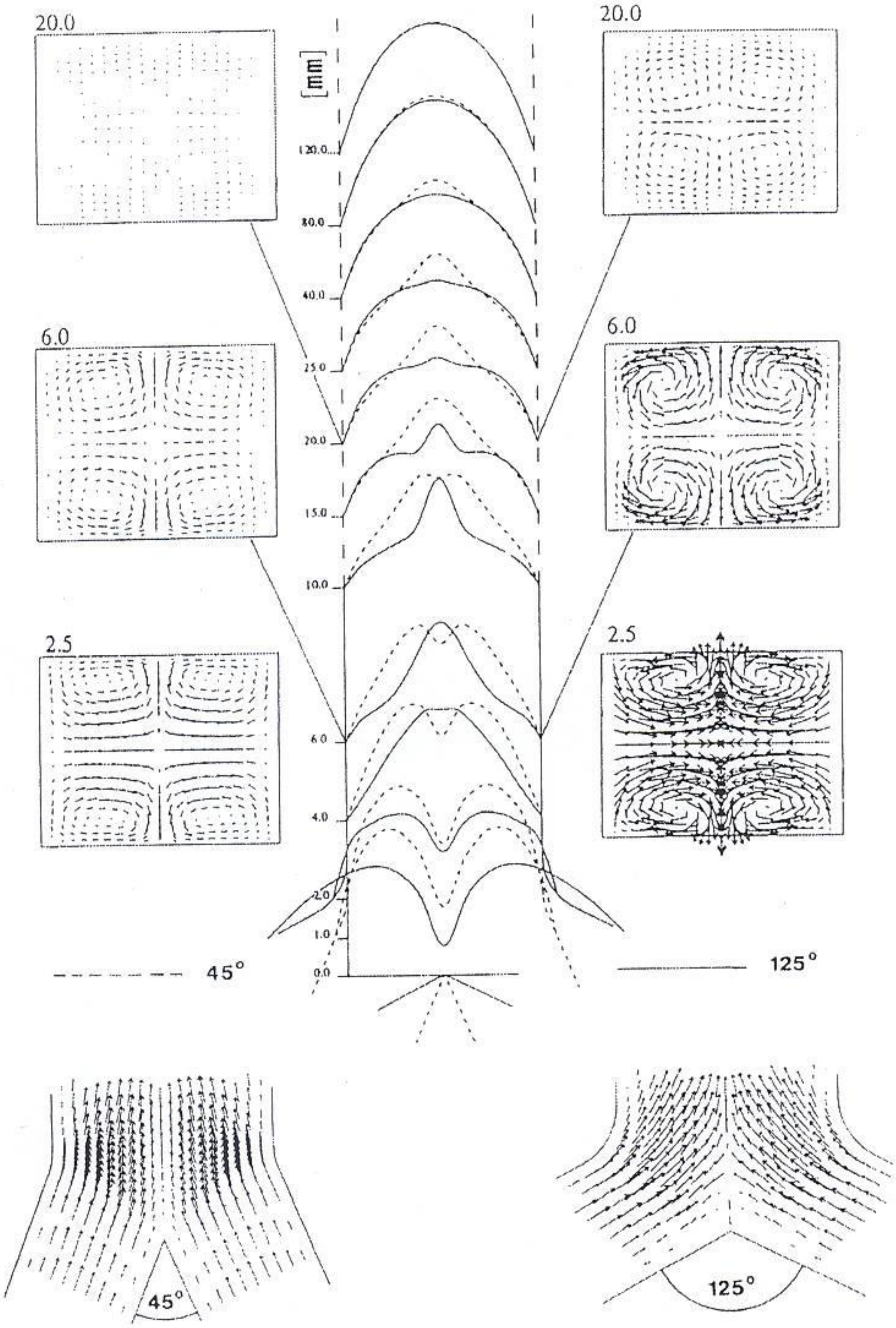
$$R_V = \frac{8\eta \ell}{\pi R^4}$$


Primer: Pretakanje krvi po žilah



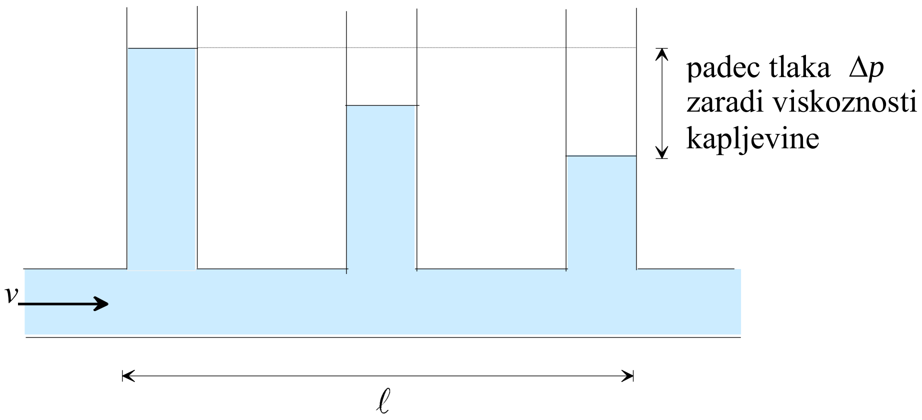
Gibanje eritrocitov v žili pri velikih hitrostih krvi (a) in pri majhnih hitrostih krvi (b



FEM (Ravensbergen in sod., J. Biomechanics, 29:281,1996)

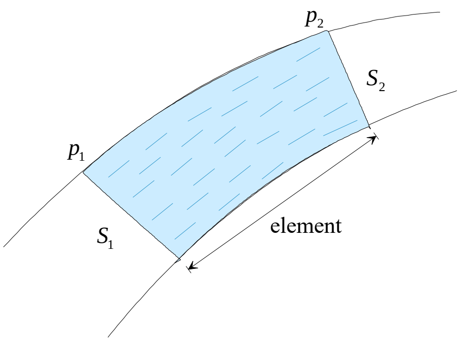
Bernoullijeva enačba (viskozne sile zanemarimo)

Poiseuille – Hagenov zakon (padec tlaka Δp je večji za bolj viskozne tekočine)

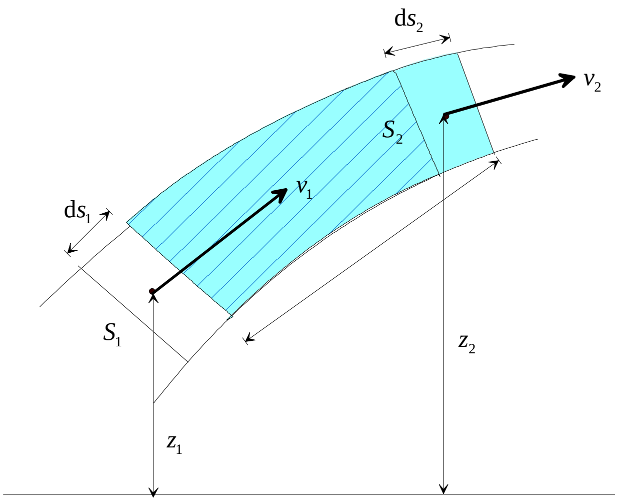


limita : $\eta \rightarrow 0 \qquad \Delta p \rightarrow 0$

$$A = p_1 S_1 \,\mathrm{d}s_1 - p_2 S_2 \,\mathrm{d}s_2 = \frac{m_2 v_2^2}{2} - \frac{m_1 v_1^2}{2} + m_2 g z_2 - m_1 g z_1$$



$$A = p_1 S_1 \mathrm{d}s_1 - p_2 S_2 \mathrm{d}s_2 = \frac{m_2 v_2^2}{2} - \frac{m_1 v_1^2}{2} + m_2 g z_2 - m_1 g z_1$$



nestisljivost : $S_2 \mathrm{d}s_2 = S_1 \mathrm{d}s_1$ $(\rho) = konst.$

$$m_1 = \rho S_1 ds_1 = m_2 = \rho S_2 ds_2$$

$$p_1 S_1 \mathrm{d}s_1 - p_2 S_2 \mathrm{d}s_2 = \frac{m_2 v_2^2}{2} - \frac{m_1 v_1^2}{2} + m_2 g z_2 - m_1 g z_1$$

