Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Numerical analysis of the asymptotic two-scale limit of the plasma-wall transition using a one-dimensional two-fluid model

IOP Conf. Series: Journal of Physics: Conf. Series **982** (2018) 012005 doi:10.1088/1742-6596/982/1/012005

Numerical analysis of the asymptotic two-scale limit of the plasma-wall transition using a one-dimensional two-fluid model

T Gyergyek [1,2], **J Kovačič**[2]

[1] University of Ljubljana, Faculty of electrical engineering, Tržaška 25, 1000 Ljubljana, Slovenia

[2] Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia

E-mail: tomaz.gyergyek@fe.uni-lj.si

Abstract. A one-dimensional two-fluid model is presented and used for numerical analysis of the asymptotic two-scale limit of the plasma-wall transition. Numerical results confirm that when the problem is treated on the pre-sheath scale, the sheath edge is determined by the electric field singularity. When the problem is approached on the sheath scale, electric field at the sheath edge must be larger than zero in order to obtain any solutions of the model equations. In this case the Bohm criterion is determined by two parameters: the electric field and the ion velocity at the sheath edge.

1. Introduction

The plasma-wall transition is one of the oldest in plasma physics [1,2]. A widely accepted of this problem can be summarized in the following way. If neutral plasma is bounded at least on one side by a planar wall that absorbs all particles that reach it and is biased negatively with respect to the plasma potential, a transition layer is formed between the "unperturbed" plasma and the wall, when the potential drop takes place. The "unperturbed" plasma is neutral, its potential is zero and there is no electric field present in such plasma. The transition layer can be divided into 2 parts. The first part is called the pre-sheath. In this region the plasma is neutral, but a finite electric field exists in this region, which accelerates positive particles (ions) towards the wall and negative particles (electrons and possibly negative ions) in the opposite direction. When the velocity of the collective ion motion towards the electrode reaches the ion sound velocity, plasma neutrality breaks down and a space-charge dominated region is formed. This region is called the sheath. In many plasmas the extension of the pre-sheath is much larger than the dimension of the sheath. The length scale L of the pre-sheath is usually some characteristic length that governs the binary processes in this region – usually the mean free path for most frequent collisions. The length scale of the sheath on the other hand is by the rule the Debye length λ_D, which gives the distance at which the potential perturbation introduced in the plasma is shielded. The ratio of this two lengths $\varepsilon = \lambda_D/L$ is sometimes called the neutrality parameters. The larger is this parameter, stronger local violations of plasma neutrality can occur. In the asymptotic 2 scale limit [1], neutrality parameters goes to zero $\varepsilon \to 0$, since $L >> \lambda_D$. In this limit the analysis of the pre-sheath and of the sheath can be completely separated. In the limit $\varepsilon \to 0$ the boundary between the pre-sheath and the sheath is defined very sharply and it is called the sheath edge. When the sheath edge is approached from the pre-sheath side, electric field gradually increases and becomes infinite (has a singularity) at the sheath edge. When the sheath edge is approached from the sheath side, electric field gradually decreases and becomes zero at the sheath edge.

In this work the asymptotic two-scale limit of the plasma-wall transition is studied numerically using a one-dimensional, steady state, two-fluid model. It is assumed that plasma consists only of electrons and just one species of singly charged positive ions. In the next section the model is presented. In section 3 complete solutions of the model for finite ε are shown first and then the pre-sheath and the sheath region are studied separately by simply inserting $\varepsilon = 0$ into the model equations. In section 4 some conclusions are presented.

2. Model

The model is based on a one-dimensional two-fluid model in steady state:

$$\begin{aligned}
&\frac{d}{dx}\left(n_i u_i\right) = S_i, \quad \frac{d}{dx}\left(n_e u_e\right) = S_e, \\
&m_i n_i u_i \frac{du_i}{dx} = n_i e_0 E - \frac{dp_i}{dx} + A_i - m_i u_i S_i, \\
&m_e n_e u_e \frac{du_e}{dx} = -n_e e_0 E - \frac{dp_e}{dx} + A_e - m_e u_e S_e, \\
&\frac{1}{2} u_i \frac{dp_i}{dx} + \frac{3}{2} p_i \frac{du_i}{dx} = M_i + B_i + \frac{1}{2} m_i u_i^2 S_i, \\
&E = -\frac{d\Phi}{dx}, \quad \frac{d^2\Phi}{dx^2} = -\frac{e_0}{\varepsilon_0}\left(n_i - n_e\right).
\end{aligned} \tag{1}$$

