MATHEMATICAL ANALYSIS OF CHIARI OSTEOTOMY

MATEMATIČKA ANALIZA OSTEOMOJE PO CHIARIJU

A. Iglić, F. Srakar, V. Antolič, V. Kralj Iglić and V. Batagelj

Ključne riječi: biomehanika, lokomotorni sustav, osteotomija po Chiariju

Key words: biomechanics, locomotor system, Chiari osteotomy

SAZETAK

Konstuiran je jednostavan statički trodimenzionalan matematički model kuka kako bi se omogućila simulacija pelvične osteotomije po Chiariju. U tom se modelu pretpostavlja da je prosječna napetost mišića u jednoj mišićnoj skupini jednak, što omogućuje da se u model uključi veći broj mišića nego što ima jedanđžbi za sile i moment ravnoteže bez uvjeta optimizacije. Proračunom je utvrđeno da se pomičanjem glavice bedrene kosti medijalno za 1 cm oko 10% smanjuje veličina reaktivne sile kuka. Stoga se zaključuje da pri izvođenju pelvične osteotomije po Chiariju glavicu femura treba maksimalno pomaknuti medijalno.


INTRODUCTION

While Chiari osteotomy (1) is performed an extended acetabular roof is obtained by shifting the femoral head medially (Figs. 1, 2). As a consequence of this shifting a femoral head weight bearing surface area is increased and also the hip joint reaction force is changed. In the presented work a simple static three-dimensional mathematical model of the hip was constructed in order to evaluate the magnitude of the hip joint reaction force before and after the Chiari osteotomy.

In the last twenty years many biomechanical models of the hip were constructed in order to calculate hip joint reaction force (R) and muscle forces in the hip and lower extremities during gait (2, 3, 4, 5, 6, 7) or in static body position (8, 9, 10, 11). These dynamic and static hip models and the old Pauwels model of the hip (8) were used for estimation of the effects of different surgical orthopaedic operations and implantations and also for evaluation of the biomechanical status of the hip with different femoral geometry (4, 8, 12, 13, 14).

* Supported in part by the Raziskovalna skupnost Slovenije (URP Staranje, progr. sklop Biomehanika)
METHODS

A simple static three-dimensional mathematical model was used in order to calculate the hip joint reaction force $\mathbf{R} = (R_x, R_y, R_z)$ in monopodal body position (see Fig. 3) before and after Chiari pelvis osteotomy.

The origin of coordinate system was chosen in the femoral head center (denoted + on Fig. 3) so that $x$ and $z$ axis lie in the frontal plane and $x$ and $y$ axis in the sagittal plane of the body. It is assumed that the origin of the hip joint reaction force $\mathbf{R}$ also lies in the femoral head center. According to the action-reaction force law, the femoral head exerts an opposite force $-\mathbf{R}$ on the acetabulum. Other forces acting on

The model of the one-legged stance. Here $W_B$ is the total body weight, $W_L$ is the weight of the supporting limb, $\mathbf{R}$ is the hip joint reaction force, $l$ is half the distance between the centers of the two acetabula, $a$ is $z$-coordinate of the force ($W_B$ $- W_L$) attachment point, $b$ is $z$-coordinate of the force $W_L$ attachment point, $c$ is $z$-coordinate of the ground reaction force $-W_B$ attachment point and $x_0$ is the femoral length. The meaning of the angles $\varphi$ and $\gamma$ can be seen from figure. Origin of the coordinate systems ($x$, $y$, $z$) coincides with the center of the hip joint (+).
the pelvis are the body weight minus the weight of the loaded leg \( \overline{W_B} - \overline{W_I} \) and the muscle vector resultant force \( \overline{F} = \sum F_i \) (1)

which is the vector sum of different muscle forces. Individual muscles have one attachment point on the pelvis and the other attachment point on the femur. The attachment of the force \( \overline{F} \) lies at the distance \( d \) in the antero-posterior direction from the frontal plane backward, which passes through both femoral head centers. According to Bombelli (12) the value \( d \) was used here. The forces acting on the loaded leg are the hip joint reaction force \( R \), the weight of the loaded leg \( W_I \) originating above the knee, reaction force exerted by the ground \(-\overline{W_B} \) (opposite to the body weight) and the resultant of the muscle forces \(-\overline{F}\).

In the applied mathematical model of the hip the muscles piriformis, gluteus medius, gluteus minimus, rectus femoris and tensor fasciae latae are included (see Tab. 1). Since gluteus medius and gluteus minimus are attached to the pelvis over a rather large area, in the presented model each of the two muscles is divided into three parts (18). These segments of gluteus medius and gluteus minimus and the remaining three muscles are classified in three groups according to their positions: anterior (a), middle (t) and posterior (p) as represented in Table 1.

Further it was assumed that any muscle force \( F_i \) can be approximately written by the following vector expression:

\[ F_i = f_i \cdot A_i \cdot s_i \quad i = 1, \ldots, 9 \]

where \( A_i \) is the relative cross-sectional area of the i-th muscle, \( f_i \) is the average tension in the i-th muscle and \( s_i \) is the unit vector in the direction of the i-th muscle. Unit vector \( s_i \) is determined by the coordinate vector of the muscle origin point on the pelvis \( \overline{r_i} = (x_i, y_i, z_i) \) and the corresponding coordinate vector of the muscle insertion on the femur \( \overline{r_i} = (x_i', y_i', z_i') \):

\[ s_i = \overline{r_i} - \overline{r_i}' \]

\[ |\overline{r_i} - \overline{r_i}'| \]

The values of coordinates \( x_i, y_i, z_i, x_i', y_i', z_i' \) for \( \varphi = 0 \) and \( \gamma = 0 \) (see Fig. 3) are given in Table 2 (18). While using the data for \( \overline{r_i} \) and \( \overline{r_i}' \), the reference coordinates given in Table 2, must be transformed by the corresponding rotational matrix for given values of angles \( \varphi \) and \( \gamma \).

