Gradient of contact stress in normal and dysplastic human hips

Borut Pompe a, Matej Daniel b, Miroslav Sochor b, Rok Vengust a, Veronika Kralj-Iglič a,c, Aleš Iglič d,*

a Department of Orthopaedic Surgery, University Medical Center, Zaloška 9, SI-1000 Ljubljana, Slovenia
b Faculty of Mechanical Engineering, Technická 4, 166-07 Praha 6, Czech Republic
c Institute of Biophysics, Medical Faculty, Lipičeva 2, SI-1000 Ljubljana, Slovenia
d Laboratory of Applied Physics, Faculty of Electrical Engineering, Tržaška 25, SI-1000 Ljubljana, Slovenia

Received 2 August 2002; received in revised form 4 December 2002; accepted 2 January 2003

Abstract

The stress gradient index (Gp) is introduced for the assessment of dysplasia in human hip joint. The absolute value of Gp is equal to the magnitude of the gradient of the contact stress at the lateral acetabular rim. The parameter Gp normalized with respect to the body weight (WB) is determined from the standard anteroposterior radiographs of adult human hips and pelvises using the mathematical model. The average value of Gp/WB was determined for the group of dysplastic hips and for the group of normal hips. In the group of normal hips the average value of Gp/WB is smaller (1/11002 0.445 × 10^5 m/H1121) than in the group of dysplastic hips (1/11002 1.481 × 10^5 m/H1121). The difference is statistically significant P/H1121 0.001. The average value of Gp/WB changes its sign at the value of the centre-edge angle JCE/H1121 20 which is usually considered as the boundary value of JCE (lower limit) for the normal hips. Accordingly we suggest a new definition for the hip dysplasia with respect to the size and sign of the normalized stress gradient index Gp/WB. The hips with positive Gp/WB are considered to be dysplastic while the hips with negative Gp/WB are considered to be normal. The statistically significant correlation between the value of the Harris hip score, used in the clinical assessment of the hip dysplasia, and the normalized stress gradient index was found.

© 2003 IPEM. Published by Elsevier Science Ltd. All rights reserved.

Keywords: Hip dysplasia; Contact stress gradient; Radiographs; Arthrosis

1. Introduction

Dysplasia of the hip refers to mechanical deformations and deviations in the size and shape or mutual proportions between the upper part of the femur and acetabulum [1]. The dysplastic hips are diagnosed according to anatomical changes in the hip that are visible in the radiographs [1–3]. Usually, the center-edge angle of Wiberg (JCE) (Fig. 1) is used as the main radiographic parameter for the assessment of the hip dysplasia [2,3]. The range of JCE from 20–25° is considered as the lower limit for normal hips, while the value of JCE below 20° is pathognomonic for the hip dysplasia [3]. The size of the angle JCE correlates with the size of the weight bearing area and may therefore serve as an indirect measure of the hip joint contact stress [3–8]. However, it was suggested that besides JCE other geometrical parameters such as the radius of the femoral head [3,4] or the pelvic shape [9–11] should be taken into account in assessment of the contact stress distribution. Therefore, the direct calculation of the contact stress in the hip joint has been introduced in the assessment of the biomechanical status of the hip [3,4,12,13]. However, it was suggested recently that high magnitude of gradient of contact stress could be even more important for the development of the degenerative processes in the hip joint than high value of peak contact stress [14,15]. Therefore in this work the gradient of the contact stress in the hip joint articular surface at the lateral acetabular rim was studied. Standard anteroposterior radiographs of hip and pelvis and the mathematical model were used to determine differences between the group of dysplastic hips and the group of normal hips that originate in different gradient of hip stress.

* Corresponding author. Tel.: +386-1-4768-235; fax: +386-1-4264-630.
E-mail address: ales.iglic@fe.uni-lj.si (A. Iglič).
Fig. 1. Input geometrical parameters for determination of the resultant hip force and the distribution of the contact stress: interhip distance \( l \), pelvis height \( H \), pelvis width \( C \), the vertical and the horizontal distance from the center of the femoral head to the top of the greater trochanter \((x\) and \(z\), respectively\), radius of the femoral head \( r \), center-edge angle of Wiberg \( \theta_{CE} \).

2. Determination of gradient of contact stress from anteroposterior radiographs

In this section we present a method for estimation of gradient of contact stress in one legged stance by using a standard antero-posterior radiograph. The stress gradient is determined in two steps.