Here m_i is the ion mass, u_i is the ion fluid velocity, n_i is the ion density, e_0 is the elementary charge, E is the electric field, p_i is the ion pressure, S_i is the ion source term, A_i is the ion elastic collision term, which gives the momentum which is transferred from ions to electrons per unit time and per unit volume because of their elastic collisions with electrons, Φ is the potential, m_e is the electron mass, u_e is the electron fluid velocity, n_e is the electron density, p_e is the electron pressure, S_e is the electron source term, A_e is the electron elastic collision term, which gives the momentum which is transferred from electrons to ions per unit time and per unit volume because of their elastic collisions with ions, M_i is the power density that the ions receive through external heating, B_i gives the density of kinetic energy that is transferred from ions to electrons per unit time because of elastic collisions and ε_0 is the permittivity of the free space. Terms A_i and A_e which describe the exchange of momentum between ions and electrons because of elastic collisions are given by:

$$A_i = -m_e n_e f_{ie}\left(u_i - u_e\right), \; A_e = -m_e n_e f_{ie}\left(u_e - u_i\right). \tag{2}$$

Here f_{ie} is the frequency of elastic momentum exchange collisions between ions and electrons (coulomb collisions) and f_{ie} is given by [3]:

$$f_{ei} = \frac{\sqrt{2}}{12\pi^{3/2}} \frac{e_0^4 n_i}{\varepsilon_0^2 \sqrt{m_e}\left(kT\right)^{3/2}} \ln \Lambda. \tag{3}$$

Here k is the Boltzmann constant, T is the mean value of the ion temperature T_i and the electron temperature T_e:

$$T = \frac{T_i + T_e}{2}, \tag{4}$$

and lnΛ is the coulomb logarithm, given by:

$$\ln \Lambda = \ln\left(\frac{12\pi\left(\varepsilon_0 k T_e\right)^{3/2}}{\sqrt{n_0}\, e_0^3}\right). \tag{5}$$

Here n_0 is the density of the unperturbed plasma far away from the electrode. The term B_i is given by [3]:

$$B_i = -\frac{1}{2} k n_e 2 \frac{m_e}{m_i} f_{ei}\left(T_i - T_e\right). \tag{6}$$

The source terms S_i and S_e give the difference between the number of created and annihilated ions and electrons per unit volume and per unit time. Since in our model there are only electrons and just one species of singly charged positive ions, it is clear that source terms for both particle species must be identical, $S_i = S_e$. The assumed form depends of the main mechanism of ionization. If the new electron ion pairs are mainly created by ionizing collisions between electrons and neutral atoms, it can be assumed that the source term is proportional to the local electron density:

$$S_e = S_i = \frac{n_e(x)}{\tau}. \tag{7}$$

The ionization time $\bar{\tau}$ takes into account also the annihilation of both particle species by recombination. The external heating term M_i can be assumed to be a given constant.

The closure of the system (1) is made by the following assumptions. First, the electrons are assumed to be isothermal, so the electron pressure gradient is expressed by:

$$p_e(x) = n_e(x) k T_e, \quad \frac{dp_e}{dx} = k T_e \frac{dn_e}{dx}. \tag{8}$$

Second, it is assumed that the ion heat flux is zero and consequently it does not appear in the fifth equation of the system (1). So the ion pressure gradient term is expressed as:

$$p_i(x) = k T_i(x) n_i(x), \quad \frac{dp_i}{dx} = k T_i \frac{dn_i}{dx} + k n_i \frac{dT_i}{dx} = \kappa k T_i \frac{dn_i}{dx}. \tag{9}$$

Following Kuhn et al [4] the polytropic function $\kappa(x)$ has been defined:

$$\kappa(x) = 1 + \frac{n_i(x)}{T_i(x)} \frac{dT_i}{dn_i}. \tag{10}$$

The following variables are introduced:

$$
\begin{aligned}
&\lambda_D=\sqrt{\frac{\varepsilon_0 kT_e}{n_0 e_0^2}},\ c_0=\sqrt{\frac{kT_e}{m_i}},\ L=c_0\tau,\ \varepsilon=\frac{\lambda_D}{L},\ N_i=\frac{n_i}{n_0},\ N_e=\frac{n_e}{n_0},\\
&V_i=\frac{u_i}{c_0},\ V_e=\frac{u_e}{c_0},\ \Theta=\frac{T_i}{T_e},\ \mu=\frac{m_e}{m_i},\ Z=f_{ei}\tau,\ \Psi=\frac{e_0\Phi}{kT_e},\ \xi=\frac{x}{L},\\
&X=\frac{x}{\lambda_D},\ P_i=\frac{p_i}{n_0 kT_e},\ G_i=\frac{M_i\tau}{n_0 kT_e},\ \eta=\frac{e_0 L}{kT_e}E,\ \chi=\frac{e_0\lambda_D}{kT_e}E.
\end{aligned}
\tag{11}
$$