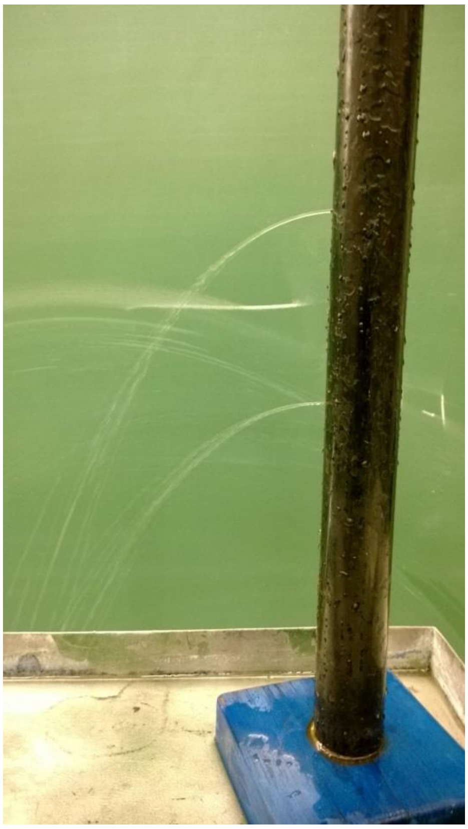
$$p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g z_2 - \rho g z_1$$

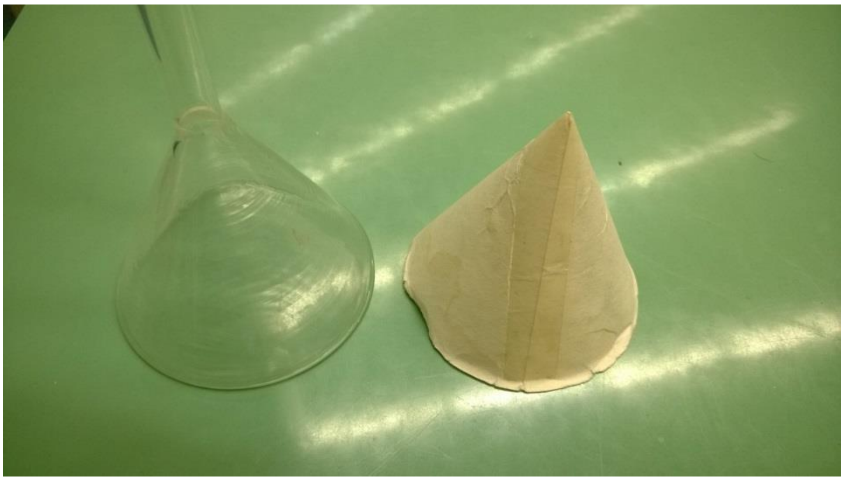
$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2$$

Bernoullijeva enačba :

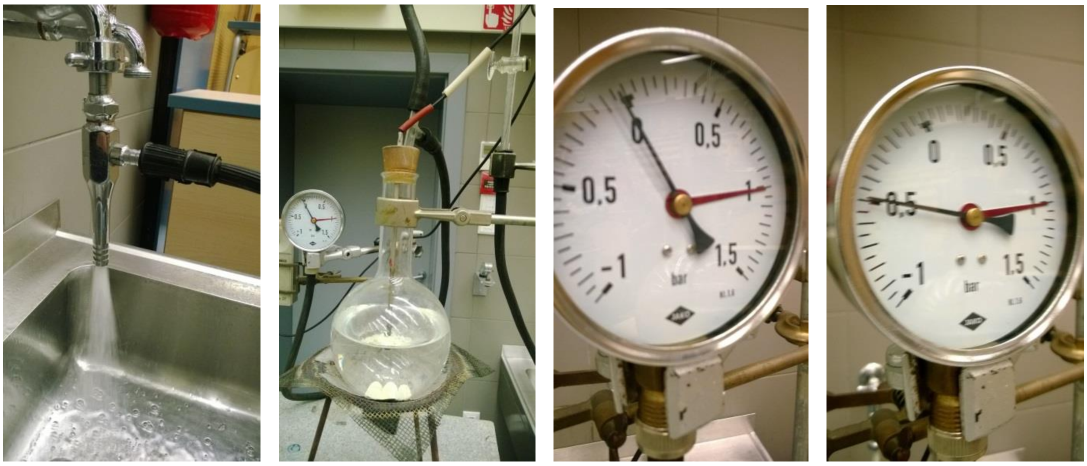
$$p + \frac{1}{2}\rho v^2 + \rho g z = \text{konst.}$$

ne deluje za :

