On the role of anisotropy of membrane constituents in formation of a membrane neck during budding of a multicomponent membrane

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Abstract

The expression for the isotropic membrane bending energy was generalized for the case of a multicomponent membrane where the membrane constituents (single molecules or small complexes of molecules–membrane inclusions) were assumed to be anisotropic. Using this generalized expression for the membrane energy it was shown that the change of intrinsic shape of membrane components may induce first-order-like shape transitions leading to the formation of a membrane neck. The predicted discontinuous membrane shape transition and the concomitant lateral segregation of membrane components were applied to study membrane budding. Based on the results presented we conclude that the budding process might be driven by accumulation of anisotropic membrane components in the necks connecting the bud and the parent membrane, and by accumulation of isotropic (conical) membrane components on the bud. Both processes may strongly depend on the intrinsic shape of membrane components and on the direct interactions between them.

Keywords: Membrane biomechanics; Anisotropy; Membrane inclusions; Membrane energy

1. Introduction

In biological cells the budding process represents the first step in the formation of the small vesicles which are needed in transport of molecules through cell membrane and between cell organelles (Holthius et al., 2003). The budding process has also been observed in simpler artificial organic systems such as one-component or multicomponent lipid vesicles with or without protein inclusions (Sackmann, 1994; Holopainen et al., 2000; Staneva et al., 2005; Yamashita et al., 2002; Hanczyc and Szostak, 2004). The driving force for membrane budding can arise from different physical and biological mechanisms (Wiese et al., 1992; Lipowsky, 1992; Iglič et al., 1999; Kralj-Iglic et al., 1999; Yamashita et al., 2002; Hanczyc and Szostak, 2004; Peeters et al., 2005; Yin et al., 2005).

In general, it goes beyond the simple mechanism of extension of a membrane but regulates the formation of a neck having a much higher (anisotropic) curvature than the bud (Kralj-Iglic et al., 1999; Huttner and Zimmerberg, 2001; Staneva et al., 2005). For example, it was observed that upon cholesterol depletion, vesicle biogenesis from the trans-Golgi network proceeds through formation of a membrane bud which does not develop a neck nor undergoes membrane fission (Wang et al., 2000). On the other hand, in some cases the tendency for formation of a very long neck connecting the daughter vesicle and the mother membrane was observed before fission (Gad et al., 1998; Huttner and Zimmerberg, 2001; Kralj-Iglic et al., 2001).

Recently, it has been indicated (Kralj-Iglic et al., 1999) that formation of the membrane neck during the
budding process might be related to the anisotropic properties of membrane constituents. Accordingly, it has been shown that the formation of the membrane neck may be induced by lateral redistribution (diffusion) and accumulation of certain membrane components (lipids, proteins) in the highly curved and anisotropic region of the membrane neck which connects the daughter vesicle (bud) with the parent membrane (Sprong et al., 2000; Holthius et al., 2003). The formation of the membrane neck can thus be driven by coupling between the local membrane curvature and the local lateral molecular composition of the membrane (Kralj-Iglič et al., 1999; Shemesh et al., 2003).

In addition to the accumulation of anisotropic membrane components (domain formation) in highly curved regions of the buds/vesicles, the budding process may also be driven by the formation of domains composed of membrane components with a preference for high spherical curvature. Such domains may be formed in the spherical regions of the buds/vesicles (Hägerstrand and Isomaa, 1994; Sackmann, 1994; Thiele et al., 1999; Laradji and Kumar, 2004; Hanczyc and Szostak, 2004; Staneva et al., 2005).

Accordingly it was recently observed that in multicomponent unilamellar lipid vesicles, the membrane components can separate in coexisting domains with distinct composition and curvatures (Holopainen et al., 2000; Staneva et al., 2005). This process may resemble raft formation in cell membranes. The observed curvature-dependent domain sorting in lipid vesicles may result in membrane budding (fission) at domain boundaries (Laradji and Kumar, 2004; Staneva et al., 2005).