If the space coordinate x is normalized to the ionization length L, and the equations (2) - (9) are taken into account, the system of the model equations reads:

$$
\begin{aligned}
&\frac{d}{d\xi}\left(N_i V_i\right)=N_e,\ \frac{d}{d\xi}\left(N_e V_e\right)=N_e,\ P_i=N_i\Theta,\\
&N_i V_i\frac{dV_i}{d\xi}=\eta N_i-\frac{dP_i}{d\xi}-\mu Z N_e\left(V_i-V_e\right)-V_i N_e,\\
&\mu N_e V_e\frac{dV_e}{d\xi}=-\eta N_e-\frac{dN_e}{d\xi}-\mu Z N_e\left(V_e-V_i\right)-\mu V_e N_e,\\
&\frac{1}{2}V_i\frac{dP_i}{d\xi}+\frac{3}{2}P_i\frac{dV_i}{d\xi}=G_i-\mu Z N_e\left(\Theta-1\right)+\frac{1}{2}V_i^2 N_e,\\
&\eta=-\frac{d\Psi}{d\xi},\ \varepsilon^2\frac{d^2\Psi}{d\xi^2}=N_e-N_i.
\end{aligned}
\tag{12}
$$

If on the other hand the space coordinate x is normalized to the Debye length λ_D, the system of equations becomes:

$$
\begin{aligned}
&\frac{d}{dX}\left(N_i V_i\right)=\varepsilon N_e,\ \frac{d}{dX}\left(N_e V_e\right)=\varepsilon N_e,\ P_i=N_i\Theta,\\
&N_i V_i\frac{dV_i}{dX}=\chi N_i-\frac{dP_i}{dX}-\varepsilon\left[\mu Z N_e\left(V_i-V_e\right)+V_i N_e\right],\\
&\mu N_e V_e\frac{dV_e}{dX}=-\chi N_e-\frac{dN_e}{dX}-\varepsilon\mu\left[Z N_e\left(V_e-V_i\right)+V_e N_e\right],\\
&\frac{1}{2}V_i\frac{dP_i}{dX}+\frac{3}{2}P_i\frac{dV_i}{dX}=\varepsilon\left[G_i-\mu Z N_e\left(\Theta-1\right)+\frac{1}{2}V_i^2 N_e\right],\\
&\chi=-\frac{d\Psi}{dX},\ \frac{d^2\Psi}{dX^2}=N_e-N_i.
\end{aligned}
\tag{13}
$$

In the asymptotic two-scale limit $\varepsilon = 0$ is inserted into systems of equations (12) and (13). The system (12) remains almost the same:

IOP Conf. Series: Journal of Physics: Conf. Series **982** (2018) 012005 doi:10.1088/1742-6596/982/1/012005

$$\begin{aligned}
&\frac{d}{d\xi}\left(N_iV_i\right)=N_e,\ \frac{d}{d\xi}\left(N_eV_e\right)=N_e,\ P_i=N_i\Theta,\\
&N_iV_i\frac{dV_i}{d\xi}=\eta N_i-\frac{dP_i}{d\xi}-\mu ZN_e\left(V_i-V_e\right)-V_iN_e,\\
&\mu N_eV_e\frac{dV_e}{d\xi}=-\eta N_e-\frac{dN_e}{d\xi}-\mu ZN_e\left(V_e-V_i\right)-\mu V_eN_e,\\
&\frac{1}{2}V_i\frac{dP_i}{d\xi}+\frac{3}{2}P_i\frac{dV_i}{d\xi}=G_i-\mu ZN_e\left(\Theta-1\right)+\frac{1}{2}V_i^2N_e,\\
&\eta=-\frac{d\Psi}{d\xi},\ 0=N_e-N_i.
\end{aligned} \tag{14}$$

The system (13) on the other hand changes considerably:

$$\begin{aligned}
&\frac{d}{dX}\left(N_iV_i\right)=0,\ \frac{d}{dX}\left(N_eV_e\right)=0,\ P_i=N_i\Theta,\\
&N_iV_i\frac{dV_i}{dX}=\chi N_i-\frac{dP_i}{dX},\ \mu N_eV_e\frac{dV_e}{dX}=-\chi N_e-\frac{dN_e}{dX},\\
&\frac{1}{2}V_i\frac{dP_i}{dX}+\frac{3}{2}P_i\frac{dV_i}{dX}=0,\ \chi=-\frac{d\Psi}{dX},\ \frac{d^2\Psi}{dX^2}=N_e-N_i.
\end{aligned} \tag{15}$$

In the next section some numerical solutions of the systems of equation (12) - (15) are examined.