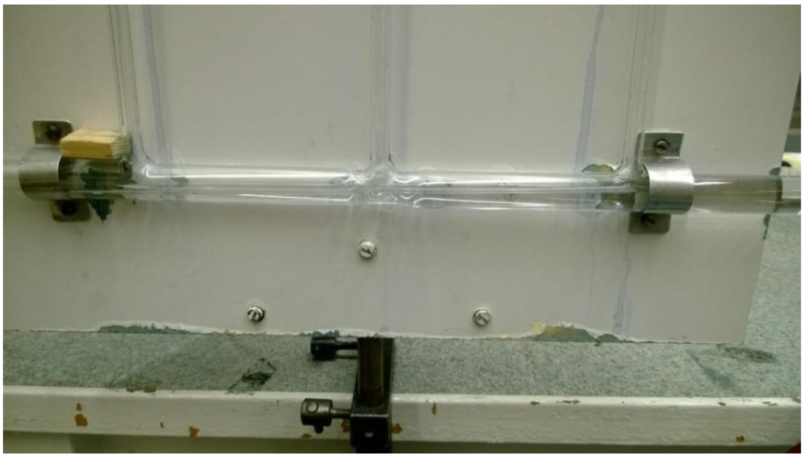


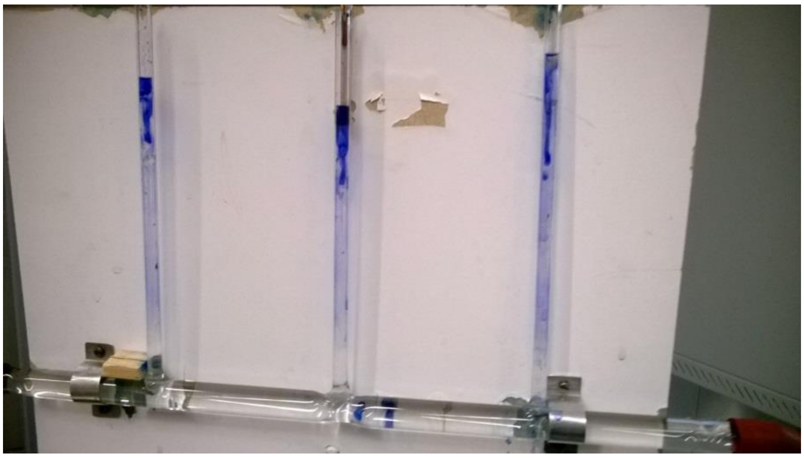
preprosta vakuumska črpalka na vodni curek

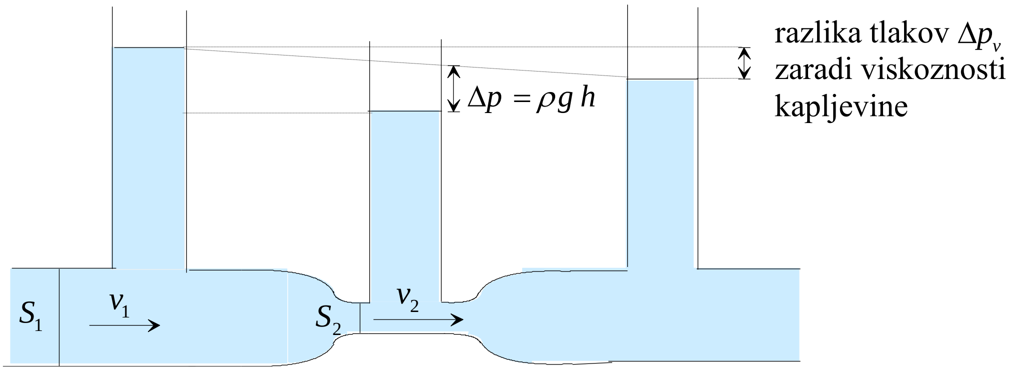


Magnusov efekt (gibajoči se papirnati valj pade z roba nagnjene plošče)