In this work, the role of coupling of the non-homogeneous lateral distribution of membrane constituents (inclusions) and the budding energetics is studied, in which the special attention is devoted to the influence of the anisotropic shape of membrane constituents (inclusions) and the direct interactions between them on neck formation during the budding process.

2. Theoretical model

For the sake of simplicity our theoretical analysis is limited to a case where buds and vesicles are depleted in membrane skeleton (Hägerstrand and Isomaa, 1994; Hägerstrand et al., 1999), therefore, the shear energy (Evans and Skalak, 1980; Iglič, 1997; Kosawada et al., 2005) of the membrane is neglected. We assume that the membrane constituents (single molecules or small complexes of molecules comprising a small number of molecules (Jacobson and Dietrich, 1999)—called membrane inclusions) as a result of their structure and local interactions energetically prefer a local geometry that is described by the two intrinsic principal curvatures ($C_{1m,i}$ and $C_{2m,i}$). The intrinsic principal curvatures are in general different ($C_{1m,i} \neq C_{2m,i}$) (Fig. 1). If they are identical ($C_{1m,i} = C_{2m,i}$), the in-plane orientation of the molecule or inclusion is immaterial. Such molecule or inclusion is called isotropic (Fig. 1). If $C_{1m,i} \neq C_{2m,i}$ the molecule or inclusion is called anisotropic (see Fig. 1). The small complex of molecules (inclusion) can have in general different intrinsic curvatures $C_{1m,i}$ and $C_{2m,i}$ from the intrinsic curvatures of the molecules which compose the inclusion (Fig. 2) (see also McMahon and Gallop, 2005).

The orientation of anisotropic molecule (inclusion) is important for its energy. It is assumed that anisotropic molecule (inclusion) will on the average spend more time
in the orientation that is energetically more favourable than in any other orientation. The free energy of a single membrane constituent (inclusion) of the \(i\)th kind can be written in the form (Kralj-Iglič et al., 1999; Fournier, 1996; Kralj-Iglič et al., 2002):

\[
F_i = \frac{\xi_i}{2} (H - H_m)^2 + \frac{\xi_i^*}{4} (D^2 + D_m^2)
- kT \ln \left( I_0 \left( \frac{(\xi_i + \xi_i^*)D_m D}{2kT} \right) \right),
\]

where \(\xi_i\) and \(\xi_i^*\) are the interaction constants, \(H = (C_1 + C_2)/2\) and \(D = |C_1 - C_2|/2 = \sqrt{H^2 - C_1 C_2}\) are the mean curvature and curvature deviator, and \(H_{m,i}\) and \(D_{m,i}\) are the intrinsic mean curvatures and intrinsic curvature deviators describing the intrinsic shape of the membrane constituents (inclusions) of the \(i\)th type. \(kT\) is the thermal energy and \(I_0\) is the modified Bessel function.

Let us consider a single (sufficiently large) lipid layer of lateral area \(A\) and local mean and deviatoric curvatures \(H\) and \(D\), respectively. The lipid layer contains three molecular (inclusion) species, \(N_1\) molecules of type \(i = 1\), \(N_2\) molecules of type \(i = 2\) and \(N_3\) molecules of type \(i = 3\). From the conservation of the overall number of constituents in the lipid layer \((N)\), we express the number of molecules of first type \(N_1\) as

\[N_1 = N - N_2 - N_3.\]

For the sake of simplicity we assume that all three species occupy the same lateral cross-sectional area \(a = A/N\) per molecule (inclusion) on the lipid layer. The nature of the lipid bilayer allows its constituents to laterally redistribute. That is, in a non-homogeneously curved lipid layer, the molecules of all species are assumed to migrate towards their energetically preferred membrane regions so as to minimize the overall free energy. We describe this degree of freedom by the local compositions of the species, namely \(m_2\), \(m_3\) and \(m_1 = 1 - m_2 - m_3\). In our model the local compositions \(m_1, m_2\) and \(m_3\) are equal to the local fractions of the membrane area covered by the molecules of type 1, 2 and 3, respectively. We denote the average compositions \(\bar{m}_i = N_i/N\) as

\[m_i = \frac{1}{A} \int_A m_i \, dA.\]