3. Results

The systems of equation (12) - (15) are strongly nonlinear, so only numerical solutions can be found. Each of the systems is a system of 8 ordinary differential equations for 8 unknown functions of X or ξ. For a unique solution also 8 boundary conditions – this means values of the unknown functions at $X = \xi = 0$ - must be specified. Unfortunately here some compromises must be made between physical requirements and limitations imposed by mathematical properties of the systems (12) - (15). In the unperturbed plasma the potential is zero and there should be no electric field. This gives the conditions: $\Psi(0) = \chi(0) = \eta(0) = 0$. At the same place the plasma must be neutral with normalized density of ions and electrons, so $N_i(0) = N_e(0) = 1$. Next are the ion and electron velocity. In the unperturbed region of the plasma there should be no directed ion or electron flow, so this would impose zero velocities at $X = \xi = 0$. But it can be shown [5] that a value $V_i = 0$ results in a singularity by division with zero. So $V_i(0) > 0$ must be selected. For the plots shown in Fig. 1 $V_i(0) = V_e(0) = 10^{-7}$ is selected. In this work we assume that ions are born at rest, so $\Theta(0) = P_i(0) = 0$ is selected. It can also be shown [5] by combining the continuity equations and equations of motion for the ions and electrons that the systems (12), (13) and (15) become singular whenever $V_i \le V_{ith}$ or $V_e \ge V_{eth}$. Here V_{ith} and V_{eth} are ion and electron thermal velocity, given by:

$$V_{ith}=\frac{1}{c_0}\sqrt{\frac{\kappa kT_i}{m_i}}=\sqrt{\kappa\Theta},\ V_{eth}=\frac{1}{c_0}\sqrt{\frac{kT_e}{m_e}}=\frac{1}{\sqrt{\mu}}. \tag{16}$$

The system (14) on the other hand becomes singular when ion and electron velocity reach the values:

$$V_i = \sqrt{\frac{1+\kappa\Theta}{1+\mu\left(\frac{\Gamma+V_e(0)}{\Gamma+V_i(0)}\right)^2}},\; V_e = \sqrt{\frac{1+\kappa\Theta}{\mu+\left(\frac{\Gamma+V_i(0)}{\Gamma+V_e(0)}\right)^2}}. \tag{17}$$

Here Γ is the indefinite integral of the source term over ξ, given by:

$$\Gamma = \int N_e(\xi)d\xi, \tag{18}$$

while $V_i(0)$ and $V_e(0)$ are the respective integration constants. If the boundary conditions $V_i(0) = V_e(0)$ are selected, both velocities (17) are equal to the ion sound velocity V_S, given by:

$$V_S = \frac{1}{c_0}\sqrt{\frac{kT_e+\kappa kT_i}{m_i+m_e}} = \sqrt{\frac{1+\kappa\Theta}{1+\mu}}. \tag{19}$$

Let us now compare the solutions of the systems (12) and (13) for a finite ε with the same parameters and boundary conditions. The boundary conditions are: $\Psi(0) = \chi(0) = \eta(0) = \Theta(0) = P_i(0) = 0$, $N_i(0) = N_e(0) = 1$ and $V_i(0) = V_e(0) = 10^{-7}$. The other parameters are: $\varepsilon = 10^{-5}$, $Z = G_i = 0$ and $\mu = 1/3670.482$ (deuterium ions). In Fig. 1 the potential, electric field and ion temperature profiles are shown. In the