Venturijeva cev (zoženje cevi na sredi)

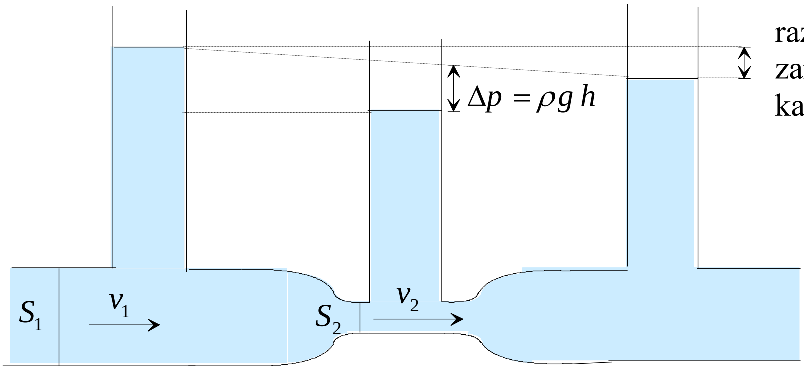






$$\Phi_v = \frac{\mathrm{d}V}{\mathrm{d}t} = S\,v$$

$$S_1 v_1 = S_2 v_2$$



$$z_1 \approx z_2 \quad : \qquad p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Delta p = p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

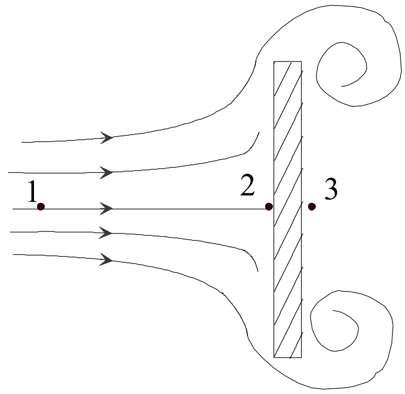
$$S_1 v_1 = S_2 v_2 \qquad v_2 = \frac{v_1 S_1}{S_2}$$

$$\Delta p = \frac{1}{2}\rho v_1^2 \left[\frac{S_1^2}{S_2^2} - 1\right]$$

$$v_1 = \left[\frac{2\Delta p}{\rho}\frac{S_2^2}{S_1^2 - S_2^2}\right]^{\frac{1}{2}}$$

$$\Phi_v = S_1 v_1 = \left[\frac{2\Delta p\, S_1^2 S_2^2}{\rho\left(S_1^2 - S_2^2\right)}\right]^{\frac{1}{2}} \qquad \Delta p = \rho g h$$

ocena velikosti zastojnega tlaka



$$p+\frac{1}{2}\rho v^2 = p+\Delta p \qquad p$$

$$\Delta p = \frac{1}{2}\rho v^2 \qquad\qquad F \approx \Delta p\, S = \frac{1}{2}\rho S v^2$$

Δp = zastojni tlak

Kvadratni zakon upora $$F_u = c_u \frac{1}{2}\rho S v^2$$

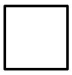
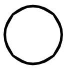
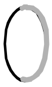
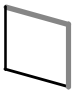
linearni zakon upora ($F_u \propto v$) velja, če je Reynoldsovo število $\mathrm{Re} = \frac{d\rho v}{\eta} < 1$

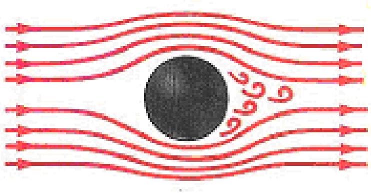
kvadratni zakon upora ($F_u \propto v^2$) velja, če je Reynoldsovo število $\mathrm{Re} = \frac{d\rho v}{\eta} > 1000$

d = linearna dimenzija telesa prečno na smer gibanja
ρ = gostota kapljevine
η = viskoznost kapljevine
v = hitrost telesa

