

Appendix B

Equilibrium lateral distribution of membrane constituents

The equilibrium lateral distribution of the membrane constituents in a particular layer for a given membrane shape in the sequence, is obtained by minimization of the free energy of the respective layer (Eq.(1)):

$$\frac{F_j}{kT} = \int_A \left\{ \sum_{i=1}^2 \left(m_{i,j} \frac{f_{i,j}}{kT} + m_{i,j} \ln m_{i,j} \right) + \frac{wz_2}{2} m_{2,j}^2 \right\} m_0 \, dA, \quad j = \text{in, out} \quad (\text{B.1})$$

The average relative densities

$$\bar{m}_{i,j} = \frac{1}{A} \int_A m_{i,j} \, dA, \quad j = \text{in, out} \quad (\text{B.2})$$

for each of the species are allowed to be different in the two membrane layers.

We construct a Lagrangian for each of the monolayers

$$L_j = \sum_{i=1}^2 \left(m_{i,j} \frac{f_{i,j}}{kT} + m_{i,j} \ln m_{i,j} + \lambda_{i,j} (m_{i,j} - \bar{m}_{i,j}) \right) + \frac{wz_2}{2} m_{2,j}^2, j = \text{in, out}, \quad (\text{B.3})$$

where $\lambda_{i,j}$ ($j = \text{in, out}$) are the Lagrange multipliers. Inserting into the Lagrangian the relations $m_{1,j} = 1 - m_{2,j}$ and $\bar{m}_{1,j} = 1 - \bar{m}_{2,j}$ ($j = \text{in, out}$), we can eliminate one Lagrange multiplier in each layer by defining $\lambda_j = \lambda_{2,j} - \lambda_{1,j}$, ($j = \text{in, out}$). Using the Euler-Lagrange equations $\partial L_j / \partial m_{2,j} = 0$ ($j = \text{in, out}$) and Eq.(B3), we can derive the relative area densities $m_{2,j}$ in both monolayers ($j = \text{in, out}$). For small direct interactions ($|w| < 1$), the relative area densities $m_{2,j}$ ($j = \text{in, out}$) can be expressed analytically (up to the first relevant term in w):

$$m_{2,j} = m_{2,j}^{(0)} \left(1 - wz_2 m_{2,j}^{(0)} (1 - m_{2,j}^{(0)}) \right), \quad j = \text{in, out}, \quad (\text{B.4})$$

where $m_{2,j}^{(0)}$ is defined as:

$$m_{2,j}^{(0)} = \frac{\bar{m}_{2,j}}{\bar{m}_{2,j} + \bar{m}_{1,j} e^{\lambda_j - (f_1 - f_2)/kT}} \quad (\text{B.5})$$

and λ_j is calculated from the condition that the average of $m_{2,j}$ over the whole membrane is equal to $\bar{m}_{2,j}$.

The orientational effects are contained in the single-constituent energies $f_{i,j}$.