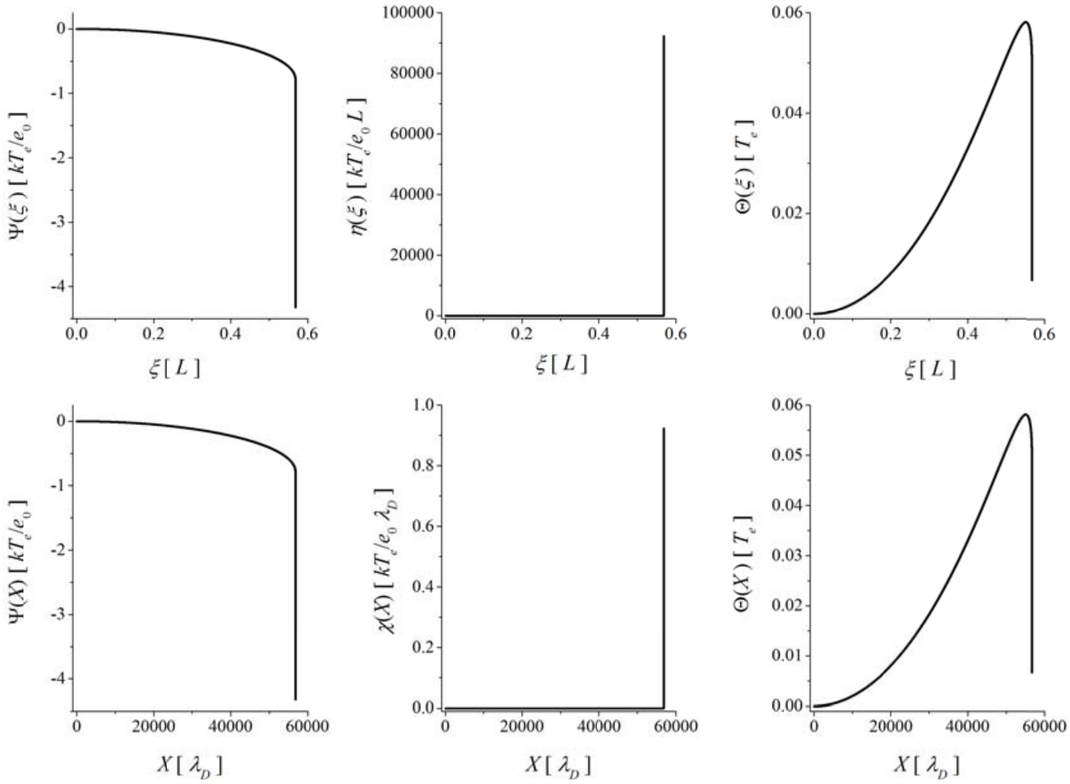

Figure 1. In the top plots the profiles of the potential $\Psi(\xi)$, electric field $\eta(\xi)$ and ion temperature $\Theta(\xi)$ obtained from the system (12) are displayed. In the bottom graphs the same profiles found from the system (13) are shown. The boundary conditions and the parameters are: $\mathbf{\Psi}(0) = \chi(0) = \eta(0) = \Theta(0) = P_i(0) = 0$, $N_i(0) = N_e(0) = 1$ and $V_i(0) = V_e(0) = 10^{-7}$, $\varepsilon = 10^{-5}$, $Z = G_i = 0$ and $\mu = 1/3670.482$.

plots the profiles of the potential $\Psi(\xi)$, electric field $\eta(\xi)$ and ion temperature $\Theta(\xi)$ obtained from the system (12) are displayed. In the bottom graphs the same profiles found from the system (13) are shown. It can be seen that the solutions of both systems of equations (12) and (13) are identical. The only difference is the scaling of the horizontal scale and of the electric field by factor ε.

Next (Fig. 2) we examine the solutions of the system (14) for the same parameters and boundary conditions, as in Fig. 1, this means: $\Psi(0) = \chi(0) = \Theta(0) = P_i(0) = 0$, $N_i(0) = N_e(0) = 1$, $V_i(0) = V_e(0) = 10^{-7}$, $Z = G_i = 0$ and $\mu = 1/3670.482$. In Fig. 2 all the unknown functions are plotted versus ξ. The electric field is shown in the logarithmic scale. It can be seen that close to the sheath edge it increases very sharply for several orders of magnitude. This is in good agreement with the asymptotic two-scale theory [1,2] which claims that in the pre-sheath scale the position of the sheath edge is characterized by the electric field singularity. Ion and electron density $N_i(\xi)$ and $N_e(\xi)$ both decrease monotonically and at the sheath edge they both drop to less than half of the boundary value at $\xi = 0$. Both velocities

