Effect of the Periacetabular Osteotomy on the Stress on the Human Hip Joint Articular Surface

Ales Iglič, Veronika Kralj Iglič, Vane Antolič, France Srakar, and Uroš Stanič

Abstract—Long-lasting over-elevated stress on the hip joint articular surface can damage the articular cartilage and is connected with arthrosis development. Periacetabular pelvic osteotomy is performed in order to prevent arthrosis development in different cases of residual hip dysplasia where the stress on the hip joint articular surface is increased because of inadequate femoral head coverage, i.e., too small hip joint weight bearing area. The aim of this work is to estimate a decrease of the stress on the hip joint articular surface after the periacetabular osteotomy. For this purpose, a three-dimensional mathematical model is constructed taking into account the spherical shape of the femoral head and of the acetabulum inner surface before and after the operation. On the basis of the presented results it is concluded that while performing the periacetabular osteotomy the proximal part of the acetabulum should be rotated over the femoral head in lateral direction with the simultaneous medial displacement of the whole joint. In this way postoperative stress on the hip joint articular surface is reduced to the greatest extent.

1. INTRODUCTION

THE importance of different factors for development of arthrosis in the hip joint (as for example humoral, constitutional, genetic, and mechanical factors) is only partially understood [1]–[3]. In this work the mechanical factors are considered. It is proposed that high magnitude of stress on the hip joint articular surface over decades leads to damage of the articular cartilage, and consequently to arthrosis development in the hip joint. This process is even more pronounced if the resistance of cartilage to the articular stress is reduced [1]–[3]. It is of great importance to reduce the over-elevated stress on the articular joint surface by means of an appropriate surgery [1], [3].

The periacetabular pelvic osteotomy [4], [5] is used for the treatment of residual hip dysplasia where the hip joint weight bearing area is small and therefore the stress on the hip joint articular surface is increased. The main purpose of the periacetabular pelvic osteotomy is to reduce the magnitude of stress on the hip joint articular surface by means of increasing the weight bearing area and thereby diminishing the development of the arthrosis.

When periacetabular osteotomy is performed (see Fig. 4) it is made all around the acetabulum in such a way that the major pelvic ring remains stable. As the acetabulum is released and is no more an integral part of the pelvis, it is rotated over the femoral head in the lateral direction in order to increase the weight bearing area (Fig. 4). In this way the lateral coverage of the femoral head is increased up to its normal value [6], [7]. The acetabulum is fixed with screws in the new position [4]. No plaster immobilization is needed after the operation, while use of crutches is recommended for three months. The described periacetabular osteotomy is possible only in the cases of residual hip dysplasia where the normal spherical shape of the femoral head and the inner surface of the acetabulum is retained. In periacetabular osteotomy no femoral head center (which coincides with the hip joint rotation center) shift is required in order to obtain a greater hip joint weight bearing area. However, sometimes due to high technical demands of the periacetabular osteotomy or also coincidentally, the hip joint rotation center may be displaced in different directions. Because the rotation center shift may considerably change the magnitude of the hip joint resultant force $R$ [5], [8] it influences also the stress on the articular joint surface. Therefore, it can be proposed that after periacetabular osteotomy the stress on the hip joint articular surface can be altered for two different reasons: due to change of the magnitude of the weight bearing area and also due to rotation center shift after the operation. The aim of this work is to investigate the influence of both these factors on the magnitude and distribution of the stress on the hip joint articular surface after the periacetabular osteotomy.

II. THEORY

In the model, the femoral head is represented by a sphere and the acetabulum is represented by a fraction of a spherical shell (of a somewhat larger radius than the femoral head), separated by a soft intermediate layer. If the hip is unloaded, the femoral head sphere and the acetabulum spherical shell are concentric, while if the hip is loaded, the femoral head sphere is displaced relative to the acetabulum spherical shell. The point of minimal distance between the sphere and the shell is called the pole. A hip joint articular surface sphere is constructed (Fig. 1) in which its radius is taken to be the mean of the radii of the femoral head sphere and the acetabulum shell. The position of the pole ($P$) on the articular surface is determined in spherical coordinates ($\theta, \phi$) by two characteristic angles $\theta = \Theta$ and $\phi = \Phi$ (Fig. 1).
Fig. 1. Schematic presentation of the hip joint articulating surface. The rectangular Cartesian coordinate system is oriented so that its $x$ and $z$ axes lie in the frontal plane of the body through the centers of both femoral heads. The weight bearing area of the articulating surface, i.e., the area where the stress is different from zero (marked by shading), is taken to be a portion of the spherical surface bounded by the lateral intersecting plane inclined for the angle $\varphi_L$ with respect to the $x = 0$ plane and the medial intersecting plane inclined for the angle $\varphi_M$ with respect to the $z = 0$ plane. Symbol $P$ denotes the pole of stress distribution determined in spherical coordinates by angles $\varphi$ and $\theta$. The resultant hip joint force $\mathbf{R}$ lies in the $y = 0$ plane. The angle $\varphi_R$ describes the inclination of the hip joint resultant force $\mathbf{R}$ with respect to the $x = 0$ plane.