We compose the free energy per molecule \(F\) of the lipid layer that contains the single-molecule energies \((f_i)\), the corresponding configurational entropies (Kralj-Iglič et al., 1999, 2002) and also direct interactions between molecules (considered within the Bragg-Williams approximation) of the second type \((i = 2)\)

\[
\frac{F}{kT} = \frac{N}{A} \int_A \left\{ \frac{3}{4} \left( m_1 f_1 + m_2 \ln \frac{m_2}{\bar{m}_2} - m_2 + \bar{m}_2 \right) + \frac{w_{22}}{2} \bar{m}_2^2 \right\} \, dA,
\]

where \(f_i = F_i/kT\), \(w_{ij}\) is the constant of direct interactions between molecules of type \(i = 2\) (in units of \(kT\)) and \(z_2\) is the corresponding coordination number. From this we can construct a Lagrangian of the form

\[
L = \sum_{i=1}^{3} \left( m_i f_i + m_i \ln \frac{m_i}{\bar{m}_i} + \bar{m}_i (m_i - \bar{m}_i) + \frac{w_{22}}{2} m_i^2 \right),
\]

where \(\bar{m}_i\) are the Lagrangian multipliers. Inserting into the Lagrangian the relations \(m_1 = 1 - m_2 - m_3\) and \(\bar{m}_1 = 1 - \bar{m}_2 - \bar{m}_3\), we can then eliminate one of the Lagrangian multipliers by defining \(\lambda_2 = \lambda_3 = \lambda_1\). Using the Euler–Lagrange equations \(\partial L/\partial m_2 = 0\) and \(\partial L/\partial m_3 = 0\), we can derive an expression for composition \(m_2\) in the form

\[
m_2 = \bar{m}_2 \left( \frac{m_2 e^{f_2 - f_1 + w_{22} m_2}}{1/\bar{A}} \int_\Omega m_2 e^{f_2 - f_3 + w_{22} m_2} \, d\Omega.\right.
\]

While \(m_1\) can be obtained from \(m_1 = 1 - m_2 - m_3\), the equation for \(m_2\) is in general an integral equation that cannot be solved easily. However, for small direct interactions \((w \ll 1)\) we can obtain (up to the first relevant term in \(w\))

\[
m_2 = m_2^0(1 - w_2 m_2^0 (1 - m_2^0)),
\]

where

\[
m_2^0 = \frac{m_2 e^{f_2 - f_1}}{m_2 e^{f_2 - f_3} + m_3 e^{f_3 - f_2} + m_1 e^{f_1}}
\]

is the value of \(m_2\) for \(w = 0\). Values for the Lagrangian multipliers \(\lambda_2\) and \(\lambda_3\) are obtained from constraints (2).

In the above theoretical model of a three-component membrane all three molecular species can in general be anisotropic (by having a non-zero curvature deviator \(D_{m,i}\)).

3. Model predictions

3.1. Formation and stability of membrane neck

First, we shall study the influence of the anisotropic membrane components on the formation of the membrane neck for the special case of the undulated tubular membrane shape given by the function

\[r(z) = R_0 + u \cdot \sin(qz),\]

where \(r\) is the radial coordinate, \(z\) is the coordinate along the longitudinal axis, \(R_0\) is the average radius, \(q\) is the wave number of the modulation and \(u\) is its amplitude.