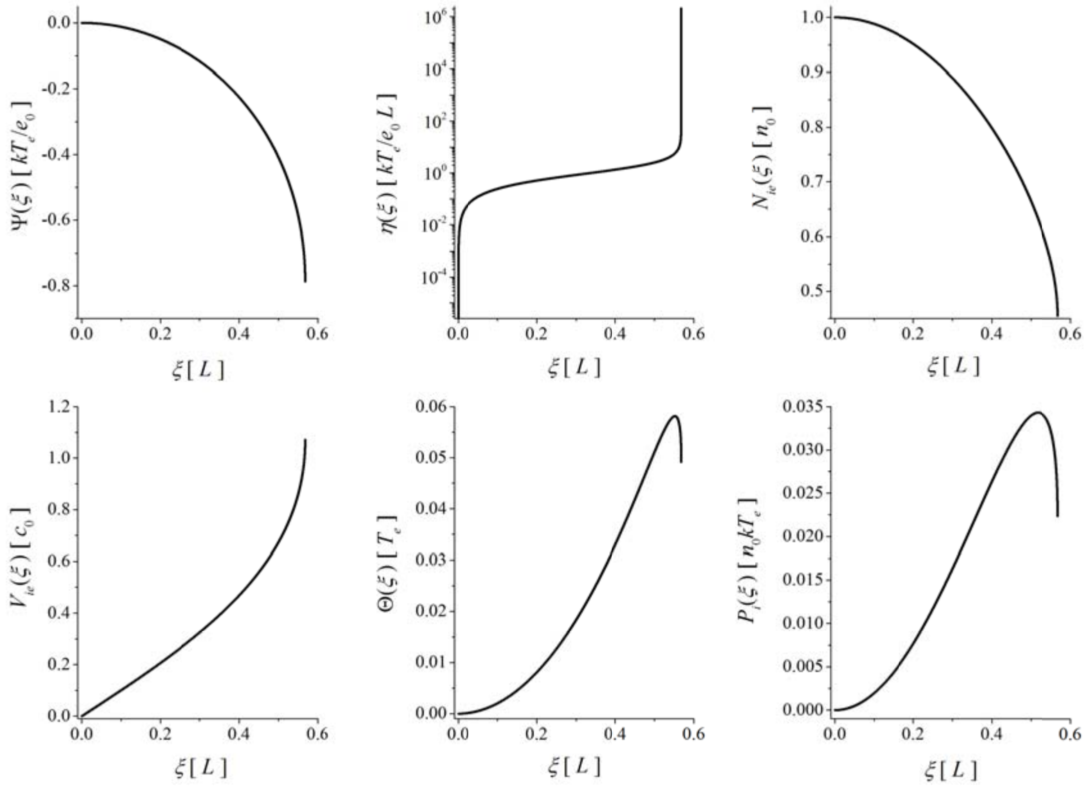

Figure 2. Solutions of the system (14) for the following parameters and boundary conditions: $\Psi(0) = \chi(0) = \eta(0) = \Theta(0) = P_i(0) = 0$, $N_i(0) = N_e(0) = 1$, $V_i(0) = V_e(0) = 10^{-7}$, $Z = G_i = 0$ and $\mu = 1/3670.482$.

$V_i(\xi)$ and $V_e(\xi)$ increase monotonically until they reach the ion sound velocity V_S, given by (19).

Finally we move to the analysis of the problem on the so-called sheath scale – this means using the system of equations (15). First the issue of boundary conditions arises. At $X = 0$ obviously the sheath edge is located. One might attempt to take the values found from the solutions of the pre-sheath system (14) as boundary conditions for the sheath system (15). The obvious problem with electric field singularity prevents this. Since at the sheath edge the plasma is still neutral the density at the sheath edge can be renormalized to unity and the boundary condition $N_i(0) = N_e(0) = 1$ is taken. Also the potential at the sheath edge can be reset to zero, $\Psi(0) = 0$. The velocities $V_i(0)$ and $V_e(0)$ need some comment. The ion velocity should fulfil the Bohm criterion, so one should take $V_i(0) = V_S$. But in order to do this one must first select the ion temperature $\Theta(0)$ and then find also the polytropic function κ using (10). This can be done only after the system (15) has already been solved. Since the ion flow at the sheath edge can be expected to be adiabatic, the value $\kappa = 3$ can be selected and then verified from the solution of the system (15) a posteriori using (10). The selection of the electron velocity $V_e(0)$ is almost arbitrary with 2 remarks. First it should not be larger than V_{eth} (see (16)) because in this case the system (15) has a singularity. Second, if $V_e(0) = 0$ is selected, $V_e(X)$ will be zero for all X. In this case the singularity (16) is never reached and the point where the integration of the system (15) is stopped must be selected arbitrarily. In order to avoid this, $V_e(0) = 10^{-7}$ is selected. The parameters from previous figures are not changed, so $Z = G_i = 0$ and $\mu = 1/3670.482$ are taken.