First, the hip joint resultant force \( \mathbf{R} \) transmitted from the acetabulum to the femur is determined by a three-dimensional biomechanical model of the human hip \([16]\). This model is based on solving of the static equilibrium equations for the forces and torques acting on the pelvis and the loaded leg in the one-legged stance \([9,16]\). In the one-legged stance the activity of the hip abductor muscles is necessary to maintain the balance of the pelvis. In our model, nine effective muscles are included \([9,16]\). It is assumed that the force of the individual muscle acts in the straight line connecting the attachment point of the muscle on the pelvis to the attachment point on the femur. The individual variations in the femoral and pelvic geometry influence the directions of the muscle forces as well as the radius vectors of the application points of the muscle forces on the pelvis and femur. Therefore the geometry of the hip should be adapted for each patient individually according to the data determined from standard anteroposterior radiographs \([11,12,18,19]\). The input geometrical parameters of the model for determination of \( \mathbf{R} \) are shown in the Fig. 1.

It was shown that the resultant hip joint force \( \mathbf{R} \) determined in one-legged stance lies nearly in the frontal plane of the body \([6,9,16]\). Therefore in the second mathematical model for determination of the contact stress distribution \([19,16]\) the force \( \mathbf{R} \) is assumed to lie in the frontal plane. The hip joint reaction force in the frontal plane can be expressed by its magnitude \( \mathbf{R} \) and by its inclination in the frontal plane with respect to the vertical plane \( \theta_R \) (Fig. 1). The angle \( \theta_R \) is taken to be positive in the medial direction from the vertical axis and negative in the lateral direction from the vertical axis \([6,19]\).

In the second step the mathematical model for calculation of the stress distribution in the hip joint \([20,19,16]\) is used. The model assumes the non-uniform distribution of the contact stress \([20]\). Area of the hip where stress differs from zero is called the weight-bearing area. The size of the weight-bearing area and distribution of the contact stress are not fixed but depend on the load and geometry of the hip \([20]\). The basic idea of the model is described below.

Besides the magnitude of the resultant hip force \( \mathbf{R} \) and inclination of the resultant hip force \( \theta_R \) the input parameters of the mathematical model for calculation of the hip joint contact stress and for calculation of the gradient of the contact stress in the hip joint are also the center-edge angle \( \theta_{CE} \) and the radius of the femoral head \( r \) (Fig. 1). The \( \theta_{CE} \) is taken to be positive in the lateral direction from the vertical axis and negative in the medial direction from the vertical axis (Fig. 1).

Within the model of stress distribution the contact stress at any point of the weight-bearing area \( p \) is taken to be proportional to strain in the cartilage layer. The cartilage fills in the cleft between the femoral head and the acetabulum. It is assumed that the femoral head has a spherical shape and acetabulum is the portion of the sphere, symmetric with respect to the frontal plane.

After loading there is one point where the spherical surfaces of the acetabulum and the femoral head are the closest. Due to symmetry of the articular surfaces with respect to the frontal plane and due to position of the hip joint reaction force in the frontal plane this point lies in the frontal plane and is called the stress pole \([20]\). Position of the stress pole can be determined by the spherical coordinate \( \Theta \) \([4,20]\) (Fig. 2) which is angular displacement of the pole from the vertical axis in the frontal plane. \( \Theta \) is positive in the lateral direction and negative in the medial direction from the vertical axis as well as \( \theta_{CE} \). The above assumptions lead to the cosine dependency of the contact stress distribution in the hip joint \([4]\):

\[
p = p_0 \cos \gamma
\]

where \( p_0 \) is the value of stress in the pole and \( \gamma \) is the angle between radius vector to the given point and the radius vector to the stress pole. The lateral border of the weight-bearing area is determined by the acetabular geometry while the medial border is determined as the curve where stress vanishes, i.e. where \( \cos \gamma = 0 \). Stress,
integrated over the weight-bearing area $S$, has to be equal to the force $R$ \[2,20\]:

$$\int_S p \, dS = R \quad (2)$$

The solution of the system of Eqs. (1) and (2) \[20\] yields equations for the spherical coordinate of the stress pole ($\Theta$), and for the value of stress at the pole ($p_0$):

$$\tan(\Theta) = \frac{\cos^2(\varphi_{CE} - \Theta)}{\frac{\pi}{2} + \varphi_{CE} - \Theta + \sin(\varphi_{CE} - \Theta) \cos(\varphi_{CE} - \Theta)} = 0 \quad (3)$$

The value of $\Theta$ was determined numerically from Eq. (3) using the Newton iteration method. If the pole of stress distribution is located within the weight bearing area, the location of the peak contact stress ($p_{max}$) coincides with the location of the pole ($p_{max} = p_0$) (Fig. 2a). This situation is characteristic for the hips with large enough values of $\varphi_{CE}$ where the stress distribution is (except at the medial border of the weight bearing area) considerably uniform (Fig. 2a). In the case of dysplastic hips (usually with small $\varphi_{CE}$) the stress pole lies outside the weight bearing area, therefore the peak contact stress is located at the point of the weight bearing surface which is closest to the pole, i.e. at the lateral acetabular rim (Fig. 2a). For small $\varphi_{CE}$ the contact stress distribution is highly nonuniform all over the weight bearing area (Fig. 2b).