For the sake of simplicity we shall assume that the membrane contains only two molecular species, lipids \((i = 1)\) and anisotropic inclusions \((i = 2)\), while the third membrane component will not be considered, i.e., \(m_3 = \bar{m}_3 = 0\). The lipid molecules which are in general anisotropic (Kralj-Iglič et al., 2002) are considered as isotropic. From the constraint for the surface area per
unit length (along the $z$-axis) $\int_0^1 dA = A$ the parameter $R_0$ can be expressed as a function of $q$ and $u$:

$$R_0 = \frac{A - 2\pi u \int_0^1 \sin qz \sqrt{1 + u^2 q^2 \cos^2 qz} dz}{2\pi \int_0^1 \sqrt{1 + u^2 q^2 \cos^2 qz} dz}.$$  

(9)

By using Eq. (8) the sum and the product of the main curvatures are calculated:

$$C_1 + C_2 = \frac{1 + u^2 q^2 + R_0 u q^2 \sin qz}{(R_0 + u \sin qz)(1 + u^2 q^2 \cos^2 qz)^{3/2}},$$  

(10)

$$C_1 C_2 = \frac{u q^2 \sin qz}{(R_0 + u \sin qz)(1 + u^2 q^2 \cos^2 qz)^2}.$$  

(11)

From Eqs. (10) and (11) the average mean curvature $H = (C_1 + C_2)/2$ and the curvature deviator $D = \sqrt{H^2 - C_1 C_2}$ are calculated. The mean curvature and the curvature deviator derived from Eqs. (10) and (11) are then inserted into above equations. The Lagrange parameter $z_2$ is calculated from constraint (2). Minimization of the free energy (Eq. (3)) with respect to the remaining two parameters $q$ and $u$ gives us values of $q$ and $u$ that correspond to the equilibrium shape having the minimal possible free energy of the membrane ($F$) for chosen values of model parameters. The minimization procedure was performed numerically.

The calculations were performed for membrane inclusions of saddle-like shape with $H_{m,2} = 0$ and $D_{m,2} \neq 0$ (Fig. 3). For small values of the intrinsic curvature deviator $D_{m,2}$, the tubular shape was shown to be energetically most favourable. With increasing $D_{m,2}$ a certain critical value of $D_{m,2}$ is reached where the tubular shape is changed discontinuously to an undulated shape with a narrow neck (Fig. 3). Fig. 3 shows a part of the calculated membrane shape for two values of the intrinsic curvature deviator $D_{m,2}$ above the critical value ($D_{m,2,\text{crit}} \approx 0.33 \text{ nm}^{-1}$). As can be seen in Fig. 3, the saddle-like anisotropic inclusions accumulate in the energetically favourable saddle-like neck regions. With increasing values of the intrinsic curvature deviator of anisotropic inclusions ($D_{m,2}$), the neck becomes thinner.

### 3.2. Membrane budding

Based on the results presented in Fig. 3 we suggest that the anisotropic (saddle-like) membrane constituents (inclusions) with a preference for regions with saddle-curvatures may induce the formation of a membrane neck in the budding process. In addition to the accumulation of anisotropic membrane components in the membrane necks, the budding process may also be driven by the formation of domains composed of membrane components with a preference for high spherical curvature (Holopainen et al., 2000; Staneva et al., 2005). These may be formed in the spherical regions of the buds/vesicles. In accordance, it was observed that in a multicomponent membrane the membrane constituents may separate into coexisting domains with distinct composition, i.e. the raft formation takes place (Hägerstrand and Isomaa, 1994; Harder and Simons, 1997; Staneva et al., 2005).

To illustrate the role of the intrinsic shape of membrane constituents and their lateral distribution in membrane budding, we calculated the lateral distribution of the membrane components for the special case of an axisymmetric vesicle shape composed of a large mother cell (only partially shown in Fig. 4) and a small