Loading of the femoral head is described by means of the stress due to the hip joint resultant force $\mathbf{R}$. The force $\mathbf{R}$ is calculated here by using a static three-dimensional mathematical model of an adult human hip in one-legged stance [8], [9]. In this model, $\mathbf{R}$ is calculated by solving the equations of mechanical equilibrium for the pelvis and the loaded lower extremity. The piriformis, gluteus medius, gluteus minimus, rectus femoris, and tensor fasciae latae muscles are included in the model. Since gluteus medius and minimus are attached to the pelvis in large areas, each of these two muscles is divided into three segments. Thus nine effective muscles are included in the model. It is assumed that the force of an individual muscle acts along a straight line connecting the attachment point of the muscle origin on the pelvis and the corresponding attachment point of the muscle insertion on the femur. The change of hip configuration after periacetabular osteotomy is described by changing the hip joint rotation center position in antero-posterior, supero-inferior, and medio-lateral directions. This is simulated in the model by changing the distance between the two femoral heads centers and by changing the coordinates of all muscle attachment points included in the model [5], [8]. Regarding the effect of the operation on $\mathbf{R}$ it was calculated that the rotation center shift in the medio-lateral direction has a much larger effect on $\mathbf{R}$ than the shifts in antero-posterior and supero-inferior directions [5], [8]. Therefore, in this work we consider only the effect of the medio-lateral rotation center shift ($\Delta z$).

Fig. 2. (a) The original coordinate system is rotated in the $y = 0$ plane for $\varphi_R$ so that in (b) the rotated coordinate system the $x$-axis points in the direction of $-\mathbf{R}$.

The weight bearing area of the hip joint articulating surface, i.e., the area where the stress on the articulating surface is different from zero (marked by shading in Fig. 1), is taken to be a portion of the spherical surface bounded by the lines of intersection of the spherical surface with two planes (the lateral and the medial intersecting plane, respectively) (Fig. 1). The lateral intersecting plane is inclined for $\varphi_L$ in the lateral direction with respect to the $x = 0$ plane and the medial intersecting plane is inclined for $\varphi_M$ in the medial direction with respect to the $x = 0$ plane (Fig. 1 and Fig. 2a). The surface, obtained in such way, has mirror symmetry with respect to $y = 0$ plane. Due to the geometry chosen and due to the force $\mathbf{R}$ lying in the $xz$ plane, simple analytical functions can be used to describe the interdependence of the two spherical coordinates $\varphi$ and $\theta$ at the line of intersection of the intersecting planes and the spherical surface. Furthermore, in this case, the pole of the stress distribution lies in the $y = 0$ plane (Fig. 1).

For the sake of simplicity, the coordinate system is chosen so that its $x$-axis is pointing in the direction of $-\mathbf{R}$,

$$\mathbf{R} = (0, 0, -R)$$  \hspace{1cm} (1)

meaning that the system is rotated in the $y = 0$ plane for $\varphi_R$ (Fig. 2). Therefore, the most medial and the most lateral point of the weight bearing surface in the new system ($\varphi'_M$ and $\varphi'_L$, respectively) are rotated in the $y = 0$ plane for $\varphi_R$ with respect to the original coordinates (Fig. 2) so that

$$\varphi'_L = \varphi_L + \varphi_R$$  \hspace{1cm} (2)

$$\varphi'_M = \varphi_M - \varphi_R.$$  \hspace{1cm} (3)
Although shear stress in the hip joint due to friction is also present, it is taken here that it is negligibly small and so that only normal stress is considered [10, 11]. This is a reasonable assumption for smooth, well-lubricated acetabular and femoral head surfaces which are spherical and congruent [10]. It was shown [11] that for small displacements of the femoral head sphere relative to the acetabular spherical shell for a system subject to the above assumptions, the radial strain in the layer between the femoral head sphere and the acetabular spherical shell is proportional to the cosine between any point on the articular surface sphere and the pole of the stress distribution. As it is assumed that stress is proportional to strain in the intermediate layer, it follows that [11]

\[ p = p_0 \cos \gamma \]  

where \( p_0 \) is the value of \( p \) at the pole and \( \gamma \) is the angle between any point on the surface and the pole (Figs. 1 and 2). In our case (in the rotated coordinate system with \( R \) pointing in the negative \( z \) direction) (Fig. 2(b))

\[ \cos \gamma' = \sin \theta' \cos \phi' \sin \phi' \cos \theta' \sin \Phi' + \cos \theta' \cos \theta' \sin \Phi' \].  