The value of the peak contact stress ($p_{max}$) does not describe how the stress distribution varies over the weight bearing area. Therefore we tried to find some new biomechanical index which would be directly connected to the shape of the stress distribution function (Fig. 2).

According to the recent suggestion \[14,15\], we have chosen the gradient of contact stress distribution in the hip joint.

In order to calculate the stress gradient the original coordinate system is rotated so that in the rotated coordinate system the radius vector to the pole of stress distribution points in the direction of the vertical axis. In the spherical coordinates of the rotated system $(r, \vartheta, \phi)$ the stress distribution can be expressed as $p = p_0 \cos \varphi$. The gradient of contact stress ($\nabla p$) is then expressed as:

$$\nabla p = \frac{\partial p}{\partial r} e_r + \frac{1}{r} \frac{\partial p}{\partial \vartheta} e_\vartheta + \frac{1}{r \sin \vartheta} \frac{\partial p}{\partial \phi} e_\phi \quad (5)$$

where $e_r$, $e_\vartheta$ and $e_\phi$ are the orthogonal spherical unit vectors. Considering $p = p_0 \cos \varphi$ and Eq. (5) we obtain:

$$\nabla p = -\frac{p_0}{r \sin \vartheta} e_\phi \quad (6)$$

It follows from Eq. (6) that the gradient of contact stress in the hip joint is tangent to meridian of the articular sphere. In the following, our analysis is limited to the frontal plane (Fig. 2) where the angle $\vartheta$ in Eq. (6) is taken to be positive in the lateral direction from the radius vector to the stress pole and negative in the medial direction from the radius vector to the stress pole.

It was observed that the degenerative changes in the hip joint usually occur at the lateral acetabular rim \[2\]. To test the hypothesis that an increased magnitude of stress gradient at the lateral acetabular rim is biomechanically unfavorable we calculated the meridional component of stress gradient at the lateral acetabular rim:
\[ G_p = \frac{P_0}{r} \sin(\theta_{CE} - \Theta) \]  

(7)

which is equal to the scalar product \( \nabla p \cdot e_p \) (see Eq. (6)) at the lateral rim of the acetabulum where \( \theta = \theta_{CE} - \Theta \). The absolute value of \( G_p \) is equal to the magnitude of stress gradient \( \nabla p \) at the lateral rim of acetabulum. We define the parameter \( G_p \) as the index of the stress gradient in the hip joint. If the pole of stress distribution lies outside the weight bearing area (i.e. if \( \Theta > \theta_{CE} \)) then \( G_p > 0 \) (Fig. 2b). If the pole of stress distribution lies inside the weight bearing area (i.e. if \( \Theta < \theta_{CE} \)) then \( G_p < 0 \) (Fig. 2a).

Another biomechanical parameter which describes the size of the weight bearing area is the functional angle \( \theta_F \) [12,16]:

\[ \theta_F = \frac{\pi}{2} + \theta_{CE} - \Theta \]  

(8)

The functional angle is equal to the size of the weight bearing area divided by \( 2r^2 \) (Fig. 2). Combination of Eqs. (7) and (8) yields:

\[ G_p = \frac{P_0}{r} \cos\theta_F \]  

(9)

We can see that the stress gradient index \( G_p \) is in a simple way connected to the size of the weight bearing area which is proportional to the functional angle of the weight bearing area \( \theta_F \).

Besides the biomechanical and the radiographical parameters for evaluating dysplasia of the hip, the functional scores are also used in clinical practice. The Harris hip score is a worldwide distributed method for clinical evaluation of the status of the hip. Harris hip score includes patient’s opinion on pain, the functional activities and the range of motion [17]. Therefore it is of interest to compare the Harris score that is based on the subjectively determined patient’s feeling to the objectively computed biomechanical parameter \( G_p \).