![Fig. 4. Equilibrium lateral distribution of membrane components in the budding region of the bilayer membrane. Figure shows the space variation of the fractions of the membrane area occupied by the molecules of type $i$ ($m_i$) (also called local compositions) for a three-component membrane (lipids: 1, anisotropic inclusions: 2, isotropic conical inclusions: 3) calculated for three values of the interaction constant describing the strength of direct interactions between anisotropic inclusions of type 2 ($w$). The values of $m_i$ are dimensionless. The axisymmetric closed membrane shape has a relative average mean curvature of 1.075 and a relative volume 0.99. The values of the model parameters are $\xi_1 = 16 \text{ kT nm}^2$, $H_{m,1} = D_{m,1} = 0$, $\xi_2 = 320 \text{ kT nm}^2$, $H_{m,2} = 0$, $D_{m,2} = 0.3 \text{ nm}^{-1}$, $m_2 = 0.01$, $z_2 = 6$, $\xi_3 = 240 \text{ kT nm}^2$, $H_{m,3} = 0.3 \text{ nm}^{-1}$, $D_{m,3} = 0$, $m_3 = 0.05$, $z_3 = 0$.](image-url)
daughter vesicle connected by a thin neck (Fig. 4). The closed membrane shape given in Fig. 4 was calculated for an inclusion-free membrane, based on minimization of the Helfrich local bending energy as explained elsewhere (Iglič et al., 1999). We assume that the presence of the anisotropic inclusions in the membrane does not substantially affect the vesicle shape. This assumption is justified by our choice of membrane shape (Fig. 4) which is close to the limiting shape composed of two spherical vesicles connected by infinitesimally thin neck, i.e. a larger mother vesicle and a smaller daughter vesicle, and also by the choice of the small concentrations of membrane constituents (inclusions) of type 2 and 3.

The membrane is assumed to contain three molecular (inclusion) species; lipid molecules (type \(i = 1\)) which for the sake of simplicity are assumed to be isotropic (i.e. \(D_{m,1} = 0\)), anisotropic molecules (inclusions) of type \(i = 2\) (\(D_{m,2} \neq 0\)) and isotropic conical molecules (inclusions) of type \(i = 3\) (\(D_{m,3} = 0, H_{m,3} > 0\)) with a preference for a high spherical curvature. The molecules (inclusions) of type 2 and 3 are assumed to be distributed only in the outer layer of the membrane bilayer.

It can be seen in Fig. 4 that the anisotropic saddle-like membrane inclusions (\(m_2\)) are predominantly distributed in the region of the membrane neck, while the isotropic inclusions with a preference for a highly isotropically curved membrane (\(m_1\)) are predominantly accumulated on the spherical daughter vesicle (bud). Increase of \(m_2\) with increasing interaction constant \(w\) indicates that the direct interactions between the inclusions may play an important role in the energetics of the budding process and clustering of membrane inclusions (i.e. raft formation).

### 4. Discussion and conclusion

The results presented in this paper may add to a better understanding of the mechanisms that are important for formation of highly curved membrane regions and for maintaining increased concentrations of certain membrane components in such regions, for example, in spherical parts of the vesicles (Thiele et al., 1999) and in vesicle necks (Holthius et al., 2003). In this work we propose that accumulation of some membrane components in membrane necks is coupled by in-plane ordering of these components in necks where the difference between the principal membrane curvatures (\(C_1\) and \(C_2\)) is very large. Also, formation of the spherical part of the vesicles may be favoured by the specific intrinsic shape of membrane components. Small protein–phospholipid–cholesterol complexes (Thiele et al., 1999; Jacobson and Dietrich, 1999) are obvious candidates for such membrane components (inclusions). Anisotropic (lipid and cholesterol binding) proteins which form small anisotropic protein–lipid membrane inclusions (raft elements) may coalesce into larger domains (rafts) (Holthius et al., 2003) upon curvature-induced clustering in spherical parts of the vesicles (Thiele et al., 1999) as shown in Fig. 4.