In order to evaluate the hip joint reaction force \( \overline{R} \) and muscle resultant \( \overline{F} \), the statistic force and momentum equilibrium equations must be solved,

\[ \sum \overline{F_i} = \overline{R} + (\overline{W_B} - \overline{W_I}) = 0. \]

\[ \sum \overline{r_i} \times \overline{F_i} + \overline{a} \times (\overline{W_B} - \overline{W_I}) = 0, \]

where

\[ \overline{a} = (0, 0, a), \] and where the distance \( a \) is defined by the expression (19)

\[ \overline{W_B} \cdot c - \overline{W_I} \cdot b \]

\[ \overline{W_B} - \overline{W_I} \]

where the meaning of the parameter \( c \) is defined in Figure 3. The values of \( b \) and \( c \) were determined according to McLeish and Charnley (19)
as a function of the half the distance between the centers of the two acetabula 1:

\[ b = 0.48 \cdot 1 \]

\[ c = 1.01 \cdot 1 \]

(7)

where the value of the angle \( \varphi = -0.5^\circ \). The angle \( \gamma \) is determined by the equation (see Fig. 3):

\[ \sin \gamma = b/x_\varphi. \]

(8)

When determining the value of \( W_L \), experimentally obtained approximative equation \( W_L = 0.161 \cdot W_B \) is used (20).

In the presented model it is assumed that the average muscle tensions of individual muscles are equal in the single muscle group (see Table 1). This presumption enables us to include in the model the number of muscles which exceeds the number of model force and momentum equilibrium equations without any optimization method.

The variation of femoral head center position \((x_A, z_A)\) was simulated by changing the inter-hip distance \( 1 \rightarrow 1_{\text{ref}} - \Delta z \) (\( 1_{\text{ref}} = 8.45 \) cm) and by changing the coordinate of all muscle attachment points included in the represented model. The value of the angle \( \varphi \) was kept constant during the variation of pelvis configuration.

RESULTS

Figure 4 shows the dependence of magnitude of the hip joint reaction force on the femoral head shifts in the medio-lateral and superior-inferior directions, where the femoral head shifts are presumed to be the consequence of the Chiari osteotomy. It can be seen from Figure 4 that the magnitude of the hip joint reaction force (and therefore also the corresponding pressure on the femoral head) decreases as the femoral head is shifted medially, while it increases as the femoral head is shifted laterally in accordance with the results of Johnston et al. (14). On the other hand it can be seen from Figure 4 that femoral head shifts in superior-inferior directions have nearly no influence on the magnitude of the hip joint reaction force.

CONCLUSIONS

It is shown that the simulated femoral head shifts in medio-lateral directions have significant effect on the hip joint reaction force and therefore also on the corresponding pressure on the femoral head bearing area. It was calculated that the shifting of the femoral head medially for 1 cm reduces the magnitude of the hip joint reaction force by approx 10\%. It seems that long lasting overexertion of the hip joint reaction force causes development of osteoarthritis. Since medialization of femoral head as a consequence of Chiari osteotomy causes a decrease of the magnitude of the hip joint reaction force, this appears to be favourable. We suggest that reduction of the hip joint reaction force after medialization at Chiari osteotomy causes supplementary protection from cartilage degeneration in addition to the primary effect of the increased weight bearing surface area after medialization. This is especially important when a hip with an incipient coxarthrosis is considered. Therefore it could be concluded that while performing the Chiari pelvic osteotomy, the femoral head must be shifted medially as far as possible in order to reduce the hip joint reaction force and the corresponding pressure on the femoral head.
Symbols

\(\bar{R}\)  
hip joint reaction force

\(R\)  
magnitude of the hip joint reaction force

\(R_i\)  
j-th component of the hip joint reaction force, \(j = x, y, z\)

\(\bar{W}_i\)  
weight of the supporting limb

\(W_B\)  
total body weight

\(F_i\)  
i-th muscle force, \(i = 1, \ldots, 9\)

\(\bar{F}_i\)  
magnitude of the i-th muscle force

\(A_i\)  
relative cross-sectional area of the i-th muscle force

\(f_i\)  
average muscle tension of the i-th muscle force

\(\hat{n}_i\)  
unit vector in direction of the i-th muscle

\(\bar{r}_i\)  
position vector of the i-th muscle pelvis attachment point

\(\bar{r}_i'\)  
position vector of the i-th muscle femoral attachment point

\(\bar{F}\)  
vector sum of the muscle forces acting on the pelvis

\(l\)  
half the distance between the centers of the two acetabula

\(a\)  
z-coordinate of the force \(\bar{W}_B - \bar{W}_i\) attachment point

\(b\)  
z-coordinate of the force \(\bar{W}_i\) attachment point

\(c\)  
z-coordinate of the ground reaction force \(- W_B\) attachment point

\(x_6\)  
femoral length

\(\Delta x, \Delta z\)  
shifts of the femoral head in superior-inferior and medio-lateral directions

LITERATURE