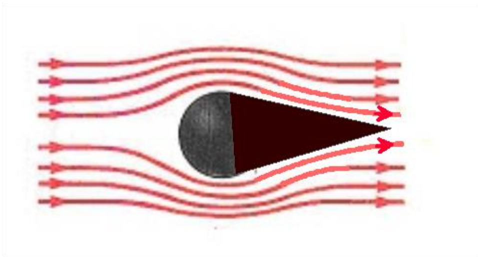
dinamični upor (kvadratni zakon upora) :

$$F_u = c_u \frac{1}{2} \rho S v^2$$

oblika		c_u
	kocka	1.1
	votla polkrogla	0.4
	votla polkrogla	1.4
	krogla	0.5
	okrogla plošča	1.1
	kvadratna plošča	1.2
	aerodinamično telo	0.03 –0.1



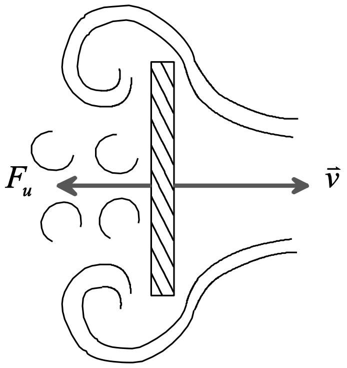
c_{u1}

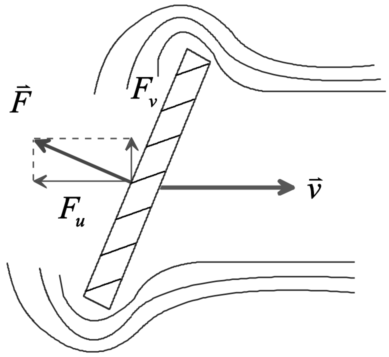


$c_{u2} < c_{u1}$

čim močnejši vrtinci za gibajočim se telesom, tem večji je koeficient **dinamičnega** upora c_u

dinamični upor : odvisnost od orientacije sim. osi telesa glede na smer vektorja hitrosti





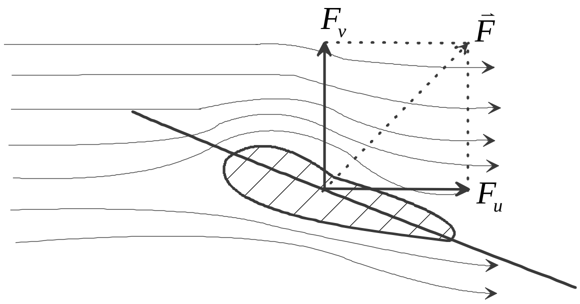
$$F_u = c_u \frac{1}{2} \rho S v^2 \qquad F_v = c_v \frac{1}{2} \rho S v^2$$

c_u = koeficient (dinamičnega)

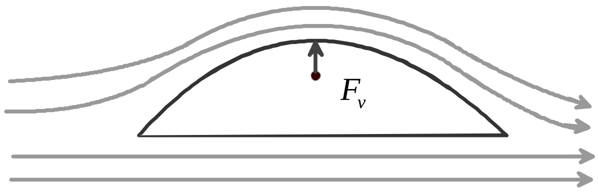
c_v = koeficient **dinamičnega vzgona**.

Dinamični vzgon je zelo pomemben pri letalskem krilu :