Blocked biogenesis of synaptic-like vesicles after limited cholesterol depletion from the membrane protrusions indicates the importance of cholesterol (Thiele et al., 1999) as the interaction partner in the formation of small protein–phospholipid–cholesterol complexes (inclusions) and their clustering into larger rafts. The clustering of the membrane inclusions (raft elements) into larger rafts is probably additionally promoted by direct interaction between the inclusions (Jacobson and Dietrich, 1999), as presented also in Fig. 4. The short-range phospholipid (and cholesterol) mediated attractive interactions between membrane inclusions (Bohinc et al., 2003) may offer a possible explanation for the nature of such interactions. In the present work, the direct interactions between the anisotropic membrane constituents (inclusions) are considered within the Bragg–Williams approximation. Alternatively, the direct interactions between the membrane components can also be described by minimization of the boundary (line) tension between membrane domains (Lipowsky, 1992). However, considering the line tension assumes that the composition of the membrane domains is fixed in advance which is not in accordance with experimental observation (Hägerstrand and Isomaa, 1994) and does not allow study of the process of formation of membrane domains where the composition may be gradually changed.

In the past, an increased local concentrations of membrane components has also been predicted on highly curved tubular membrane protrusions (Kralj-Iglič et al., 2000; Corbeil et al., 2001). Coupling between the non-homogeneous lateral distribution of membrane components and the local anisotropic membrane curvature has also been recently indicated in Golgi bodies (Sprong et al., 2000; Iglič et al., 2004) in which some of the membrane components are concentrated mainly on the bulbous rims of the Golgi vesicles, where the difference between the two principal membrane curvatures (\(C_1\) and \(C_2\)) is very high. Similar phenomena have also been observed in the photoreceptor discs (Corbeil et al., 2001) and flattened endovesicles of the erythrocyte membrane (Hägerstrand et al., 2004), indicating that the coupling between the non-homogeneous lateral distribution of membrane (generally anisotropic) components and specific membrane shapes may be a general mechanism of stabilization of highly curved membrane structures (flattened disc-like vesicles, spherical buds, necks, tubular protrusions) (Thiele et al., 1999; Hägerstrand et al., 2004; Markin, 1981; Iglič et al., 2004).

The predicted membrane shape changes due to the changed (increased) intrinsic curvature deviator of
anisotropic membrane components \(D_{m,i}\) can among other factors be applied to explain the membrane shape changes (inward and outward bending, budding) arising from conformational changes of membrane proteins induced by ligand binding or variation of ionic strength and pH (Gimsa and Ried, 1995).

In the above described theoretical consideration of the membrane budding we assumed that buds and released daughter vesicles are depleted in membrane skeleton. The presence of the membrane skeleton in the budding region would decrease the lateral mobility of the membrane constituents and thereby their ability to sort due to membrane curvature and/or direct interactions. In addition, the in-plane shear energy of the membrane skeleton would increase during the budding process (Hägerstrand et al., 1999) and the calculated shapes of membrane buds would be different from those presented in Fig. 4, especially in the vicinity of the neck region (Kosawada et al., 2005).

In predicting the equilibrium lateral distribution of membrane components in multicomponent membrane, we did not take into account different cross-sectional areas of membrane components (Shah, 1998). However, considering the different cross-sectional areas of membrane components, for example, within lattice statistics model (Manciu and Ruckenstein, 2002), would considerably increase the complexity of our theoretical model, but would not alter the basic conclusions of our work.

To conclude, based on the results presented in this work we suggest that the budding process might be driven by at least three different but complementary mechanisms: (1) by accumulation of anisotropic membrane components in the neck connecting the bud and the parent membrane; (2) by accumulation of isotropic conical membrane constituents (inclusions) in the region of the bud (i.e. daughter vesicle) (Holopainen et al., 2000; Sprong et al., 2001; Holthius et al., 2003) and (3) by a local increase of the area difference between the outer and inner lipid layer (Iglić and Hägerstrand, 1999; Staneva et al., 2005). Reports on the clustering of rafts and membrane proteins in highly curved membrane regions (invaginations) and vesicles \(H \approx \frac{1}{2} \text{nm}\) (Hägerstrand and Isomaa, 1994; Harder and Simons, 1997) support these conclusions.

References


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