The articular stresses integrated over the weight bearing area yields the resultant hip joint force

\[ \int p \, dA = R \]  

where

\[ dA = (\sin \theta' \cos \phi', \sin \theta' \sin \phi', \cos \theta') r^2 \sin \theta' \, d\theta' \, d\phi' \]  

and \( \pi \) is the radius of the sphere. The unknown quantities: the value of the stress at the pole \( p_0 \) and the coordinates of the pole \( \theta' \) and \( \Phi' \) are determined by solving the three integral equations for the components of the force \( R \). Considering (4), (6), and (7), the components of the force are

\[ \int p \, dA = 0 \]  

\[ \int p \, dA = 0 \]  

\[ \int p \, dA = R \]  

where \( \cos \gamma' \) is expressed by (5). The integration is performed over the weight bearing area. This area is distributed over four octants of the sphere in a coordinate system with the \( z \)-axis pointing in the direction of \( -R \) (Fig. 3). In a particular octant, it consists of a fourth of a spherical cap and a portion of the area of the sphere bounded by \( x = 0 \) and \( y = 0 \) planes and by the intersecting planes inclined for \( \theta' \) or \( \Phi' \) (Fig. 3). The interdependence of coordinates \( \theta \) and \( \phi' \) at the line of intersection between the intersecting planes and the spherical surface are given: in the first octant \( (0 < \phi' < \pi/2) \) by

\[ \phi' = \arccos \left( \frac{\cos \theta' \cos \Phi'}{\sqrt{\cos^2 \theta' - \cos^2 \Phi' - \cos^2 \Phi' \sin \theta'}} \right), \quad \theta > \theta' \]  

in the second octant \( (\pi/2 < \phi' < \pi) \) by

\[ \phi' = \pi - \arccos \left( \frac{\cos \theta' \cos \Phi'}{\sqrt{\cos^2 \theta' - \cos^2 \Phi' - \cos^2 \Phi' \sin \theta'}} \right), \quad \Phi > \Phi' \]  

in the third octant \( (\pi < \phi' < 3\pi/2) \) by

\[ \phi' = \pi - \arccos \left( \frac{\cos \theta' \cos \Phi'}{\cos \theta' \cos \Phi'} \right), \quad \theta > \theta' \]  

and in the fourth octant \( (3\pi/2 < \phi' < 2\pi) \) by (see Fig. 3)

\[ \phi' = 2\pi - \arccos \left( \frac{\cos \theta' \cos \Phi'}{\cos \theta' \cos \Phi'} \right), \quad \Phi > \Phi' \]  

While the lateral angle \( \phi' \) is determined from the hip geometry, the medial angle \( \phi' \) is determined at the point where the cosine function of the stress distribution reaches the value 0. In other words, \( \phi' \) is rotated in the clockwise direction from the pole of the stress distribution for \( \pi/2 \).

After some calculation, it follows from (8), which gives the first component of the resultant force \( R \), that

\[ \cos \theta' \cos \Phi' = \frac{1}{\cos \theta' \cos \Phi'} \]  

As the pole of the stress distribution lies in the \( y = 0 \) plane the second component of \( R \) (9) turns out to be proportional to \( \sin \Phi' \), which leads to the requirement that

\[ \sin \Phi' = 0 \]  

and therefore to two different solutions for \( \Phi' \) which determine the location of the pole relative to the \( x = 0 \) plane: \( \Phi' = 0 \) (medially relative to the \( x = 0 \) plane), and \( \Phi' = \pi \) (laterally relative to the \( x = 0 \) plane).

As \( \Phi' \) is shifted for \( \pi/2 \) clockwise with respect to the pole, it can be written

\[ \Phi' = \pi/2 + \Phi' \cos \Phi' \]
Fig. 4. The calculated distribution of stress on the human hip joint articular surface before (A) and after (B) the periacetabular osteotomy. The coverage of the femoral head (θ_L) is increased in the lateral direction. After the operation, the hip joint rotation center is not shifted. The calculated values of the coordinates of the pole (θ and Φ) and of the magnitude and direction of the resultant hip joint force (R and θ_R, respectively) are: (a) Θ = 31.3°, Φ = 180°, R = 1906 N, θ_R = 21°; and (b) Θ = 7°, Φ = 180°, R = 1906 N and θ_R = 9°. The value of p_max = 1.72 · 10^6 Pa coincides with the value of p_0. The values of model parameters used are: (a) θ_L = 18°, (b) θ_L = 33°, radius of the articular surface r = 2.5 cm and body weight W_R = 800 N.