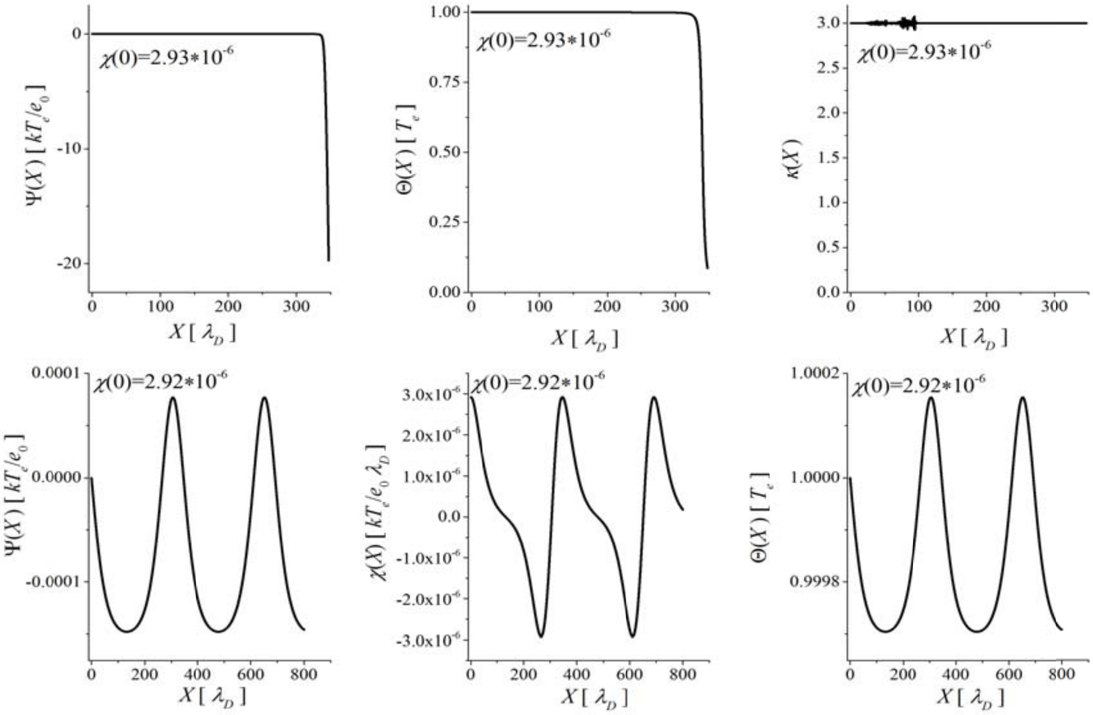

Figure 3. In the top plots the potential $\Psi(X)$, the ion temperature $\Theta(X)$ and the polytropic function $\kappa(X)$ are shown for $\chi(0) = 2.93\times10^{-6}$. In the bottom graphs the potential $\Psi(X)$, electric field $\chi(X)$ and ion temperature $\Theta(X)$ are presented for $\chi(0) = 2.92\times10^{-6}$. The other parameters and boundary conditions are: $Z = G_i = 0$ and $\mu = 1/3670.482$, $\Psi(0) = 0$, $V_i(0) = 1.99973$, $V_e(0) = 10^{-7}$, $P_i(0) = 1$, $\Theta(0) = 1$, $N_i(0) = N_e(0) = 1$

The only remaining problem is the electric field $\chi(0)$ at the sheath edge. According to the asymptotic two scale limit [1,2] in the sheath scale the electric field at the sheath edge should be zero. But if $\chi(0)$ is selected exactly 0, the solutions of the system (15) do not move from the boundary values. A small but finite positive value should be selected. So for the top graphs of Fig. 3 the value $\chi(0) = 2.93\times10^{-6}$ is selected. The ion temperature and pressure are $\Theta(0) = 1$, $P_i(0) = 1$ and $\kappa = 3$ is taken. In addition $V_i(0) = 1.99973$ is selected, which corresponds to V_S found from (19). In the top graphs of Fig. 3 the potential $\Psi(X)$, ion temperature $\Theta(X)$ and polytropic function $\kappa(X)$ are presented. It can be seen that the solutions are monotonic and the polytropic function has a constant value 3. The curves shown in the bottom plots of Fig. 3 are also obtained from the system (15) for the same parameters and boundary conditions, only the electric field at the sheath edge is slightly decreased.

The value $\chi(0) = 2.92\times10^{-6}$ is selected. In this case the solutions are oscillatory. This means that at $X = 0$ the Bohm criterion is not fulfilled anymore. For the system (15) the Bohm criterion is determined by two quantities $V_i(0)$ and $\chi(0)$. If one of these boundary conditions is decreased, the other should be increased if one wants to get monotonic solutions.

In Fig. 4 the smallest $V_i(0)$ which results in a monotonic solution of the system (15) is plotted versus $\chi(0)$. The boundary condition $\chi(0)$ is varied over 10 orders of magnitude, so the horizontal scale is logarithmic. The parameters and boundary conditions are: $Z = G_i = \Psi(0) = 0$, $\mu = 1/3670.482$, $V_e(0) = 10^{-7}$ and $N_i(0) = N_e(0) = 1$. Four values of $\Theta(0)$ are selected. The horizontal lines in the plots indicate the values of the respective ion sound velocities, calculated with equation (19) using $\kappa = 3$. It can be seen that for boundary electric fields up to roughly $\chi(0) \sim 2\times10^{-6}$ the boundary ion velocity $V_i(0)$ must slightly exceed the respective ion sound velocity V_S in order to obtain monotonic solutions. For larger $\chi(0)$ the velocity $V_i(0)$ can be slightly smaller than the respective V_S.