bistvena je nagnjenost krila

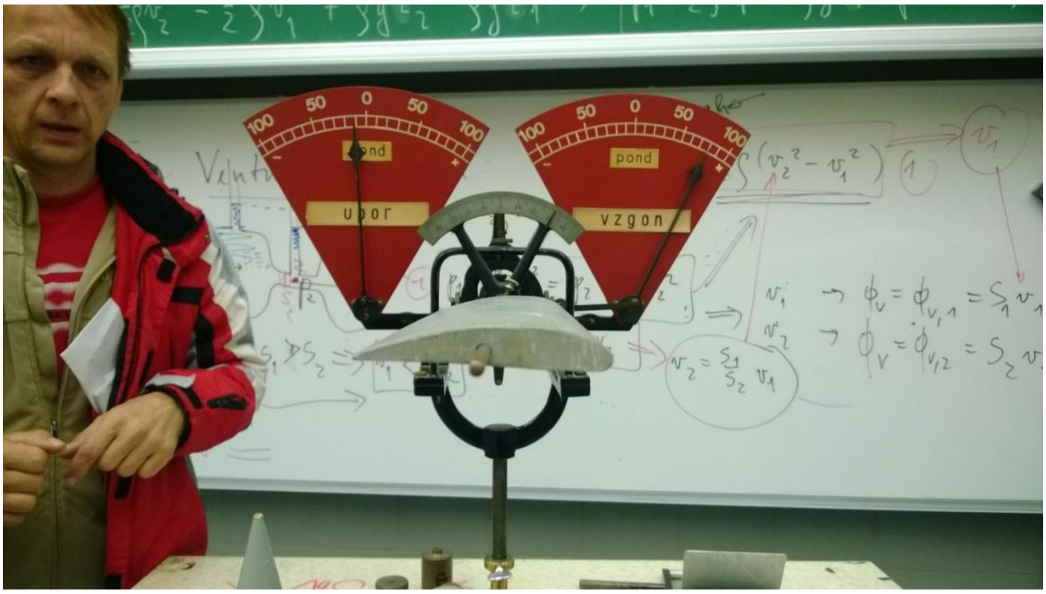


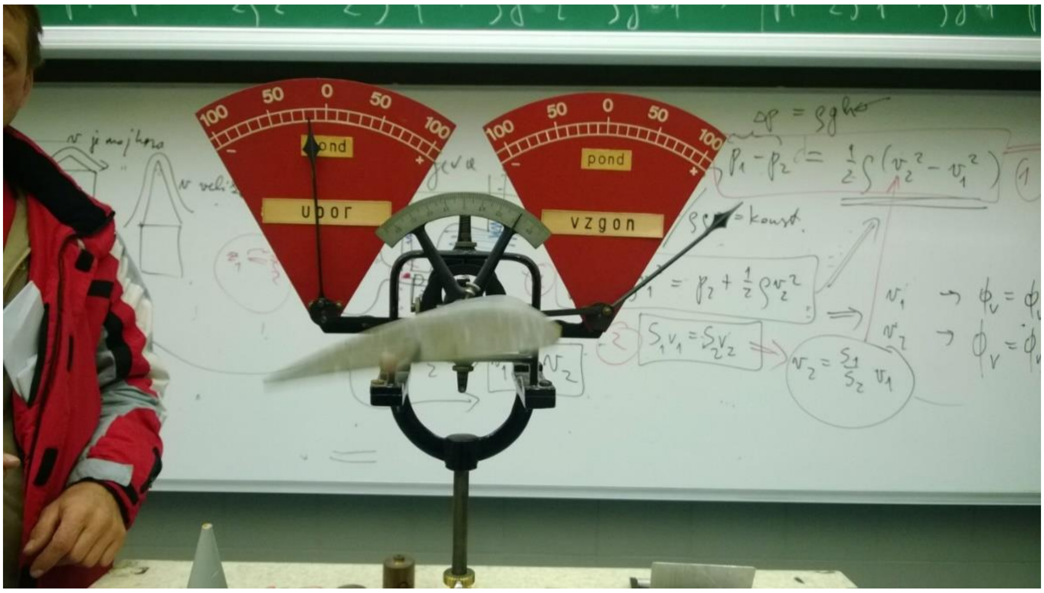
Bernoulli :