3. Results

The group of dysplastic hips and the group of normal hips were examined with respect to the above defined stress gradient index \( G_p \) (Eq. (7)). The correlation between the index \( G_p \) and the center-edge angle \( \theta_{CE} \) is studied. The standard anteroposterior radiographs of the hips were taken from the medical records of the Department of Traumatology and Department of Orthopaedic Surgery, Medical Center, Ljubljana. In total we have 56 dysplastic hips of 20 subjects with unilateral dysplasia and 18 subjects with bilateral dysplasia. In the group of dysplastic hips, nine hips belonged to male persons and 47 belonged to female persons, 32 hips were right and 24 were left. The normal hips belonged to 146 persons who were subject to the X-ray examination of the pelvic region for reasons other than degenerative diseases of the hip joints. These radiographs showed no signs of the hip pathology. In the group of normal hips only one hip was taken into account from each subject.

For testing the relationship between the gradient index and the clinical score, we considered patients who were not included in the previous groups. The new group consists of 27 patients. In total we have 45 dysplastic hips. The Harris hip score was evaluated for these patients. It was obtained that the average Harris hip score was 88.4 (standard deviation 18.40).

In the present work the normalized stress gradient index \( G_f/W_B \) was studied. It can be seen from Eqs. (4) and (7) that the value of the index \( G_f/W_B \) depends only on the normalized magnitude of the resultant hip joint force \( R/W_B \) and not on the body weight \( W_B \). Since \( R/W_B \) is independent of the body weight \( W_B \) [9,11,16,19] also the index \( G_f/W_B \) does not depend on \( W_B \). This was particularly useful in our study since the body weight of some of the patients was not known.

The contours of the bony structures in each anteroposterior radiograph were put into digital form and the measurements of the geometrical parameters were performed by a computer program HIJOMO [11,12,18]. The program HIJOMO was also used to determine the center-edge angle \( \theta_{CE} \) and the femoral head radius \( r \) that are needed for calculation of the normalized stress gradient index \( G_f/W_B \).

The interdependence between the center-edge angle \( \theta_{CE} \) and the normalized stress gradient index \( G_f/W_B \) is shown in Fig. 3. The shape of the numerically obtained fitting curve is consistent with the above described mathematical model, i.e. for lower \( \theta_{CE} \) the values \( G_f/W_B \) are large and positive, while for higher \( \theta_{CE} \) the values \( G_f/W_B \) become small and negative. The normalized stress gradient index \( G_f/W_B \) changes its sign at \( \theta_{CE}=20^\circ \). The scattering of \( G_f/W_B \) as the function of \( \theta_{CE} \) shows that in determining \( G_f/W_B \), the geometrical parameters of the hip other than \( \theta_{CE} \) (like for example the interhip distance [9,16]) are also important. The scattering is higher for lower \( \theta_{CE} \) (i.e. for \( \theta_{CE} < 20^\circ \)).

To characterize the role of the normalized stress gradient index \( G_f/W_B \) in the assessment of the hip dysplasia the statistical significance of the difference in the average value of \( G_f/W_B \) between the group of the normal hips and the group of the dysplastic hips was calculated by the two-tailed pooled \( t \)-test (Table 1). The null hypothesis [21] assuming the equal average values is rejected at the level lower than 0.001. It can be therefore concluded that the above defined normalized index of the stress gradient \( G_f/W_B \) is an appropriate parameter for the assessment of the hip dysplasia.

Since the hip joint is usually considered as dysplastic for \( \theta_{CE} < 20^\circ \) [3] and we observed that \( G_f/W_B \) changes
Fig. 3. The correlation between the index of the stress gradient \( (G_p) \) normalized with respect to the body weight \( (W_B) \) and the center-edge angle \( \theta_{CE} \). The values for the normal hips are denoted by the symbol \( \diamond \) and the values for the dysplastic hips are denoted by \( \Delta \). The correlation coefficient \( R^2 = 0.897 \) \( (P < 0.001) \).

Table 1
The average values of the normalized stress gradient index \( (G_p/W_B) \) in normal and dysplastic hips

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Dysplastic</th>
<th>Difference (( P ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_p/W_B ) ( (10^5 \text{ m}^{-3}) )</td>
<td>-0.445</td>
<td>+1.481</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

its sign at approximately 20° we suggest a new definition of hip dysplasia. It is based on the sign of the stress gradient index \( G_p \). Hips with positive normalized stress gradient index \( G_p/W_B > 0 \) are taken to be dysplastic and hips with negative normalized stress gradient index \( G_p/W_B < 0 \) are taken to be normal.

In the clinical practice the patient’s subjective feeling of pain and the patient’s mobility are often considered to be more important than the outcome of the biomechanical analysis of the hip. Therefore, the relationship between the Harris hip score and stress gradient index was tested by the linear regression (not shown). Statistically significant correlation between the \( G_p/W_B \) and Harris Hip Score at the significance level lower than 0.01 was found \( (R = -0.426) \).