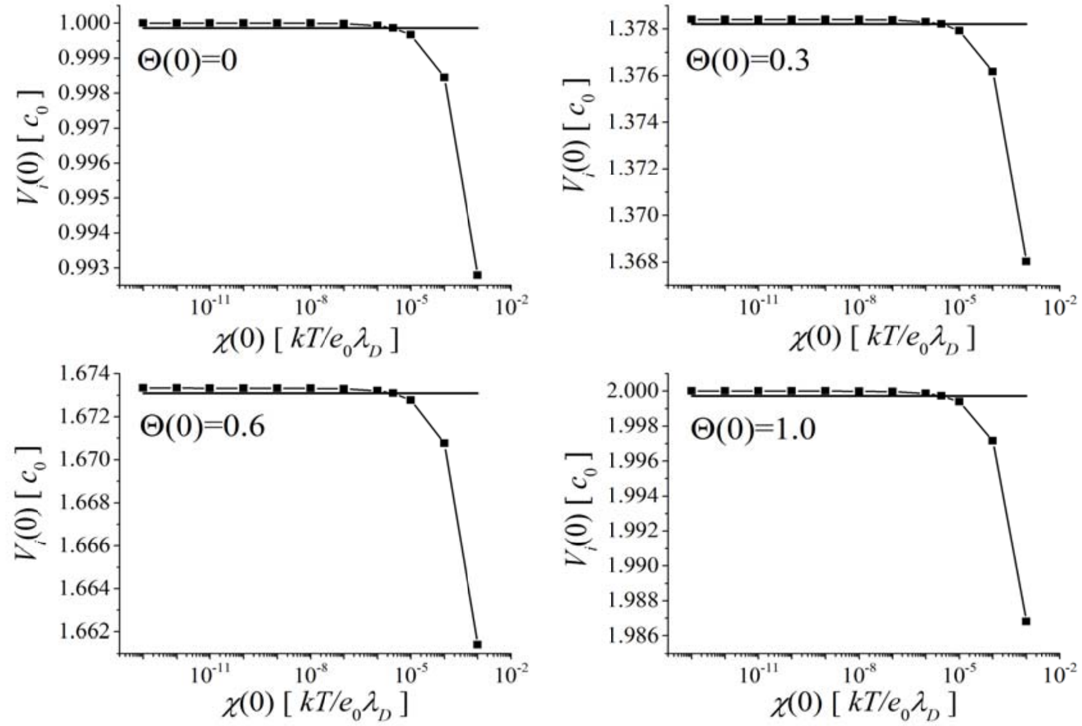

Figure 4. The minimum ion velocity $V_i(0)$ which results in a monotonic solution of the system (15) versus $\chi(0)$. The parameters and boundary conditions are: $Z = G_i = \Psi(0) = 0$, $\mu = 1/3670.482$, $V_e(0) = 10^{-7}$ and $N_i(0) = N_e(0) = 1$ and 4 values of $\Theta(0)$ are selected. The horizontal lines indicate the values of the respective V_S, calculated with equation (19) using $\kappa = 3$.

4. Conclusions

A one-dimensional two-fluid model has been presented and used for numerical analysis of the asymptotic two-scale limit [1,2]. Numerical results confirm that when the problem is treated on the pre-sheath scale, the sheath edge is determined by the electric field singularity. When the problem is approached on the sheath scale, electric field at the sheath edge must be larger than zero in order to obtain any solutions of the model equations. In this case the Bohm criterion is determined by two parameters: the electric field and the ion velocity at the sheath edge. If a very small value of the electric field at the sheath edge is selected, the ion velocity at the sheath edge must be increased

IOP Conf. Series: Journal of Physics: Conf. Series **982** (2018) 012005 doi:10.1088/1742-6596/982/1/012005

slightly above the corresponding ion sound velocity if monotonic solutions are to be obtained. For larger values of the electric field at the sheath edge the ion velocity at the sheath edge can be even slightly smaller than the corresponding ion sound velocity.

Acknowledgements
This work has been partially supported by the grant P2-0073 of the Slovenian Research Agency.

References

[1] Riemann K U 1991 *J.Phys. D: Appl. Phys.* **24** 493
[2] Riemann K U 2000 *J. Tech. Phys. Special Issue* **41** 89
[3] Freidberg J »Plasma Physics and Fusion energy«, Cambridge University Press, Cambridge, New York, (2007).
[4] Kuhn S, Riemann K U, Jelić N, Tskhakaya D D, Tskhakaya D Jr, Stanojević M 2006 *Phys.Plasmas* **13** 013503
[5] Gyergyek T, Kovačič J 2016 *Phys. Plasmas* **23** 063510