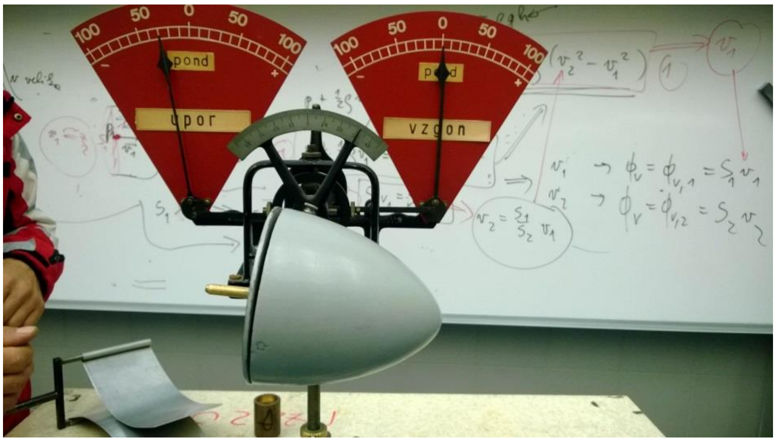
VETROVNIK

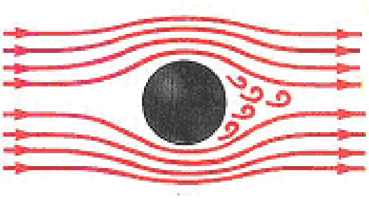
LETALSKO KRILO (vpliv nagiba krila na dinamični vzgon)

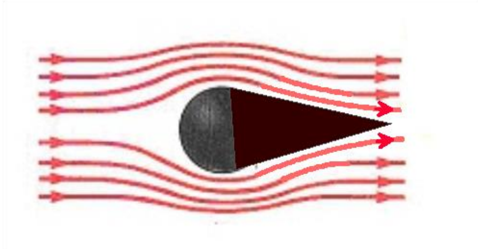

100 50 0 50 100
pond
upor
100 50 0 50 100
pond
vzgon


100 50 0 50 100
pond
upor
100 50 0 50 100
pond
vzgon

RIBJA OBLIKA in njeni sestavni deli

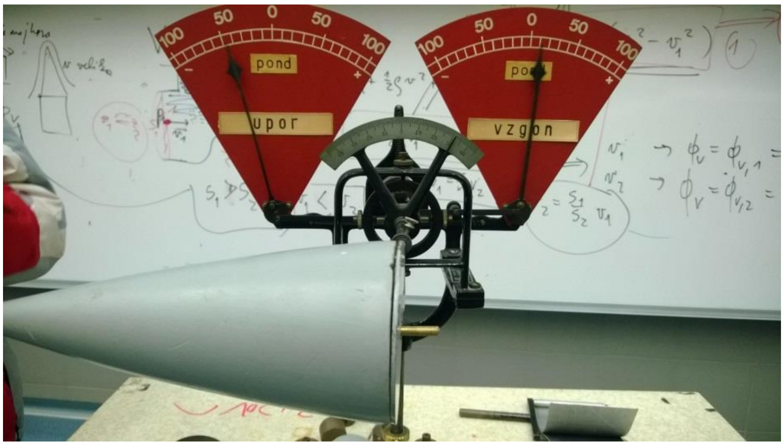




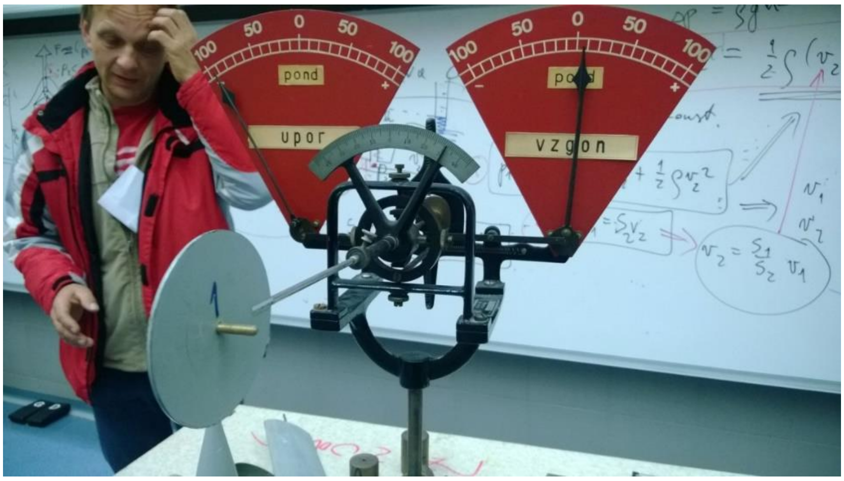


c_{u1}

$c_{u2} < c_{u1}$



OKROGLA PLOŠČICA



žoga lebdi na zračnem curku

dve loputi iz aluminija v zračnem curku
(na sliki loputi nista v zračnem curku)