According to our suggestion of estimating the hip dysplasia based on the sign of the normalized stress gradient index, the hips were divided into two groups: the group of dysplastic hips with positive stress gradient index \( G_p/W_B > 0 \) which consists of 16 hips and the group of normal hips with negative normalized stress gradient index \( G_p/W_B < 0 \) which consists of 29 hips.

For comparison, the method for estimating hip dysplasia based on the center-edge angle was used. In clinical practice, hips with center-edge angle \( \theta_{CE} \) lower than 20° are considered to be dysplastic [3]. According to this classification, the control hips were divided into two groups: a group of dysplastic hips with \( \theta_{CE} < 20° \) which consists of 10 hips and a group of normal hips with \( \theta_{CE} > 20° \) which consists of 35 hips.

The difference in Harris hip score between the corresponding groups of normal and dysplastic hips was estimated by a non-parametrical statistical test (Kolmogorov-Smirnov test). If the hips were grouped according to the value of \( G_p/W_B \), then the null hypothesis assuming equal average values of Harris hip scores in both groups was rejected at the level lower than 0.05 \( (P = 0.031) \), i.e. statistically significant difference in Harris hip score exists between normal and dysplastic hips. If the hips were grouped according to the value of \( \theta_{CE} \), no statistically significant difference between the groups of normal and dysplastic hips was observed \( (P = 0.233) \).

4. Discussion and conclusions

It was indicated that high hip joint contact stress is an important factor accelerating the degenerative processes in the hip joint [12,22,23]. It was also shown that the contact stress in the hip joint is higher in the female population than in the male population [11]. Since women have higher incidence of arthrosis these results favor the hypothesis that elevated stress in the hip joint could be one of the reasons for the development of arthrosis [10].
The normal hip has uniform contact stress distribution (Fig. 2a) which is reflected in equal thickness of the bone condensation layer in the acetabular roof [13]. On the other hand nonuniform contact stress distribution in the dysplastic hip (Fig. 2b) leads to a triangular shape of the bone condensation layer on the lateral side of the acetabular roof [13]. In accordance, it was suggested recently that high magnitude of the gradient of the contact stress distribution could be even more important than high magnitude of the contact stress [14].

Therefore in this work we introduce a new parameter for the assessment of hip dysplasia from the anteroposterior radiographs, i.e. the stress gradient at the lateral rim of the acetabulum (Eqs. (7) and (9)). The gradient of the contact stress was calculated at the lateral rim of the acetabulum since it was observed that usually the degenerative changes in the hip joint occur at the lateral acetabular rim [2]. In the population study it was indicated that the normalized stress gradient index \( G_p/W_B \) changes its sign around \( \theta_{\text{EF}}=20^\circ \). Accordingly we suggest a new definition for the hip dysplasia according to the size and sign of the normalized stress gradient index \( G_p/W_B \). The hips with positive \( G_p/W_B \) are considered to be dysplastic while the hips with negative \( G_p/W_B \) are considered to be normal (see also Fig. 3).

The proposed criterion to differentiate dysplastic hips on the basis of the normalized stress gradient index was tested on the hips with evaluated Harris hip score. Statistically significant difference in Harris hip score between the group of normal and dysplastic hips determined after the normalized stress index gradient \( G_p/W_B \) was found. Therefore it could be concluded that the normalized stress gradient index is related also to the subjective state of the patient. For further clarification of this statement additional clinical studies should be carried out.

In this paper we assumed that the hip joint resultant \( \mathbf{R} \) lies in the frontal plane. In general the force \( \mathbf{R} \) lies in the frontal plane only in the special case of the one-legged stance \([2,16]\), used in this work as the representative body position. However, it has been shown that the one-legged stance is important, not only in its own right, but also more generally due to its resemblance to the stance phase of slow gait \([24]\). In addition, it was also indicated that stress in the hip joint during midstance phase of gait is related linearly to corresponding stresses in all phases of gait, as well as in some other activities, such as adduction, external rotation and flexion \([8]\).

In conclusion, we present a new method for the assessment of hip dysplasia where the normalized contact stress gradient at the lateral acetabular rim is determined from the standard anteroposterior radiographs using the mathematical model. The method is simple and can therefore be used in everyday clinical practice in planning of surgical interventions as well as in the population studies where a large number of data in the form of the standard anteroposterior radiographs are available from the archives.

**Acknowledgements**

The authors thank Simon Herman, Oskar Zupanc, Boštjan Kersnič, Blaž Mavčič and Matej Drobnič for gathering the radiographs from the archives and determining the profiles of the hip and pelvis. The computer programs (in TURBO PASCAL or in VISUAL BASIC) can be obtained from the authors free of charge in order to use them for scientific purposes according to ethical principles.

**References**