Fig. 5. The calculated distribution of stress on the human hip joint articular surface after the periacetabular osteotomy for lateral (a) and medial (b) hip joint rotation center shift. The state before the operation is presented in Fig. 4(b). The rotation center shift Δx is positive when the femoral head is shifted medially and negative when it is shifted laterally. The calculated values of the coordinates of the pole (θ and Φ) and of the magnitude and direction of the resultant hip joint force (R and θ_R, respectively) are: (a) Θ = 0.3°, Φ = 0, R = 2331 N, θ_R = 12.7°; and (b) Θ = 15.1°, Φ = 180°, R = 1192 N, θ_R = 4.8°. The value of p_max = 2.01 · 10^6 Pa coincides with the value of p_0. The values of model parameters used in calculations are: (a) Δx = −1.5 cm, (b) Δx = 1.9 cm, θ_L = 38°, radius of the articular surface r = 2.5 cm and body weight W_R = 800 N.

In order to obtain the coordinates of the pole, (17) is inserted into (15) yielding a nonlinear equation for Φ' at a given choice of Φ'. The nonlinear equation is solved numerically and the choice of Φ' is made such that the value of the solution Φ' is positive, as required by definition of the spherical coordinates.

The value of the stress at the pole p_0 is then obtained from

\[ p_0 = 3R/(2Ir^2) \]  

where

\[ I = \sin \Theta \cos \Phi' (\cos^2 \theta_L' - \cos^2 \theta_M') + \cos \Theta (\sin (\theta_M' + \theta_L') \cos (\theta_M' - \theta_L') + (\theta_M' + \theta_L')). \]

We consider maximal value of the stress on the hip joint articular surface p_max as a measure of stress. The value of p_max is taken to be p_0 if the pole is located inside the surface bounded by the intersecting planes, while if the pole is located outside this surface, the maximal value of the stress distribution p_max is taken at the acetabular rim.

III. RESULTS

Presentation of the pelvic configuration before and after periacetabular osteotomy for different simulated postoperative situations and corresponding calculated distributions of the stress on the hip joint articular surface are shown in Figs. 4 and 5. Fig. 4 shows how the rotation of the acetabulum around the center of femoral head in the lateral direction (an increase of the lateral coverage of the femoral head) renders the stress distribution much more favorable. Note that in this case there is no hip joint rotation center shift according to the state before the osteotomy (Δx = 0). Fig. 5 shows the effect of the rotation center shift after periacetabular osteotomy in the lateral (a) and medial (b) directions, respectively. It can be seen from Figs. 4 and 5 that in addition to the rotation of the acetabulum over the femoral head, the hip joint rotation center shift also has considerable influence on the postoperative stress distribution on the hip joint articular surface. This should be taken into account while performing periacetabular osteotomy. The continuous dependence of the maximal value of stress on the hip joint articular surface (p_max) on the acetabulum coverage in the lateral direction (θ_L) as well as the continuous dependence of p_max on the hip joint rotation center shift in medio-lateral direction (Δx) are presented in Fig. 6. The normal values of θ_L for adults are approximately in the range from 25° to 45° [6], [7]. The values of Δx after the periacetabular osteotomy vary approximately in the range from −1.5 to 1.5 cm [4], [5] and it seems that this represents no problem for the stability and postoperative recovery of the operated hip.
IV. CONCLUSIONS

In this paper the effect of periacetabular osteotomy on the distribution of normal compressive stress on the hip joint articular surface is considered. For this purpose the mathematical model was constructed. Our results show that rotation of the proximal part of the acetabulum over the femoral head in the lateral direction after the periacetabular osteotomy substantially decreases stress on the hip joint articular surface. In addition, it was also shown that the hip joint rotation center shift in the medial direction as a consequence of periacetabular osteotomy also considerably decreases stress on the hip joint articular surface. On the contrary, lateral displacement of the hip joint rotation center increases stress on the joint articular surface. By taking into account that too high stress on the joint articular surface can encourage osteoarthrotic process in the hip joint, it can be concluded that while performing the periacetabular osteotomy, acetabulum should be rotated over the femoral head and hip joint rotation center should be shifted medially as far as it is technically possible and anatomically reasonable. Medialization of the hip joint rotation center is also favourable because in this way the hip abductor muscles' strength is increased due to translocation of hip muscle attachment after the operation [12]. On the other hand, the lateral shift of the hip joint rotation center decreases the strength of the patient's hip abductor muscles strength which is unfavorable [12].

As in some other triple pelvic osteotomies the acetabulum is also rotated over the femoral head [13]. It can be expected that the presented conclusions are valid also for these osteotomies.

REFERENCES